Further Investigation of
the Jet Mixing Layer Using
a 138 Hot-wire probe and the POD

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Background

- previous work by Citriniti & George (2000) using 138 hot-wire probes and the POD suggested the structure of the shear layer at $x/D = 3.0$

![Diagram of shear layer with vortices]

Objective of this investigation

- to quantify how the energy modes change with $Re_D$
- to determine when or if ever the asymptotic trends become Reynolds number independent
- to investigate how the structures in the flow change with downstream position
What is the POD?

(a) the original velocity

(b) decomposition

(c) reconstruction
The Proper Orthogonal Decomposition

- Lumley (1967) define POD as a method finding functions to represent the velocity vector, $u_i(\vec{x}, t)$, in an optimal way from an energy sense

- functions are determined from the eigenvalue equation

$$\int_D R_{ij}(\vec{x}, t, \vec{x}', t') \phi_i(\vec{x}', t') d(\vec{x}', t') = \lambda \phi_j(\vec{x}, t)$$

- reconstruct the velocity from only most energetic modes

$$u^N_i(\vec{x}, t) = \sum_{n=1}^{N} a_n \phi^{(n)}_i(\vec{x}, t)$$

$$a_n = \int u_i(\vec{x}, t) \phi^{(n)*}_i(\vec{x}, t) d(\vec{x}, t)$$

- The turbulent kinetic energy:

$$E = \sum_{n=1}^{\infty} \lambda^{(n)}$$
Jet facility

- used by Glauser (1987), Citriniti (1996)
- nozzle diameter, $D : 0.098 \, m$
  - contraction ratio : 10 : 1

<table>
<thead>
<tr>
<th>$U, , m/s$</th>
<th>12</th>
<th>18</th>
<th>24</th>
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<tbody>
<tr>
<td>$Re_D$</td>
<td>78400</td>
<td>117600</td>
<td>156800</td>
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Potential core
Axisymmetric mixing layer
138 hot-wire probe array

- simultaneous sampling at all 138 positions
- to resolve large scales, remove the smaller ones to avoid the spatial aliasing problems (George & Taulbee 1992)
- 138 hot-wires: long sensing element
Eigenvalue contribution from all the POD modes

- To decide which POD mode is dominant in the turbulent kinetic energy.

\[ E^{(n)} = \sum_{m} \int_{-\infty}^{\infty} \hat{\lambda}^{(n)}(m, f) \, df \]

- Clearly the first POD mode is dominant.
Energy distribution of the first POD mode, $\hat{\lambda}^{(1)}(m, f)$

$Re_D = 78400$  \hspace{1cm}  $Re_D = 117600$  \hspace{1cm}  $Re_D = 156800$
Normalized eigenvalues as a function of $m$

- The normalized eigenvalue, $\xi^{(1)}(m)$; i.e.,

$$\xi^{(1)}(m) = \frac{\int f \hat{\lambda}^{(1)}(m, f) \, df}{\sum_n \sum_m \int f \hat{\lambda}^{(n)}(m, f) \, df}$$
Variation in eigenvalues with Reynolds number

![Graph showing variation in eigenvalues with Reynolds number.](image)
Change in energy in azimuthal mode 0 with $x/D$

![Graph showing the change in energy in azimuthal mode 0 with $x/D$. The graph includes data points for different Reynolds numbers ($\text{Re}_D = 78400$, $\text{Re}_D = 117600$, $\text{Re}_D = 156800$) and shows the trend $0.5 / (x/D)$ for comparison. The graph is labeled with $n=1$, $n=2$, and $n=3$.](image)
Change in energy in higher azimuthal mode with $x/D$

- scaled by the shear layer variable, $x/D$

$Re_D = 78400$

$\eta^{(0)}/(x/D)$

$m \cdot x/D$

$n=1\times$
$n=2\times$
$n=3\times$

$\times x/D = 2.0$
$\times x/D = 2.5$
$\circ x/D = 3.0$
$\square x/D = 3.5$
$\diamond x/D = 4.0$
$\triangle x/D = 4.5$
$\bigtriangleup x/D = 5.0$
$\bigtriangledown x/D = 5.5$
$\blacktriangle x/D = 6.0$
Normalized eigenvalues for all Reynolds number

- azimuthal mode > 0

- scaled by the shear layer variable, $x/D$
Velocity reconstruction

- As a linear combination of the eigenfunctions and coefficient:

\[
r^\frac{1}{2} \hat{u}_1^N(r, m, f) = \sum_{n=1}^{N} \hat{a}_n(m, f) \hat{\phi}_1^{(n)}(r, m, f)
\]

\[
\hat{a}_n(m, f) = \int r^\frac{1}{2} \hat{u}_1(r, m, f) \hat{\phi}_1^{(n)*}(r, m, f) \, dr
\]

with inverse Fourier transform:

\[
\hat{u}_1^N(r, m, t) = \int e^{i2\pi ft} \hat{\hat{u}}_1^N(r, m, f) \, df
\]

\[
u_1^N(r, \theta, t) = \sum_{m=0}^{M} e^{-im\theta} \hat{u}_1^N(r, m, t)
\]

- Reconstructed velocity can be partial sum of POD mode, n, and azimuthal mode, m.
Velocity reconstruction at a point with POD mode
Velocity reconstruction at a field

- The original velocity

- First POD mode, all mode

- First POD mode, mode=0, 3-7
Animation of the reconstructed velocity

- $x/D = 2.0$
- $x/D = 4.0$
- $x/D = 6.0$
Summary, discussion, and conclusions

Similarity of the energy distribution

- the energy distribution of the first POD mode has **no dependence** on $Re_D$ over the range of $78400 \leq Re_D \leq 156800$.
- Mode-0 behaves entirely different than the higher modes.
- The eigenspectra collapse when scaled in shear layer similarity variables.  $\Rightarrow$ **structures are same as flow traverse downstream but different size**

The velocity reconstruction

- Azimuthally coherent vortex rings dominate the dynamics and the interactions of the structures until about $x/D \approx 4$.
- Beyond $x/D \approx 4$, the volcano-like eruptions die off rapidly.
- For $x/D \geq 4.0$, a “propeller-like” structure appears and dominates the pattern.
Bibliography


