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Far downstream development of POD modes in a turbulent disk wake

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1 Introduction

The Proper Orthogonal Decomposition (POD) technique has recently become very popular for investigating the energetic structures of turbulent free shear flows. This is because the POD can describe the energetic structures of such flows with only a few modes. It has become popular only recently because of the development of high-speed computers that can handle massive amounts of data; in particular the large quantity of two-point velocity correlations required to produce the kernel for the POD.

This paper is part of an ongoing investigation of the disk wake using a 'slice' version of this technique. The developing region $x/D = 10$ to 50 was presented in Johansson *et al.* [4]. This was followed up with a sensitivity study of the POD by Johansson and George [5] where the influence of wake generator support and spatial resolution was evaluated. The POD was proven to be extremely robust and insensitive to external disturbances (such as the wakes of supporting wires). In this paper, the investigation is extended to cover downstream distances all the way to $x/D = 150$ to study the far downstream development of the POD modes.

2 Experimental Setup

Unlike the earlier experiments which were conducted at Chalmers University of Technology, these experiments were conducted in the Minimum Turbulence Level (MTL) wind tunnel at Royal Institute of Technology, Stockholm, Sweden. The free-stream streamwise turbulence intensity over the span of velocities related to this work was less than 0.02%. The measuring cross-section in the tunnel is 0.8×1.2 m^2 and the downstream length is 7.0 m. The tunnel velocity was kept constant at 20.5 m/s during the experiment.

The disk was made of plastic with a diameter of 20 mm, and the Reynolds number based on the free stream velocity and disk diameter was 27,000. The investigated region spanned from 30 to 150 disk diameters downstream. A total number of 15 hot-wires were used in two arrays. The probes were used to obtain the two-point velocity cross-spectra in the manner of Glauser and George [3] for all possible combinations of probe locations. The upper array of probes was movable, and traversed from a 15° separation up to 180° with 15° increments in $\Delta\theta$. Half the cross-section of the wake at a fixed downstream position was scanned, and pairs of instantaneous velocity correlations for a fixed angle separation computed. The measurement grid was chosen following the guidelines of Glauser and George [2] to avoid as much spatial aliasing as possible when making the azimuthal Fourier decompositions to obtain the cross-spectra. The cross-spectra corresponding to the remaining half-plane were available from the azimuthal symmetry of the flow.

3 POD

The POD results from a projection of the velocity field, u_i, into a coordinate system, ϕ_i, optimal in terms of energy. If the field has finite total energy, Hilbert-Schmidt theory assures that the solution exists and consists of a denumerable, infinite, set of eigenvalues, $\lambda^{(n)}$, and corresponding eigenfunctions, $\phi_i^{(n)}$. For an axisymmetric shear flow such as a jet or a wake, this is true in the radial direction at a single downstream location, hence the term 'slice POD'.

For the axisymmetric wake considered here, the turbulent velocity field is *stationary* in time and *periodic* in the azimuthal direction. Therefore, the Hilbert-Schmidt theory does not apply to them, but instead Fourier modes are appropriate. If only the streamwise velocity component is considered, the following integral equation(s) must be solved:

$$\int_0^\infty B_{1,1}(m,f,r,r';x)\psi_1^{(n)}(m,f,r';x)r'\,dr' = \lambda^{(n)}(m,f;x)\psi_1(m,f,r;x) \qquad (1)$$

where $B_{1,1}(m,f,r,r';x)$ is the two-point velocity correlation Fourier transformed in time and expanded in Fourier series in the azimuthal direction, and ψ is the corresponding eigenfunction. Note that ψ and λ are now functions of frequency, f, and azimuthal mode number, m, More detailed descriptions of the POD and the actual computational procedure can be found in Johansson *et al.* [4].

4 Results

The eigenspectra can be integrated over frequency to illustrate a key property of the POD, its ability to show how the kinetic energy of the flow is distributed among the various azimuthal modes. To visualize the energy distribution per azimuthal mode number, m, we computed for each downstream position the quantity $\xi(m;x)$ where:

$$\xi(m;x) = \frac{\int_f \lambda(m,f;x)\,df}{\sum_m \int_f \lambda(m,f;x)\,df} \tag{2}$$

Here, the denominator is the total kinetic energy in the field. The resulting normalized eigenspectra, $\xi(m;x)$ are plotted in Fig. 1.

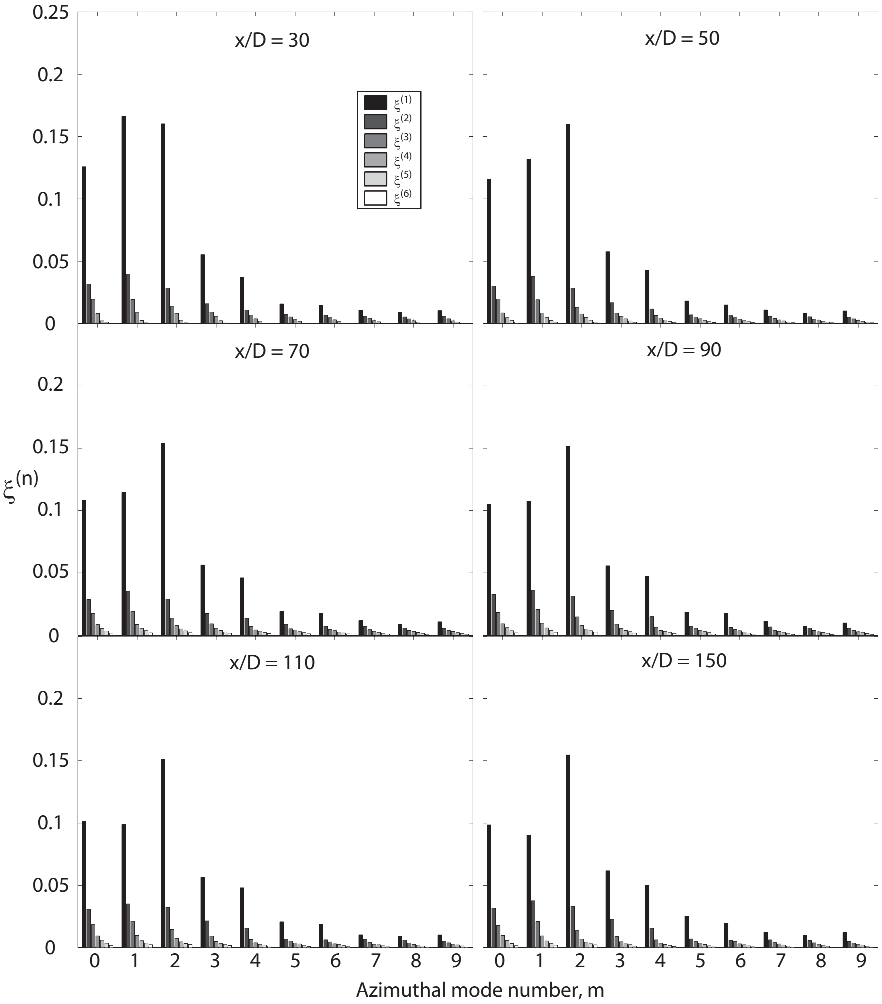

Figure 1: Eigenspectrum integrated over frequency as function of azimuthal mode number (m) at different downstream positions.

It is clear from this picture that the most dominating azimuthal mode at $x/D = 30$ is azimuthal mode number 1. But the importance of azimuthal mode 1 decreases as the flow evolves downstream, and it becomes the third most important mode behind mode 2 and mode 0 by $x/D = 110$. The picture hardly changes at all beyond $x/D = 110$. Note that mode 0 stays nearly constant and the slight decrease is due to the fact that the hot-wire probes do not cover as much of the wake at the farthest downstream position, a matter that is addressed in Johansson and George [5].

5 Summary and Conclusions

This study confirms the earlier results of Johansson *et al.* [4]. In particular, it makes clear that mode 1 does not dominate the energy of the far downstream wake. Instead, mode 2 does. In fact, it is the evolution of the eigenspectrum from mode 1 to mode 2 dominance that characterizes the evolution from near wake to far wake.

The present results are also strikingly similar to results obtained recently in the axisymmetric jet by Gamard *et al.* [1]. Despite being two different flows, the axisymmetric wake and the jet share many common features. It is reasonable to expect that the modes in these two flows can behave the same only if they are governed by similar equations, whatever they might be.

References

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