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THE INTEGRAL SCALE IN ISOTROPIC TURBULENCE

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1 Introduction

There have been numerous attempts over the past four decades to determine the integral scales of isotropic decaying turbulence, both from experiment and more recently using DNS. Of particular interest has been how the integral scale varies with time during decay. Most use a power law variation, $L \propto t^m$ with values of m ranging from 2/7 to 1/2 [3]. The latter value is of some special interest since it was originally derived by Dryden from an extension of the von Karman/Howarth [5] similarity hypothesis, and subsequently from a more general equilibrium similarity theory by George [4]. In fact, both of these approaches conclude that the integral scale and Taylor microscale, λ, remain in constant ratio throughout decay. George [4] showed that while the experimental lateral integral scales obtained by Comte-Bellot and Corrsin [3] appeared to confirm this, the longitudinal integrals scales were problematical. The ratio of L/λ is definitely not constant for recent DNS data [8].

2 The problem and solution

The problem is that both the energy and integral scale are determined in part by the largest scales of the turbulence which are either not simulated at all by DNS or experiments, or can not be estimated because of an insufficient statistical sample. Figure 1 and 2 show plots of the energy spectra, $E(k,t)$ and $E(k,t)/k$ of the recent 512^3 DNS by de Bruyn Kops and Riley [2]. The integrals under these are proportional to the energy and integral scale respectively. Clearly the missing spectral estimates at low wavenumber can significantly affect the integrals estimated from them.

A simple spectral model is used to examine what is required to determine the energy and integral scale in homogeneous, isotropic turbulence (see Wang and George [9] for details). The model is a modified von Karman type spectrum given by:

$$E(k) = u^2 L \frac{C_p (kL)^p}{[1 + (k/k_e)^2]^{p/2+5/6}} \tag{1}$$

where $1 \leq p \leq 4$. The values of C_p and k_e depend on p, and are determined by requiring E and E/k to integrate to the energy and integral scale respectively; i.e.,

$$\frac{3}{2}u^2 = \int_0^\infty E(k)dk \tag{2}$$

$$L = \frac{\pi}{2u^2} \int_0^\infty \frac{E(k)}{k} dk \tag{3}$$

3 The results

Figures 3 and 4 are produced from the spectral model for integer values of p, and show the ratios of the integrals estimated from above the lowest wavenumber, k_L, to the complete integral for the energy and integral scale respectively. It is clear that the absence of scales more than an order of magnitude below the peak in the energy spectrum, k_p, significantly affect the determination of the integral scale, and the error depends significantly on p. The energy is similarly affected but much less so. Since the spectral energy peak shifts to lower wavenumbers as the flow evolves, the problem becomes progressively worse during decay. This progressive deterioration can significantly affect the estimates of decay exponents of power law fits and make them appear to be time dependent, even if they are not. The effect is to overestimate the energy decay rate and to underestimate the growth of the integral scale.

By an iterative process [9], it is possible to approximately correct the DNS data using the spectral model. Considerable effort was applied using the theory of homogeneous turbulence [1] to ensure that the correction did not imply the result. Of particular importance is the behaviour of the time derivative of the Taylor microscale squared (shown in Figure 5), which was nearly constant for the corrected data (unlike the uncorrected data), implying a power law decay with constant exponent. Figure 6 shows the corrected and uncorrected integral scale for the de Bruyn Kops/Riley DNS data. The results are consistent with $L \propto t^{1/2}$ to within a few percent, and the ratio of integral scale to Taylor microscale is constant to within less than 0.4 % for the entire simulation.

4 Conclusions

The results imply (at least for this simulation and the Comte-Bellot/Corrsin experiments as well) that for the decay of turbulence in time for fixed initial conditions, $L/\lambda = \textit{constant}$, exactly as argued by George [4] from equilibrium similarity considerations. Moreover these results imply that the Kolmogorov relation is not satisfied; i.e., $L\epsilon/u^3 \neq \mathit{constant}$ (c.f., Batchelor [1] or Sreenivasan [6]). This is contrary to popular belief, but not surprising theoretically, since no experiments or simulations satisfy the high Reynolds number conditions for Kolmogorov's equilibrium range hypothesis from which it derives [1]. Figure 7 shows the corrected data for L/λ, along with the constant value and the variation that would be followed were Kolmogorov correct (i.e., $L/\lambda \propto R_\lambda^{-1}$). Similarly, Figure 8 shows the corrected data for Lu^3/ϵ, the constant value were Kolmogorov correct, and the variation predicted by George ($Lu^3/\epsilon \propto R_\lambda$ for fixed initial conditions).

The differences between the competing theories are small, but so is the variation of R_λ, even for this extensive data set. Even so, the results are clearly more consistent with equilibrium similarity than with the traditional view. This could have profound implications for our view of turbulence, since equilibrium similarity implies the energy decay is entirely determined by the initial conditions as preserved in p and the constant ratio L/λ.

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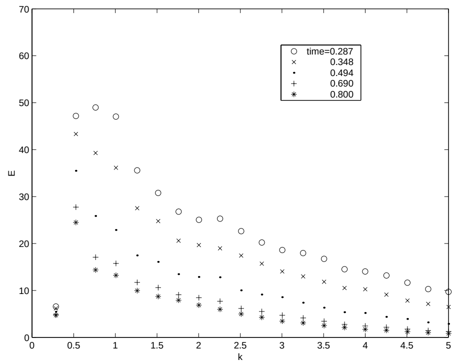


Figure 1

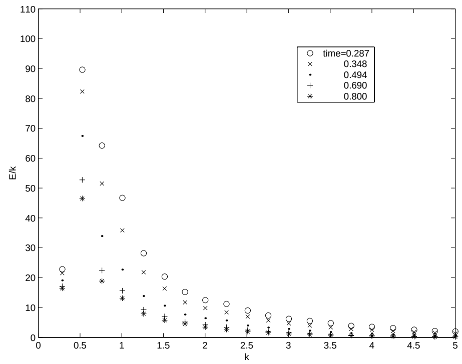


Figure 2

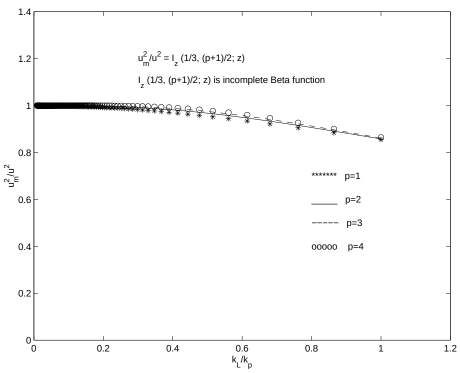
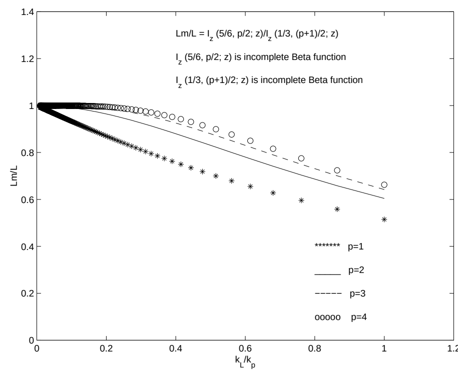


Figure 3



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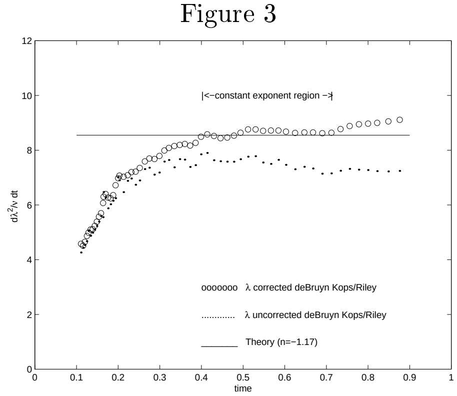

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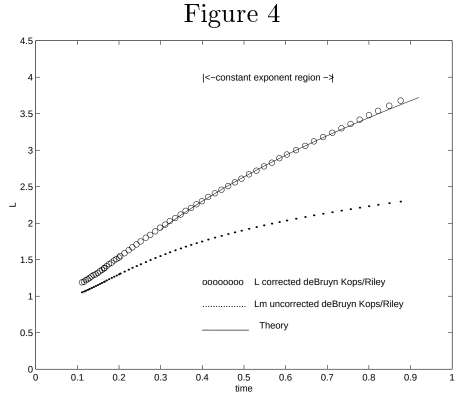

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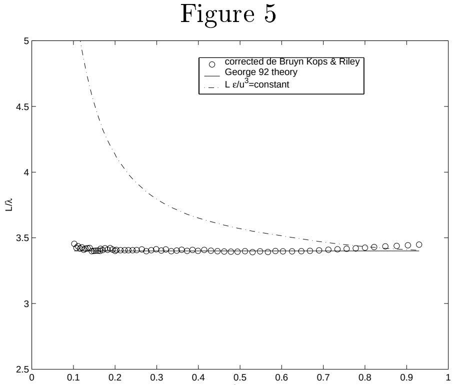

Figure 7

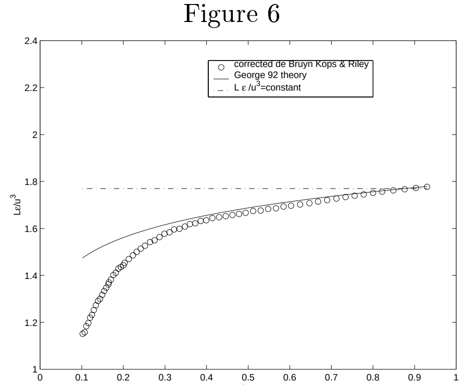

Figure 8