Further Investigation of the Jet Mixing Layer Using a 138 Hot-wire probe and the POD

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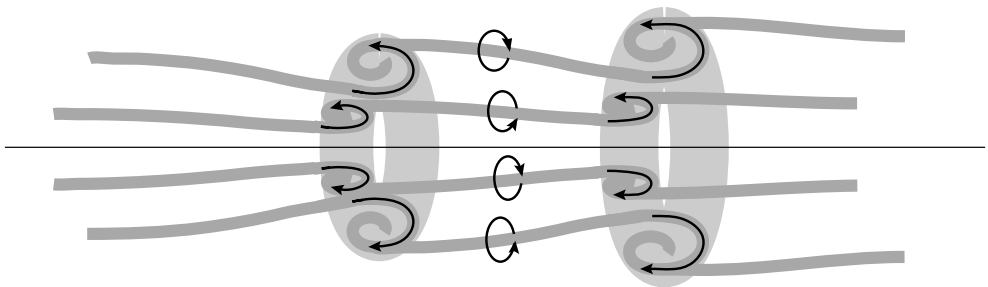
Background

- previous work by Citriniti & George (2000) using 138 hot-wire probes and the POD suggested the structure of the shear layer at $x/D = 3.0$

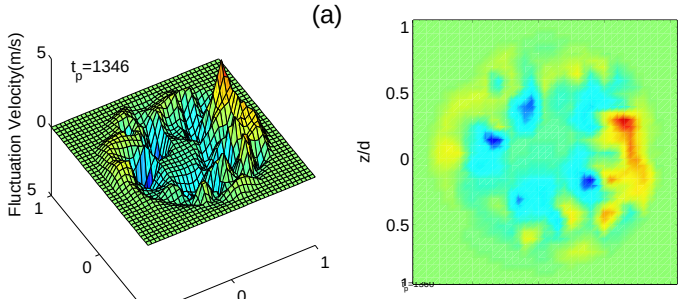


Objective of this investigation

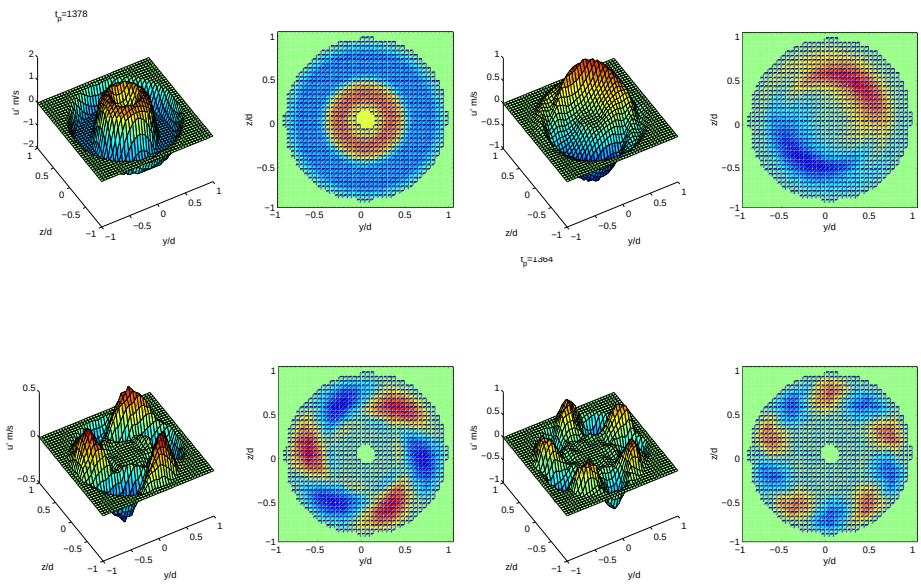
- to quantify how the energy modes change with Re_D
- to determine when or if ever the asymptotic trends become Reynolds number independent
- to investigate how the structures in the flow change with downstream position

What is the POD?

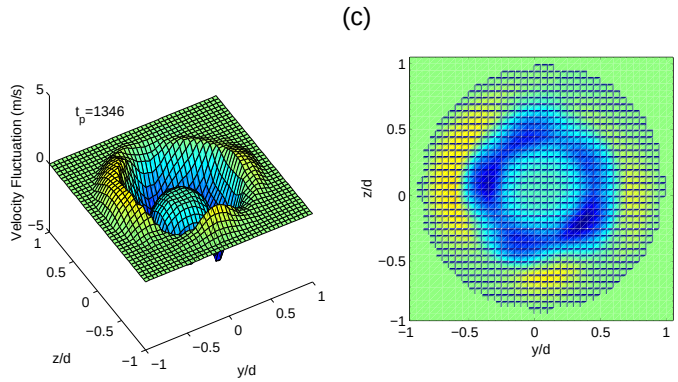
the original velocity



decomposition



reconstruction



The Proper Orthogonal Decomposition

- Lumley (1967) define POD as a method finding functions to represent the velocity vector, $u_i(\vec{x}, t)$, in an optimal way from an energy sense
- functions are determined from the eigenvalue equation

$$\int_D R_{ij}(\vec{x}, t, \vec{x}', t')\, \phi_i(\vec{x}', t')\, d(\vec{x}', t') = \lambda \phi_j(\vec{x}, t)$$

- reconstruct the velocity from only most energetic modes

$$u_i^N(\vec{x}, t) = \sum_{n=1}^{N} a_n\, \phi_i^{(n)}(\vec{x}, t)$$

$$a_n = \int u_i(\vec{x}, t)\, \phi_i^{(n)*}(\vec{x}, t)\, d(\vec{x}, t)$$

- The turbulent kinetic energy:

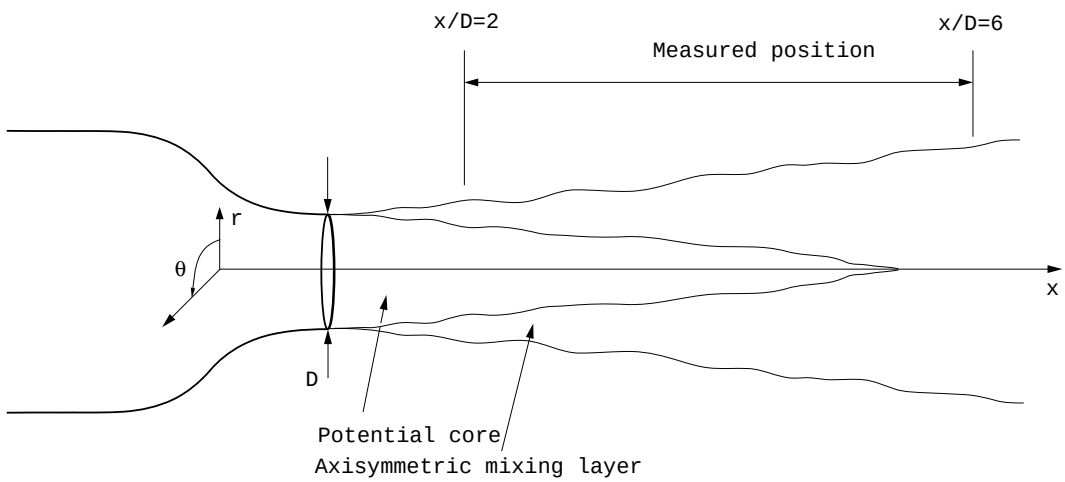
$$E = \sum_{n=1}^{\infty} \lambda^{(n)}$$

Jet facility

- used by Glauser (1987), Citriniti (1996)
- nozzle diameter, D : $0.098\,m$

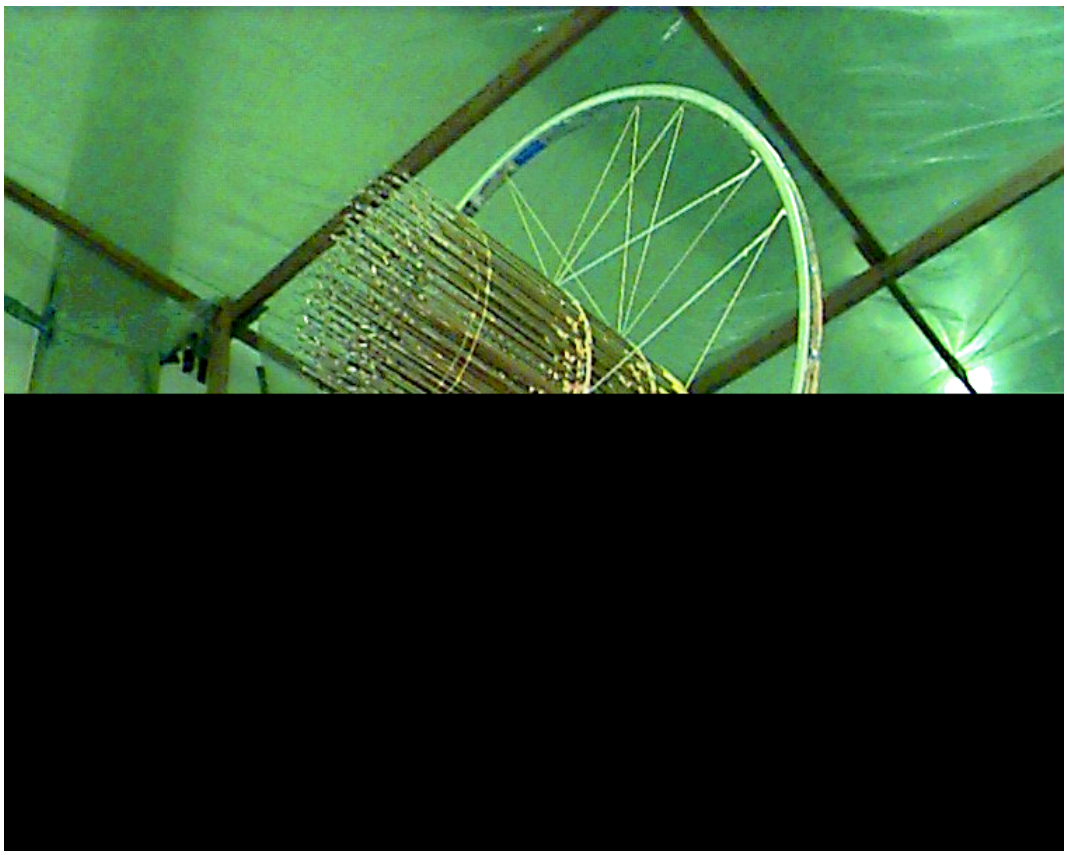
  contraction ratio : 10 : 1

U, m/s	12	18	24
Re_D	78400	117600	156800



138 hot-wire probe array

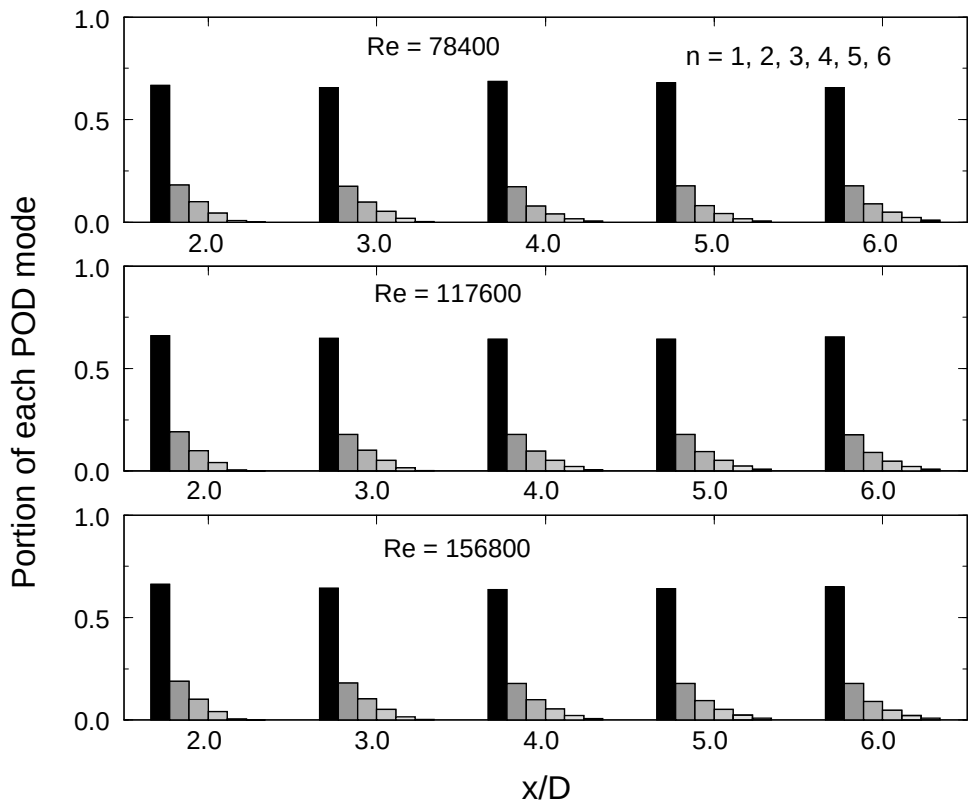
- simultaneous sampling at all 138 positions
- to resolve large scales, remove the smaller ones to avoid the spatial aliasing problems (George & Taulbee 1992)
- 138 hot-wires: long sensing element



Eigenvalue contribution from all the POD modes

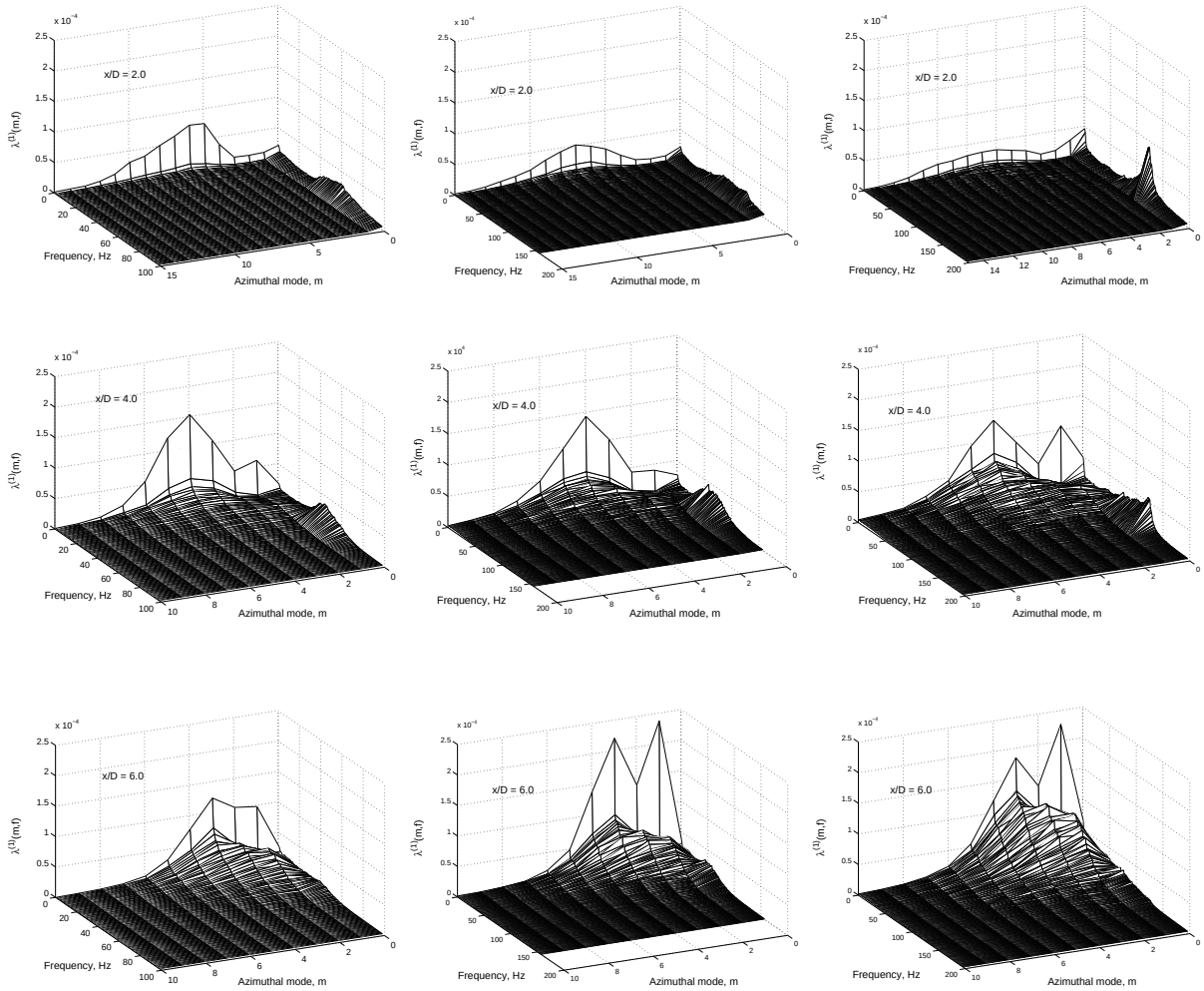
- To decide which POD mode is dominant in the turbulent kinetic energy.

$$E^{(n)} = \sum_m \int_{-\infty}^{\infty} \hat{\hat{\lambda}}^{(n)}(m, f)\, df$$



- Clearly the first POD mode is dominant.

Energy distribution of the first POD mode, $\hat{\hat{\lambda}}^{(1)}(m, f)$

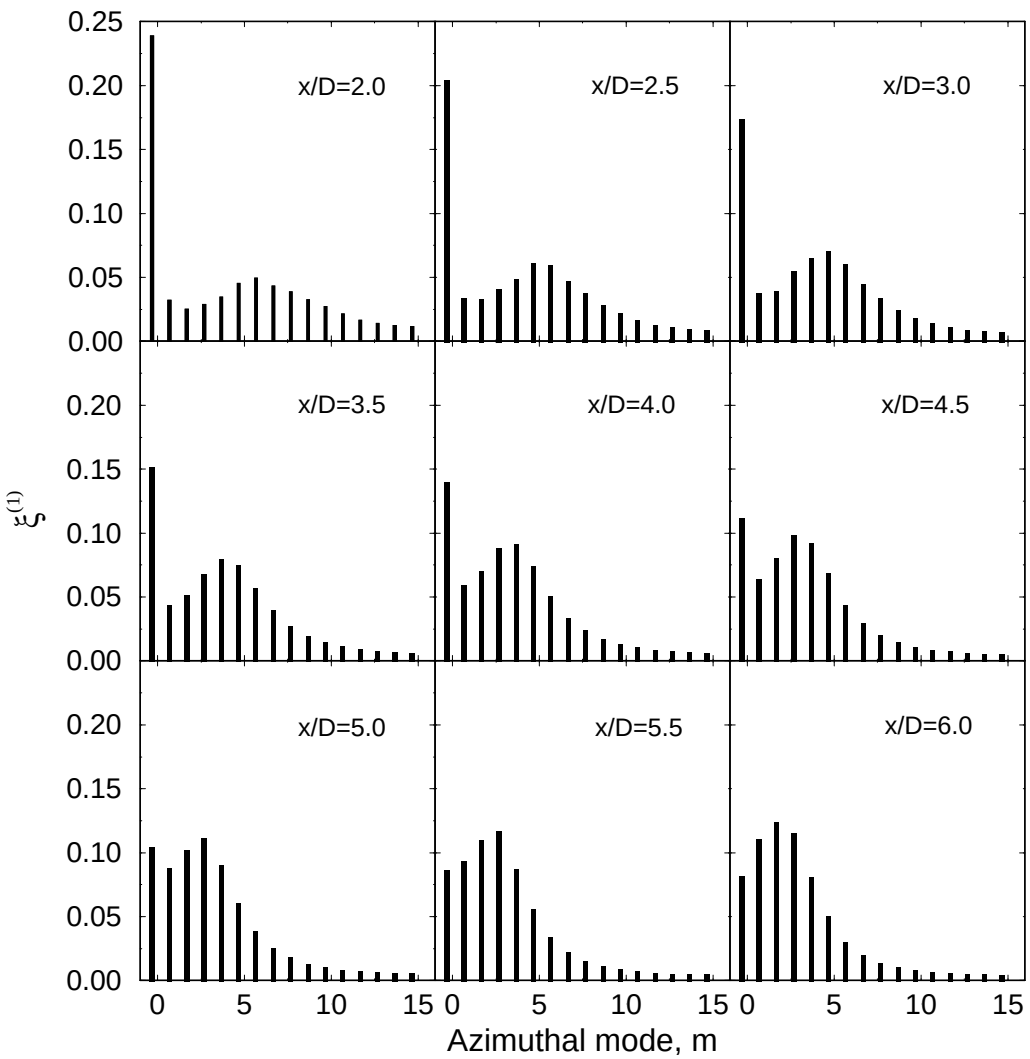


$Re_D = 78400$ $Re_D = 117600$ $Re_D = 156800$

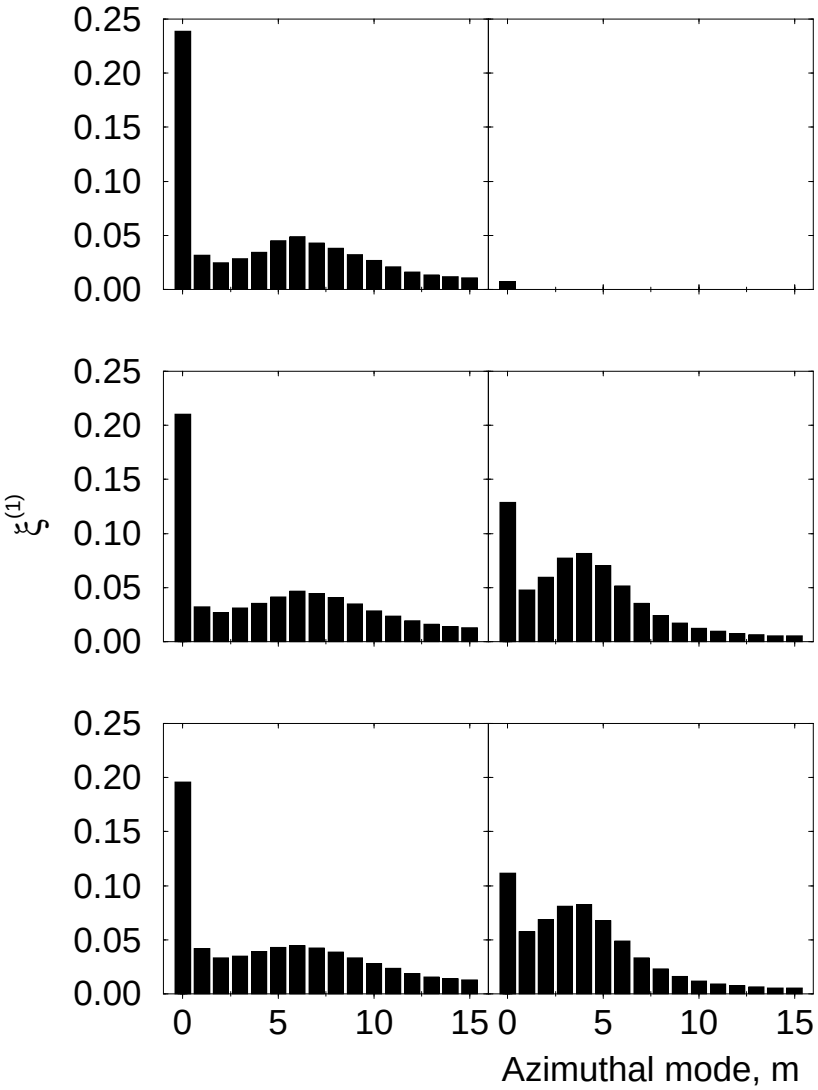
Normalized eigenvalues as a function of m

- The normalized eigenvalue, $\xi^{(1)}(m)$; i.e.,

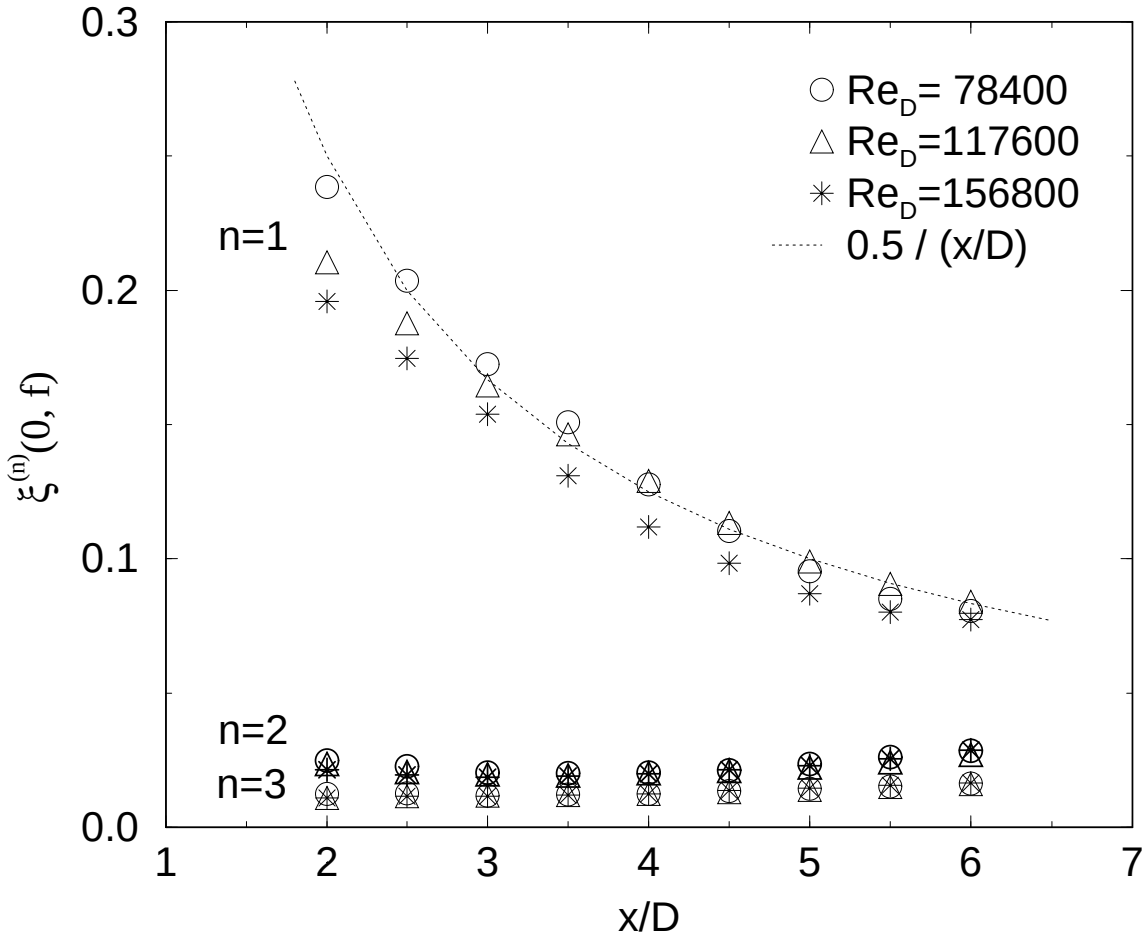
$$\xi^{(1)}(m) = \frac{\int_f \hat{\hat{\lambda}}^{(1)}(m,f)\,df}{\sum_n \sum_m \int_f \hat{\hat{\lambda}}^{(n)}(m,f)\,df}$$



Variation in eigenvalues with Reynolds number

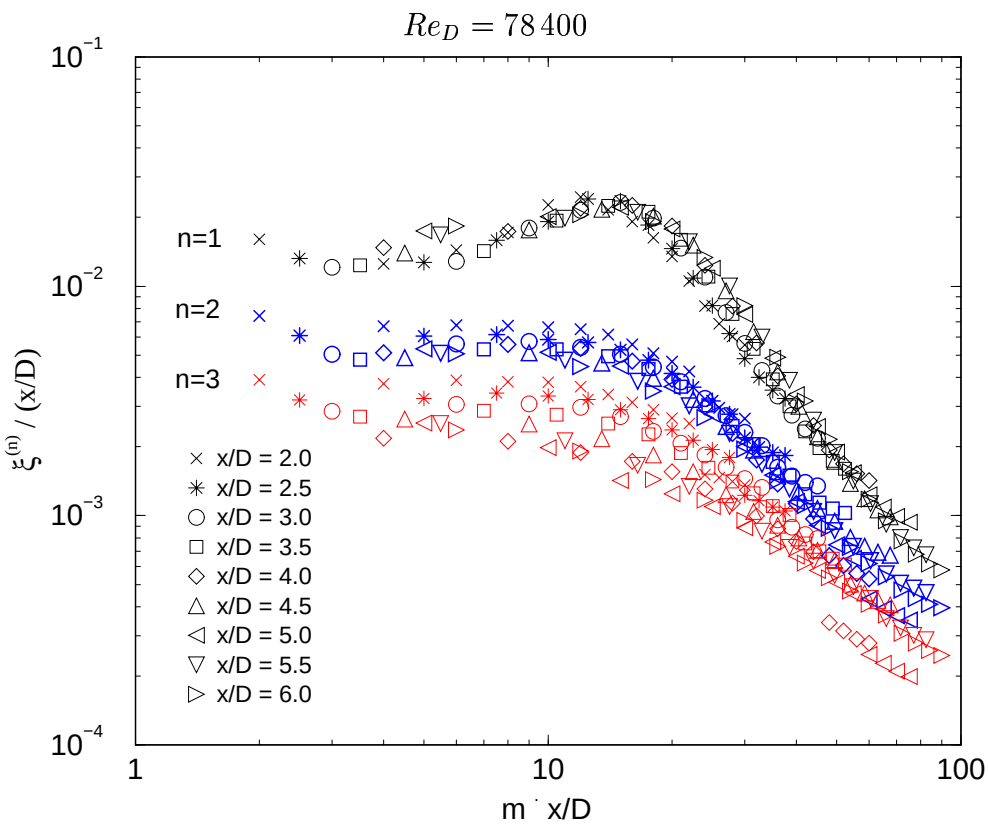


Change in energy in azimuthal mode 0 with x/D



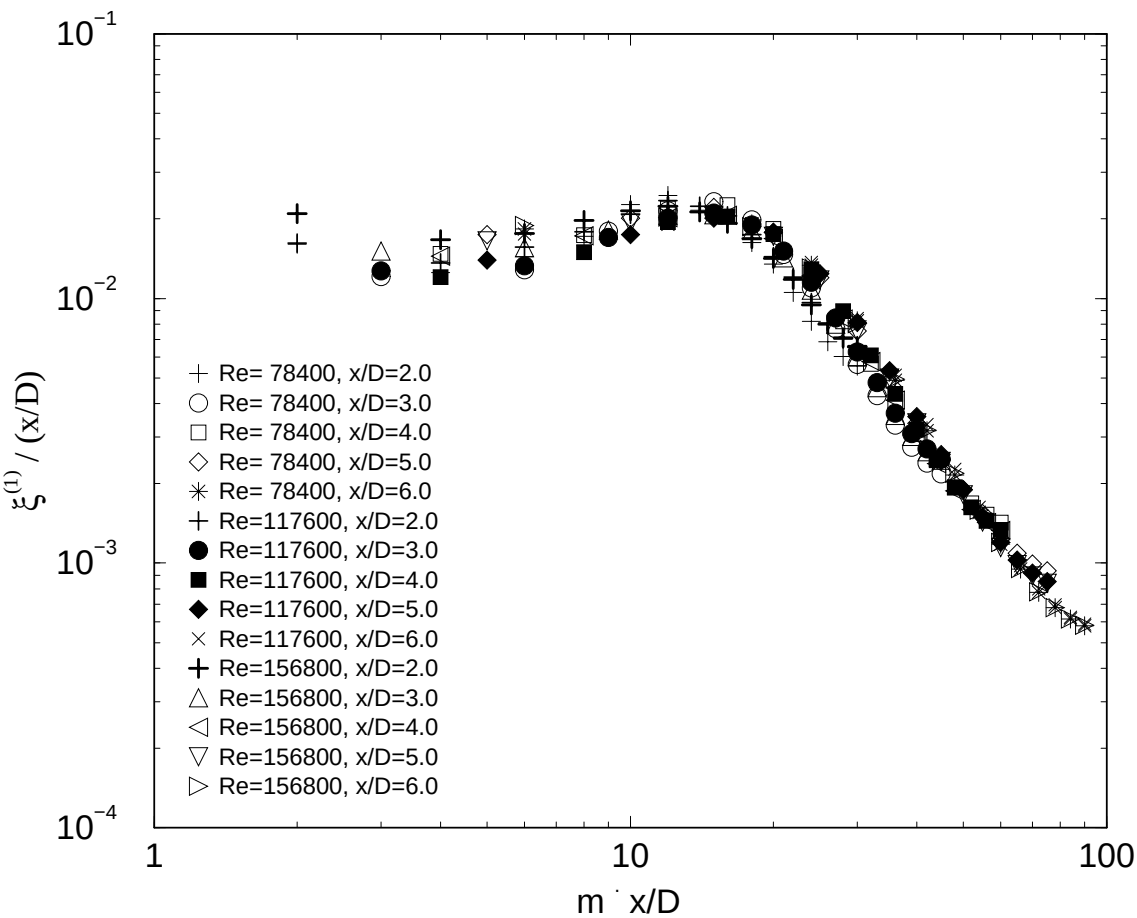
Change in energy in higher azimuthal mode with x/D

- scaled by the shear layer variable, x/D



Normalized eigenvalues for all Reynolds number

- azimuthal mode > 0
- scaled by the shear layer variable, x/D



Velocity reconstruction

- As a linear combination of the eigenfunctions and coefficient:

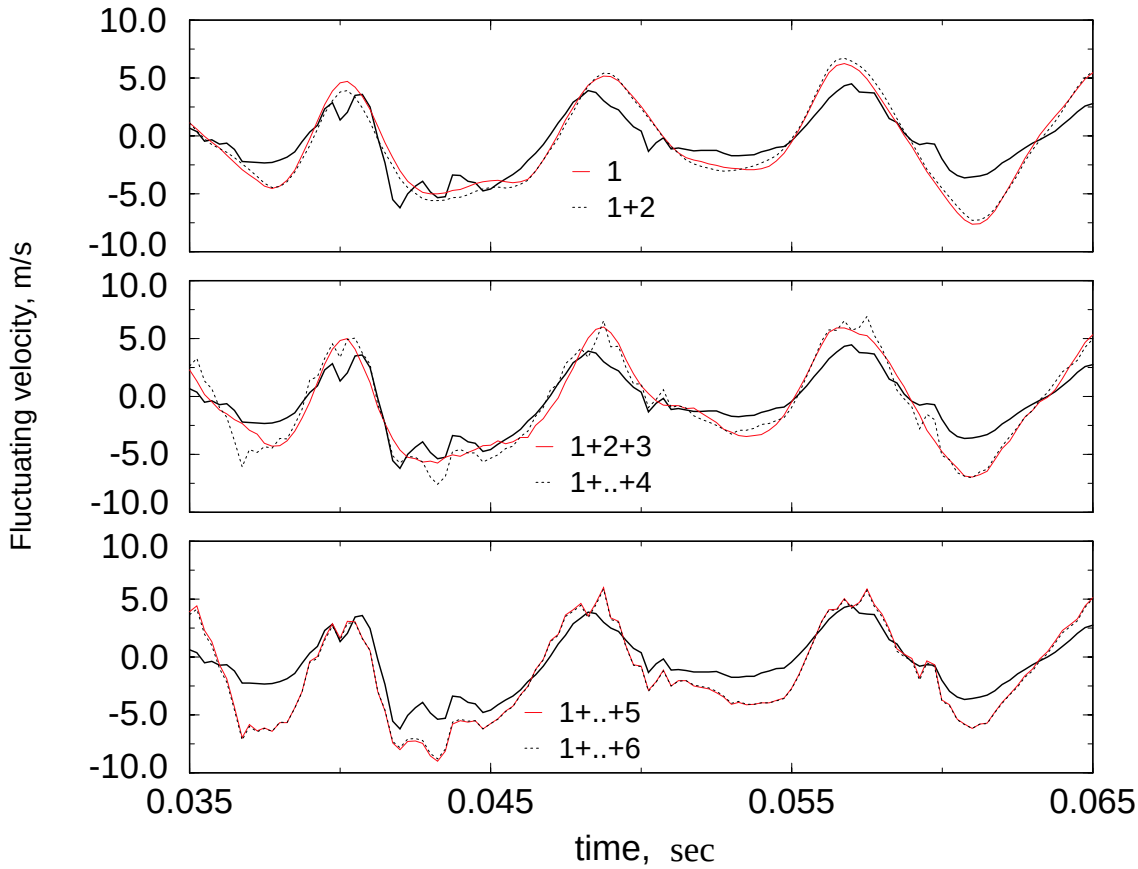
$$
\begin{aligned}
r^{\frac{1}{2}}\hat{\hat{u}}_1^N(r,m,f) &= \sum_{n=1}^{N} \hat{\hat{a}}_n(m,f)\,\hat{\hat{\phi}}_1^{(n)}(r,m,f) \\
\hat{\hat{a}}_n(m,f) &= \int r^{\frac{1}{2}}\hat{\hat{u}}_1(r,m,f)\,\hat{\hat{\phi}}_1^{(n)*}(r,m,f)\,dr
\end{aligned}
$$

  with inverse Fourier transform:

$$
\begin{aligned}
\hat{u}_1^N(r,m,t) &= \int e^{i2\pi ft}\,\hat{\hat{u}}_1^N(r,m,f)\,df \\
u_1^N(r,\theta,t) &= \sum_{m=0}^{M} e^{-im\theta}\,\hat{u}_1^N(r,m,t)
\end{aligned}
$$

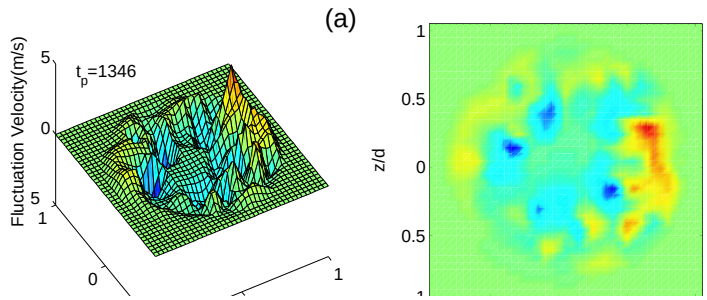
- Reconstructed velocity can be partial sum of POD mode, n, and azimuthal mode, m.

Velocity reconstruction at a point with POD mode

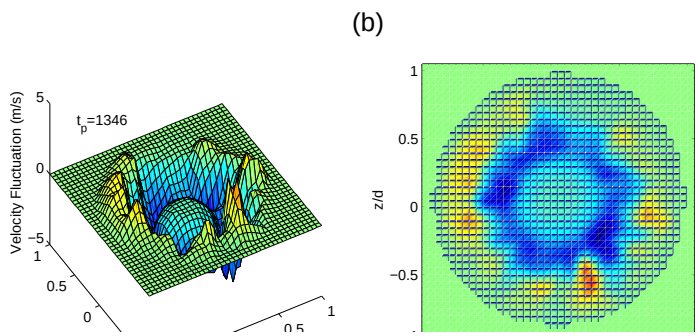


Velocity reconstruction at a field

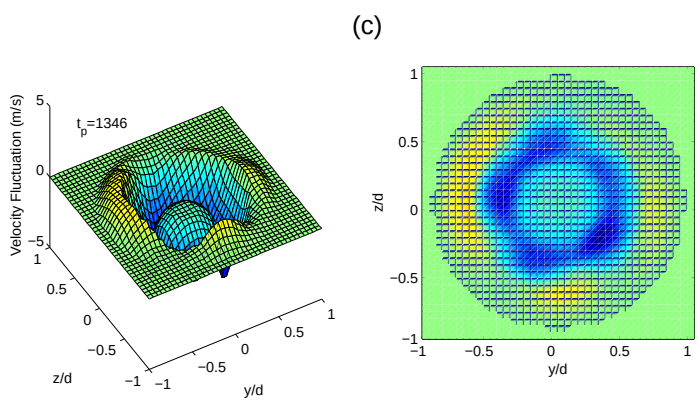
the original velocity



first POD mode, all mode



first POD mode, mode=0, 3-7



Animation of the reconstructed velocity

- $x/D = 2.0$
- $x/D = 4.0$
- $x/D = 6.0$

Summary, discussion, and conclusions

Similarity of the energy distribution

- the energy distribution of the first POD mode has **no dependence on** Re_D over the range of $78400 \leq Re_D \leq 156800$.
- Mode-0 behaves entirely different than the higher modes.
- The eigenspectra collapse when scaled in shear layer similarity variables. $\Rightarrow$ **structures are same as flow traverse downstream but different size**

The velocity reconstruction

- Azimuthally coherent vortex rings dominate the dynamics and the interactions of the structures until about $x/D \approx 4$.
- Beyond $x/D \approx 4$, the volcano-like eruptions die off rapidly.
- For $x/D \geq 4.0$, a "propeller-like" structure appears and dominates the pattern.

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