A THEORY FOR THE DECAY OF HOMOGENEOUS ISOTROPIC TURBULENCE

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ABSTRACT

A new theory for the decay of homogeneous, isotropic turbulence is proposed in which truly self-preserving solutions to the spectral energy equation are found which are valid at all scales of motion. The characteristic velocity scale is defined from the turbulence kinetic energy and the characteristic length scale is shown to be the Taylor microscale which grows as the square root of time (or distance). The decay rate is shown to be of power law form, and to depend on the initial conditions so that the decay rate constants cannot be universal except possibly in the limit of infinite Reynolds number. Another consequence of the theory is that the velocity derivative skewness varies during decay as R_λ^{-1} for fixed initial conditions.

Two different regimes for self-preservation are identified, each characterized by its own integral invariant. The first characterizes the turbulence from initiation until a transition to a second period of decay is entered which is characterized by the constancy of the Loitsiansky integral and includes the final period of decay. The exact decay coefficients and exponent cannot be determined from the theory, except for the second period of decay where the Loitsiansky integral enforces a $t^{-5/2}$ decay as observed experimentally. The theoretical predictions are found to be consistent with the abundant experimental data on grid turbulence decay.

It is further shown that the second period of decay consists of two parts: the first where the Loitsiansky integral becomes constant, but the inertial terms remain important; and the second (and final period) where the inertial terms vanish and the linear theory is applicable. A consequence of this is that a paradox regarding the observed increase of the velocity derivative skewness with decreasing R_λ and its subsequent decrease for very small R_λ is resolved.

Finally, the relation of the full self-preservation proposed here to Kolmogorov's theory for the local similarity of the small scale motions is examined. Kolmogorov's theory is shown to be strictly valid only in the limit of infinite Reynolds number, and compatible with the theory proposed here in that limit. It is further argued that the energy decay may reflect the life cycle of large scale structures, and that the 'energy cascade' is the evolution of those structures.

1. Introduction

Since the first measurements of turbulence behind a grid by Taylor (1935), there has been considerable interest in the decay of homogeneous and isotropic turbulence. Batchelor (1953) and Hinze (1959) provide excellent reviews of the abundant theoretical and experimental work over the first two decades following Taylor's pioneering work. Monin and Yaglom (1975) include much of the more recent work.

The self-preservation of decaying isotropic turbulence was first studied by von Karman and Howarth (1938) using the equation for the two-point correlation function which bears their name. In brief, they argued that the velocity correlation, or its counterpart the velocity spectrum, could be written in self-preserving form using the mean square velocity, u^2, and an integral scale defined from the dissipation and velocity as $\ell \equiv u^3/\epsilon$. A consequence of their theory was that the turbulence energy could be expected to decay as $u^2 \sim t^{-1}$ and the length scale to grow as $t^{1/2}$ so that the turbulence Reynolds number, $R=u\ell/\nu$ was constant during decay.

Batchelor (1948) reconsidered the self-preservation of decaying turbulence and argued that full self-preservation at all scales of motion was possible only in the limit as $R = u\ell/\nu \to 0$, the final period of decay. At large Reynolds numbers, Batchelor (1948,1953) argued that more than one scale was necessary to characterize the turbulence decay, invoking Kolmogorov's arguments of local similarity of the fine structure for the small scales (Kolmogorov 1941), and the von Karman-Howarth scaling for the energy containing scales. Stewart and Townsend (1951) tested these ideas experimentally with extensive measurements behind a grid in a wind tunnel, and were generally supportive of Batchelor's theories.

Concurrent with the interest in self-preservation was an interest in invariants of the decay process. Loitsiansky (1939) argued that the fourth moment of the two-point longitudinal correlation function represented an invariant of the decay. This was not confirmed by experiment, and was questioned on theoretical grounds by Proudman and Reid (1954), and by Batchelor and Proudman (1956) who studied the dynamical evolution of an assumed distribution of turbulence for which the velocity correlations went to zero exponentially as $r \to \infty$. Saffman (1967) considered an assumed distribution of turbulence for which vorticity correlations vanished exponentially with separation at the initial instant, and concluded that the second moment of the longitudinal correlation was an invariant.

None of the theories were particularly well confirmed by experiment except for the predicted $t^{-5/2}$ energy decay in the final period of decay. Even there the experiments of Bennett and Corrsin (1978) raised questions about the linearized theory (neglect of inertial terms) on which the predictions were based. Careful measurements by Uberoi (1963), Comte-Bellot and Corrsin (1966) and others made it clear that the turbulence Reynolds number was not constant during decay. The predicted $k^{-5/3}$ of the Kolmogorov theory was not observed in grid turbulence, presumably because of the low Reynolds number. And even when finally observed by Kistler and Vrebalovich (1966), the Kolmogorov constant did not accord with the value measured earlier by Grant, Stewart and Molliet (1962) in a tidal channel. Schedvin et al. (1978) did obtain an estimate of the Kolmogorov constant for grid turbulence at high Reynolds number which seemed reasonable, but only by considerably manipulating the spectral data. Finally, the combined evidence left little doubt that the turbulence behind a grid was influenced by the manner in which it was generated, and did not achieve a universal state independent of initial conditions, an implicit expectation of all the theories.

This paper seeks to reopen the question addressed by von Karman and Howarth (1938) and Batchelor (1948): Can decaying turbulence achieve a self-preserving state at finite Reynolds numbers? The answer will be seen to be an unqualified yes--at all Reynolds numbers and all scales of motion! The equations of motion will be shown, in fact, to admit to self-preserving solutions for which all of the dynamical terms in them achieve an equilibrium state and they evolve together.

The theoretically predicted self-preserving state will be shown to be consistent with the extensive and carefully documented data of Comte-Bellot and Corrsin (1971), and with the bulk of the experimental work of others. The consequences of self-preservation will be extended to consider the existence of invariants of the decay process and the multi-region character of the final period of decay. The Kolmogorov theory will be shown to be consistent with the full self-preservation proposed here, at least for infinite Reynolds numbers, Finally, some observations will be offered on the possible role of coherent structures.

PART I: SELF-PRESERVING ANALYSIS OF THE GOVERNING EQUATIONS

2. The Dynamical Equations

The spectral energy equation for a homogeneous isotropic turbulence is given by (Batchelor 1953)

$$\frac{\partial E}{\partial t} = T - 2\nu k^2 E \tag{2.1}$$

where E and T are functions of both the wavenumber k and time t. The three dimensional energy spectrum function, E(k,t), is defined as the integral over spherical shells of radius k of the contracted velocity spectrum tensor, ϕ_{ij}. Thus

$$E(k) \equiv \frac{1}{2} \iint \phi_{ii}(\underline{k}) d\sigma(\underline{k}) \tag{2.2}$$

where

$$k = |\underline{k}| \tag{2.3}$$

and where

$$\phi_{ij}(k) = \iiint_{-\infty}^{\infty} e^{i\underline{k}\cdot\underline{r}}\,\overline{u_i(\underline{x})u_j(\underline{x}+\underline{r})}\,d\underline{r} \tag{2.4}$$

Hereafter, the three-dimensional spectrum function, E(k,t), will simply be referred to as the spectrum.

It follows from the definitions that the integral of the three dimensional spectrum function can be integrated to obtain the turbulence kinetic energy, i.e.

$$\frac{3}{2} u^2 \equiv \frac{1}{2} \overline{u_i u_i} = \int_0^{\infty} E(k,t) dk \tag{2.5}$$

The spectral energy transfer T(k,t) arises directly from the transformed convective terms of the equations of motion. Following Batchelor (1953), if the two point triple velocity correlation is defined as

$$S_{ij,k}(\underline{r}) \equiv \overline{u_i(\underline{x})u_j(\underline{x})u_k(\underline{x}+\underline{r})} \ , \tag{2.6}$$

its Fourier transform can be defined as

$$\Gamma_{ij,k}(\underline{k}) = \frac{1}{8\pi^3} \iiint_{-\infty}^{\infty} S_{ij,k}(\underline{r}) e^{i\underline{k}\cdot\underline{r}} d\underline{r} \tag{2.7}$$

From the isotropic relations and incompressibility, $\Gamma_{ij,k}$ can be related to a single scalar function, $\gamma(k)$, by

$$\Gamma_{ij,\ell}(\underline{k}) = i\gamma(k)\left[k_i k_j k_\ell - \frac{1}{2} k^2 k_i \delta_{j\ell} - \frac{1}{2} k^2 k_j \delta_{i\ell}\right] \tag{2.8}$$

The transformed inertial terms in the spectral equations can be shown to be given by

$$T(k) = 4\pi k^6 \gamma(k) \tag{2.9}$$

Thus $T(k,t)$ contains the essential non-linearity of the Navier Stokes equations, and represents the transfer of energy from all other wavenumbers.

The spectral energy equation can be integrated with respect to k to yield the energy equation for the turbulence as

$$\frac{d}{dt}\left(\frac{3}{2} u^2\right) = -\epsilon \tag{2.10}$$

where ϵ is the rate of dissipation of turbulence energy per unit mass given by

$$\epsilon = 2\nu \int_0^\infty k^2 E(k,t)\,dk \tag{2.11}$$

and where the fact has been used that the net spectral transfer over all wave numbers is identically zero, i.e.

$$\int_0^\infty T(k,t)\,dk \equiv 0 \tag{2.12}$$

3. Similarity Analysis of the Spectral Equations

Self-preserving forms of the spectrum and spectral transfer functions are sought for which

$$E(k,t) = E_s(t)\, f(\eta) \qquad (3.1)$$

and

[redacted] (3.2)

where $\eta = kL$ (3.3)

and $L = L(t)$. (3.4)

[redacted].

Differentiating equation (3.1) yields

$$\frac{\partial E}{\partial t} = \left[\dot{E}_s\right] f(\eta) + \left[\frac{E_s \dot{L}}{L}\right] \eta f'(\eta) \qquad (3.5)$$

where $\cdot$ denotes time differentiation and $'$ differentiation with respect to η. Substituting into the spectral equation (eqn. 2.1) leads immediately to the transformed equation

$$\left[\dot{E}_s\right] f(\eta) + \left[\frac{E_s \dot{L}}{L}\right] \eta f'(\eta) = \left[T_s\right] g(\eta) - \left[\frac{\nu E_s}{L^2}\right] 2\eta^2 f(\eta) \qquad (3.6)$$

It is convenient to divide by $\nu E_s/L^2$ so that the transformed equation reduces to

$\left[\frac{\dot{E}_s L^2}{\nu E_s}\right] f + \left[\frac{L\dot{L}}{\nu}\right] \eta f'$ [redacted] $g - [1] 2\eta^2 f$ (3.7)

Now since the coefficient of the last term is time-independent, self-preserving solutions are possible only if the other bracketed terms are also time-independent. Thus the conditions for self-preservation are

$$\left[\frac{\dot{E}_s L^2}{\nu E_s}\right] \sim \text{const} \tag{3.8}$$

$$\left[\frac{L\dot{L}}{\nu}\right] \sim \text{const} \tag{3.9}$$

and

$$\left[\frac{T_s L^2}{\nu E_s}\right] \sim \text{const} \tag{3.10}$$

From equation (3.9) it follows immediately that

$$L^2 \sim \nu t \ , \tag{3.11a}$$

or

$$L^2 = 2A\nu t \tag{3.11b}$$

where the constant of proportionality, 2A, must be determined from other considerations and the initial condition can be absorbed by an appropriate choice of origin in time.

Equation (3.8), together with equation (3.11) reduces to

$$\frac{\dot{E}_s t}{E_s} \sim \text{const} \tag{3.12a}$$

or

$$\frac{\dot{E}_s t}{E_s} = p \tag{3.12b}$$

where p is a constant.

Equation (3.12b) can be integrated to yield

$$\frac{E_s}{E_{so}} = \left(\frac{t}{t_o}\right)^p \tag{3.13}$$

where E_{so} and t_o denote an arbitrary reference state. Thus the spectrum undergoes a power law decay (assuming p to be negative).

The third equation (equation 3.10) can be satisfied if and only if

$$T_s \sim \nu E_s / L^2 \tag{3.14a}$$

or using equation (3.11),

$$T_s \sim t^{-1} E_s \tag{3.14b}$$

4. The Energy and Dissipation Integrals

From equation (2.5) it follows by substitution that the energy integral can be written as

$$\frac{3}{2} u^2 = \left[E_s L^{-1}\right] \int_0^\infty f(\eta) d\eta \tag{4.1}$$

Since the integral is time-independent,

[redacted] (4.2)

It follows from equations (3.14a) and (4.1) that

[redacted] (4.3)

From equations (3.11),(3.13), and (4.2) a decay law can be immediately obtained as

$$\frac{u^2}{u^2_o} \sim \left[\frac{t}{t_o}\right]^p \left[\frac{L_o}{L}\right] \sim \left[\frac{t}{t_o}\right]^{p-1/2} \tag{4.4}$$

or

$$u^2 \sim t^n \tag{4.5}$$

where $n \equiv p - \frac{1}{2}$ (4.6)

and either p or n must be determined. Thus the kinetic energy also undergoes a power law decay.

It remains to relate L to a physically observable length scale. This can be accomplished by considering the rate of dissipation of turbulence energy given by equation (2.11). In similarity variables this becomes

$$\epsilon = \left[\nu E_s L^{-3}\right] 2 \int_0^\infty \eta^2 f(\eta) d\eta \tag{4.7}$$

or using equation (4.2)

$$\epsilon \sim \nu u^2 L^{-2} \tag{4.8}$$

For isotropic turbulence

$$\epsilon = 15\nu \overline{\left(\frac{\partial u}{\partial x}\right)^2} = 15\nu \frac{u^2}{\lambda^2} \tag{4.9}$$

where λ is the Taylor microscale, defined from the mean square derivative by

$$\overline{\left(\frac{\partial u}{\partial x}\right)^2} \equiv \frac{u^2}{\lambda^2} \tag{4.10}$$

By comparing equations (4.8) and (4.9), <u>the similarity length scale, L, is readily recognized to be the Taylor microscale, λ.</u> Therefore,

[redacted] (4.11a)

Thus from equation (3.11b)

[redacted] (4.11b)

The coefficient A can be related to the decay law exponent, n, by equations (2.10) and (4.9) with the result that

[redacted] (4.12)

which is the result obtained by von Karman and Howarth (1938). (Note that this result does not depend on self-preservation, but only on a power law energy decay, Batchelor 1953.)

A consequence of equations (4.11) and (4.3) is that the scale function for the spectral transfer is given as

[redacted] (4.13)

where R_λ is the Reynolds number defined from the Taylor microscale as

$$R_\lambda \equiv \frac{u\lambda}{\nu} \tag{4.14}$$

This result will be seen below to represent the principal point of departure of the analysis presented here from the earlier analyses of von Karman and Howarth (1938) and Batchelor (1948).

Thus self-preserving solutions to the spectral energy equations are possible and have the following characteristics:

(i) The characteristic length scale for the <u>entire</u> spectrum is the Taylor microscale, λ.

(ii) The Taylor microscale increases as the square root of time, i.e.

$$\lambda \sim t^{1/2}.$$

(iii) The spectrum and spectral transfer functions collapse when plotted as

$$\frac{E(k,t)}{u^2\lambda} \text{ vs } k\lambda$$

and

$$\frac{\lambda T(k,t)}{\nu u^2} \text{ vs } k\lambda.$$

(iv) The energy undergoes a power law decay

$$u^2 \sim t^n$$

where n is a constant.

(v) The turbulence Reynolds number characterizing the motion is $R_\lambda = u\lambda/\nu$ and it varies as

$$R_\lambda \sim t^{(n+1)/2} \tag{4.15}$$

[redacted]

5. Determination of the Constants and the Spectral Shape

What determines the value of the coefficients and the decay exponent? Except at infinite Reynolds number and for the final period of decay which are discussed below, this question cannot be answered in general except to say that they must be determined by the initial conditions. However, since these coefficients directly enter the spectral equations, they are closely related to the shapes of the spectrum and spectral energy transfer. This can be easily seen by substituting into equation (3.7) the appropriate form of the bracketed terms. From equation (4.11b),

$$\frac{\lambda\dot{\lambda}}{\nu} = A \tag{5.1}$$

and from equation (3.12b) and (4.6),

$$\frac{\dot{E}_s t}{E_s} = p = n + \frac{1}{2} \tag{5.2}$$

From equation (3.14a) a coefficient B can be defined so that

$$B \equiv \frac{T_s \lambda^2}{\nu E_s} \tag{5.3}$$

Thus the spectral equation reduces to

$$[2A(n+1/2)]f + [A]\eta f' = [B]g - 2\eta^2 f \tag{5.4}$$

or using $-A=5/n$ from equations (4.11) and (4.12),

$$n(10f + Bg - 2\eta^2 f) + 5\eta f' + 5f = 0 \tag{5.5}$$

Clearly the shape of the functions f and g satisfying equation (5.5) will depend on the values of B and n. Thus turbulence generated in different ways can be expected to have different spectral shapes during decay if their decay constants are different.

6. The Velocity Derivative Skewness and the Inertial Terms

The velocity derivative skewness is defined as

$$S_{\partial u/\partial x} = \overline{(\partial u/\partial x)^3} \Big/ \overline{\left[\frac{\partial u}{\partial x}\right]^2}^{\,3/2} \tag{6.1}$$

and can be related to the spectral energy transfer by (Tavoularis et al. 1978)

$$S_{\partial u/\partial x} = \frac{3(30)^{1/2}}{14} \; \frac{\int_0^\infty k^2 T(k)\,dk}{\left[\int_0^\infty k^2 E(k)\,dk\right]^{3/2}} \tag{6.2}$$

Thus the velocity derivative skewness is a direct measure of the importance of the inertial terms in the dynamical equations.

By substituting equations (5.2), (5.3) and (4.12) into equation (6.2) it follows that

$$S_{\partial u/\partial x} = \frac{3(30)^{1/2}}{14} \; \frac{B\left[\nu\frac{u^2}{\lambda}\,\lambda^{-3}\right] \int_0^\infty \eta^2 g(\eta)\,d\eta}{\left[u^2\lambda.\lambda^{-3}\right]^{3/2} \left[\int_0^\infty \eta^2 f(\eta)\,d\eta\right]^{3/2}} \tag{6.3}$$

This in turn implies

$$S_{\partial u/\partial x} \sim R_\lambda^{-1} \tag{6.4}$$

where the constant of proportionality is a function of the initial conditions. [redacted]. Alternately, equation (6.4) can be rewritten as

$$S_{\partial u/\partial x}\, R_\lambda = \text{constant} \tag{6.5}$$

where the constant is determined by the initial conditions.

The value of the constant in equation (6.5) can be related to the decay exponent n and the spectral shape by substituting for the spectral transfer using the spectral energy equation. From equations (5.5) and (6.3) it follows after some manipulation that

$$S_{\partial u/\partial x} R_\lambda = \left[\frac{30}{7}\right] \left[\frac{n-1}{n}\right] - \frac{4}{35} \int_0^\infty \eta^4 \; f(\eta) d\eta \tag{6.6}$$

Since both the decay exponent, n, and the spectral shape depend on the initial conditions, so must $S_{\partial u/\partial x} R_\lambda$.

Both equations (6.4) and (6.5) are important consequences of the proposed theory of self-preservation, and provide a crucial experimental test of its validity. These results can be contrasted with those of Kolmogorov (1941) which require that the velocity derivative skewness itself be a constant (v. Batchelor 1953), or the modified theory of Kolmogorov (1963) in which the velocity derivative skewness increases with Reynolds number. Note that neither of Kolmogorov's theories suggest or even allow for a separate dependence on initial conditions.

7. Comparison with the Von Karman-Howarth and Batchelor Results

The key assumption of the earlier von Karman-Howarth analysis (see also Monin and Yaglom (1975)) is the arbitrary choice, $T_s \sim u^3$, which can be contrasted with that obtained here in equation (4.13). This choice dictates that the energy decays as inverse time and that the turbulence Reynolds number is constant during decay. Thus both these important results (which have presented so much difficulty to the turbulence community) follow not from the similarity analysis, but from the assumptions which went into it. Neither of these predictions is confirmed by experiment, nor can the assumption be justified by an analysis of the equations as shown above.

Batchelor (1948), approaching the self-preservation analysis from the correlations functions using the von Karman-Howarth equation, in essence makes the same assumption by arguing that the triple correlation function k(r) defined by

$$\overline{u^2(x)u(x+r)} = u^3k(r) \tag{7.1}$$

can be written in self-preserving form as

$$k(r) = \tilde{k}(r/\lambda). \tag{7.2}$$

The full implications of this can be seen by writing the entire triple correlation in self-preserving form as

$$\overline{u^2(x)u(x+r)} = u^3k(r) = K_s(t)\tilde{k}(r/\lambda) \tag{7.3}$$

In effect, Batchelor has also arbitrarily chosen

$$K_s \sim u^3 \tag{7.4}$$

which implies that the velocity skewness is independent of Reynolds number. A further consequence of this assumption is that a factor of R_λ is left in the equation from which it is concluded (as in the von Karman-Howarth analysis) that the Reynolds number must be a constant for self-preservation at all scales. Batchelor then argues that this can only occur in the limit as $R_\lambda \to 0$, the final period of decay (see below).

If an analysis of the von Karman-Howarth equation is carried out using equation (7.3) leaving K_s to be determined (as in the spectral analysis presented above), the result is

$$K_s \sim \nu \frac{u^2}{\lambda} \sim R_\lambda^{-1} u^3 \tag{7.5}$$

Thus the skewness retains a Reynolds number dependence and no factor of R_λ remains in the equation. Most importantly, true self-preservation is possible at <u>all</u> scales of motion, independent of R_λ.

It is instructive to examine why the choice $T_s \sim u^3$ (or $K_s \sim u^3$) might have been made and what physics it implies. If it is assumed (as in Monin and Yaglom, 1975) that at some moment during decay the turbulence spectrum can be completely characterized by two parameters, say u and ℓ, then it follows from dimensional analysis that $E_s \sim u^2\ell$ and $T_s \sim u^3$ and a Von Karman-Howarth analysis is indicated. What kind of turbulence would be characterized only by its energy and a single length scale? <u>Only one which is completely independent of its initial conditions</u>, precisely the conditions previously believed to be necessary for a flow to be asymptotically* self-preserving.

As shown by the analysis above, independence of initial conditions is <u>not</u> essential for a flow to achieve a self-preserving state. However, because of the dependence on initial conditions, there is no longer possible a single <u>universal</u> solution to which all flows must be asymptotic, at least at finite Reynolds number (see Section 8). Unfortunately, self-preservation has often been confused with the existence of a single universal state. Clearly such is not the case. It has recently been argued by George (1986, 1988) that most turbulent shear flows relax to a self-preserving state determined by their initial conditions, and not to a single universal state as previously believed. The same can be said of isotropic turbulence.

8. The Relation to Kolmogorov's Theory of Local Similarity

Before comparing Kolmogorov's theory for the local self-preservation of the dissipative scales of motion to that proposed here, it is worth examining the precise conditions under which it can be expected to hold. For a given

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*Asymptotic refers to times (or distances) sufficiently far removed from the generation so that not all of the details of generation are important.

set of initial conditions of isotropic turbulence, the governing equations make it clear that the spectrum at all wavenumbers must be a function of the kinetic energy, $3u^2/2$, the rate of dissipation, ϵ, and the kinematic viscosity, ν. In the limit of infinite Reynolds number, it can be argued that the spectrum at low wavenumbers makes no contribution to the dissipation. Also in this limit, the spectrum at high wavenumbers contains no energy. Thus, in the limit of infinite Reynolds number the high wavenumber spectrum must be uniquely determined by ϵ and ν (which is Kolmogorov's 1941 proposal). Similarly, the spectrum at low wavenumbers can only depend on u^2 and ϵ, the latter entering because it is equal to the spectral energy flux from the low to the high wavenumber regions (but only at infinite Reynolds number). (This was first recognized by Batchelor 1948). It is clear from the above that neither scaling can be entirely correct at finite Reynolds numbers since the high wavenumber region will always contain some of the energy while the low wavenumber region contributes some of the dissipation, thereby invalidating the arguments on which the scaling is based.

In spite of the above, it is clear that the high and low wavenumber scaling laws might be approximately valid at very large, but finite, Reynolds numbers. If so, then there must be a region where both scaling laws are valid and they can be matched. This matching in the limit of infinite Reynolds number yields the familiar $k^{-5/3}$ inertial subrange (Tennekes and Lumley 1972). Thus the existence (or non-existence) of a $k^{-5/3}$ range in a particular experimental spectrum provides a reasonable clue as to whether or not the Kolmogorov or von Karman-Howarth scaling laws might reasonably describe the appropriate spectral regions. Certainly, if there is no $k^{-5/3}$ range, then there is no reason to believe that the assumptions for either scaling law are even approximately satisfied.

It is clear from the above that Kolmogorov's theory is at best an approximation for turbulence at finite Reynold's number, even though it may be high. Thus it is quite unlike the theory of full self-preservation proposed here which places no restriction on Reynold's number. Therefore it is only in the limit of infinite Reynold's number that these two theories must be compatible. This requirement for compatibility in the limit of infinite Reynolds number will be seen to determine the decay rate exponent in this limit.

The essence of the Kolmogorov (1941) theory is that at infinite Reynolds numbers, the small scale turbulence is in local equilibrium and the spectrum scales with only the dissipation rate, ϵ, and kinematic viscosity, ν. Thus the spectrum in Kolmogorov variables is given by

$$E(k) = (\nu^{5/4}\epsilon^{1/4})\ \tilde{f}\ (k\eta_k) \tag{8.1}$$

where η_k is the Kolmogorov microscale defined by

$$\eta_k = (\nu^3/\epsilon)^{1/4} \tag{8.2}$$

It has been generally believed that the spectrum $\tilde{f}$ is universal in that it is independent of how the turbulence is generated (v. Batchelor 1953, Monin and Yaglom 1975).

There would appear at first sight to be a major conflict between the Kolmogorov theory and the one proposed here. If the spectrum for isotropic turbulence in fact collapses in Taylor variables u and λ as required for full self-preservation (and as the experimental evidence seems to indicate), then the spectrum in Kolmogorov variables is given by

$$\nu^{5/4}\epsilon^{1/4}\ \tilde{f}(k\eta) = u^2\lambda\ f(k\lambda) \tag{8.3}$$

so that

$$\tilde{f}(k\eta) = (15)^{-1/4}\ R_\lambda^{3/2}\ f(k\lambda) \tag{8.4}$$

Thus the Kolmogorov scaled spectrum is directly dependent on R_λ, contrary to the hypothesis that it should not be. Moreover, since $f(k\lambda)$ is determined by the initial conditions, so must be $\tilde{f}(k\eta)$ and the Kolmogorov spectrum can not therefore be universal, even at high wavenumbers.

In spite of the above, it is easy to show that the two theories are not incompatible, Lin (1948) showed the equivalence between the von Karman–Howarth theory and that of Kolmogorov. In particular, he showed that Kolmogorov scaling was consistent with full self-preservation of the type proposed by von Karman and Howarth only if the energy decayed as t^{-1} and the length scale increased as $t^{1/2}$. For the special case where $n = -1$, the theory of full self-preservation presented here yields an R_λ which is constant during decay. When R_λ = const, η/λ is also constant so that both η and λ evolve together, with the result that the spectra scaled in Kolmogorov variables and Taylor variables are equivalent. Thus for the special case of $n=-1$, all three theories (including the one proposed here) are equivalent.

A heuristic argument for a decrease in the energy decay rate with increasing initial Reynolds number can be made by noting that energy is removed from a given wavenumber in the energy–containing range by both the non–linear spectral transfer and by the direct action of viscosity. If the spectral transfer is relatively insensitive to the Reynolds number, (as is usually assumed in spectral closure models, v. Lesieur 1987), then the rate at which energy is removed from a given spectral component decreases with increasing Reynolds number since the viscous term is less. In the limit of infinite Reynolds number, no energy is removed by direct viscous action on the energy–containing scales, and the decay rate is entirely determined by the spectral transfer. It follows immediately that if the turbulence settles into a self-preserving state in which the energy decay can be represented by a power law, then that power must vary with initial Reynolds number and approaches an asymptotic value in the limit of infinite Reynolds number.

The theory of full self-preservation proposed herein can only predict that there is a decay exponent, that it does not vary during decay, and that it is determined by the initial conditions. It is entirely consistent that this dependence on initial conditions vanishes in the limit as the Reynolds number characterizing the initial conditions (usually the grid mesh Reynolds number) becomes infinite. If so, then the decay exponent must approach some asymptotic value. The applicability of the Kolmogorov theory in the limit of infinite grid Reynolds number can be interpreted to require that this limit be

$$n_\infty \equiv \lim_{R_M \to \infty} n = -1 \tag{8.5}$$

It is not hard to imagine that the shape of the spectrum appropriate to this limit might also be independent of the details of the initial conditions (at least at high wavenumbers). In fact, it could even be the same as for other turbulent flows. If so all of the apparent points of conflict between the competing theories vanish and, in fact, the theories are seen to be complementary.

Before leaving this section it is interesting to examine the behavior of the derivative skewness in the limit of infinite Reynolds number if $n \to -1$. First note from equation (4.15) that the closer n approaches the limiting value, the less the variation of R_λ and the less the increase of the derivative skewness during decay. In the limit of infinite Reynolds number, neither R_λ nor S will vary at all during decay. By expressing the integral of equation (6.6) in terms of the Kolomogorov spectrum using equation (8.4), the resulting limiting value of the derivative skewness can be obtained as

$$S_\infty = -(2)^{3/2} \int_0^\infty \tilde{k}^4 \, \tilde{f}\,(\tilde{k})d\tilde{k} \tag{8.6}$$

If the limiting spectrum is independent of Reynolds number (as in Kolmogorov's

1941 theory), then S_∞ is a constant. On the other hand, if the limiting spectrum retains a Reynolds number dependence (v. Champagne 1978) or a dependence on the other initial conditions (v. George 1988), then S_∞ will reflect this. [redacted].

PART II COMPARISON WITH EXPERIMENT

9. The Relation of the Experiments to Decaying Isotropic Turbulence

The theory of self-preservation for decaying isotropic turbulence presented here makes a number of explicit predictions which are subject to experimental verification. In particular, the theory can be said to be confirmed if:

(i) The turbulence energy, u^2, decays as a power law in time.

(ii) The Taylor microscale, λ, increases as the square root of time.

(iii) The integral scale determined from the spectral intercept at the origin or from the correlation function is proportional to λ.

(iv) The spectrum scales with u^2 and λ.

(v) The non-linear terms (as measured by the velocity derivative skewness or the peak in the triple velocity correlation) vary during decay as R_λ^{-1}.

(vi) The decay exponent, coefficients and spectral shapes are determined by the initial conditions throughout the decay.

In the following sections the available experimental data will be used to evaluate whether these predictions are an accurate description of the behavior of real turbulence.

Almost all of the attempts to simulate the turbulence described by equation (2.1) have (following the example of Taylor 1935) utilized a grid in a wind tunnel. In such experiments, the wakes generated by obstructions (most relations even with the slight anisotropy, as long as the distance behind the grid was sufficient (typically $x/M > 40$, where M is the grid mesh length).

The preceding considerations alone do not insure that the spectral equations for grid-generated wind-tunnel turbulence are the same as for temporally decaying turbulence. However, since the flow is homogeneous in planes perpendicular to the mean flow direction and varies only slowly in the streamwise direction, it is reasonable to suppose the turbulence to behave as homogeneous turbulence in an infinite environment, <u>at least for scales of motion much smaller than the size of the tunnel dimensions.</u> Thus spectral and correlation measurements in such flows can be expected to satisfy those for homogeneous (and in most cases isotropic) turbulence (v. Batchelor 1953) for all but the largest scales of motion.

Because of the difficulty in making sufficient measurements in space to compute wavenumber spectra, Taylor (1935) introduced the idea of a frozen field for which time variations at a fixed point could be treated as spatial variations moving by the probe. For such a frozen field the temporal correlations measured at a fixed point, $C_{i,j}(\tau)$ are related to the spatial correlations $B_{i,j}(\underline{r})$ by:

$$B_{i,j}(r,o,o) = C_{i,j}(\tau) \tag{9.5}$$

where τ is the time delay and r is the streamwise separation and they are related by

$$r = U\tau \tag{9.6}$$

The one-dimensional wavenumber spectrum, $F_{i,j}(k_1)$, and the frequency spectrum measured by a fixed probe can be similarly related, i.e.,

$$F_{i,j}(k_1) = \frac{U}{2\pi} S_{i,j}(f) \tag{9.7}$$

where f is the circular frequency and k_1 is the wavenumber in the streamwise direction, and they are related by

$$k_1 = \frac{2\pi f}{U} \tag{9.8}$$

In the following sections the wealth of experimental data on grid-generated turbulence acquired over the past 50 years will be examined. The major difficulty which will be encountered is the <u>limited range of variation of R_λ for a particular set of initial conditions.</u> (Note that this is quite different from the usual lament of turbulence theoreticians that the Reynolds number is not high enough.) Previous investigators appear to have believed that the character of turbulence behind a grid should be independent of its initial conditions. As a consequence, <u>most of the Reynolds number variation reported in the literature has been obtained by changing the initial conditions</u> either by varying the velocity or changing the grid mesh length. These experiments (which unfortunately constitute the bulk of those performed) are of little use in evaluating the theory proposed herein.

Only a few of the experiments in the literature report detailed measurements over a wide enough range of decay times (distances downstream) for R_λ to have varied significantly. The problem is that R_λ depends only weakly on distance from the grid. From equation (4.15) it follows that

$$R_\lambda \sim t^{(n+1)/2} \sim (x)^{(n+1)/2} \tag{9.9}$$

For the experiments considered below n ~ -1.2 so that

$$R_\lambda \sim (x)^{-0.1} \tag{9.10}$$

Thus a factor of ten variation in x/M is necessary for even a 20% variation in R_λ.

Most of the early experiments (Batchelor and Townsend 1946, Mills et al. 1958 for example) only report data for $20 < x/M < 70$. A few (Uberoi 1963, Frenkiel and Klebanoff 1971, and Wahrhaft and Lumley 1978) report data to about $x/M = 110\text{-}170$. Only Comte-Bellot and Corrsin (1966), (1971) report streamwise variations even approaching an order of magnitude, $45 < x/M < 385$. (Note the very low grid Reynolds number experiments of Batchelor and Townsend (1948) and Bennett and Corrsin (1978) will be considered later in Part III when the final period of decay is discussed). Particular attention will be paid to the experiments of Comte-Bellot and Corrsin (1971) for which the spectral and decay data have been conveniently tabulated, and the data can be shown to be internally consistent.

In spite of the limited variation of R_λ in the individual experiments, it will be seen that the data is consistent with the theoretical predictions, including the proposed spectral scaling in Taylor variables and the variation of velocity derivative skewness with R_λ. Unfortunately, it will not be possible to demonstrate conclusively with the existing spectral data that the new theory is superior to the earlier theories, all of which collapse the data reasonably well because of the limited variation of R_λ. However, the observed variation of the velocity derivative skewness with R_λ, even though slight, will be seen to be in agreement with the new theory and in contradiction to the earlier expectations.

10. The Turbulence Intensity Variation Behind the Grid

The variation of the turbulence intensity with distance downstream of a grid has been investigated many times since the experiments reported by Simmons and Salter (1934) and Dryden et al. (1937). These early experiments seemed consistent with the von Karman/Howarth predictions that $u^2 \sim t^{-1}$ or $U^2/u^2 \propto (x-x_o)/M$. However, subsequent studies by Corrsin and coworkers indicated that a better fit to the experimental data could be obtained by a relation of the form

$$\frac{U^2}{u^2} = A_1\left[\frac{x-x_o}{M}\right]^{-n} \tag{10.1}$$

where the coefficient A_1 and the exponent n appeared to depend on the particular geometry and Reynolds number of the grid. The most comprehensive of these studies is due to Comte-Bellot and Corrsin (1966) from which Figure 1, is taken.

So well established is equation (10.1) on empirical grounds that few would doubt its validity. However, <u>prior to the analysis presented herein, it has not been possible to justify this relation theoretically, nor has it been possible to explain the observed dependence on initial conditions.</u> Thus the turbulence intensity measurements would appear to strongly support this prediction of the proposed theory.

A second aspect of the turbulence intensity variation concerns the possible dependence of the exponent n on grid Reynolds number R_M where

$$R_M \equiv \frac{UM}{\nu} \quad , \tag{10.2}$$

and whether or not $n \to -1$ in the limit as $R_M \to \infty$. From the definitions of R_M and R_λ and equation (10.1), the coefficient A_1 can be obtained as

$$A_1 = \left(-\frac{10}{n}\right) \frac{R_M}{R_\lambda^2} \left[\frac{Ut}{M}\right]^{n+1} \tag{10.3}$$

(Note that the explicit time dependence is cancelled by the time variation of R_λ.). If $n \to -1$ in the limit as R_M and $R_\lambda \to \infty$, then the limiting value of A_G is given by

$$A_{1\infty} = 10 \left[\frac{R_M}{R_\lambda^2}\right]_\infty \tag{10.4}$$

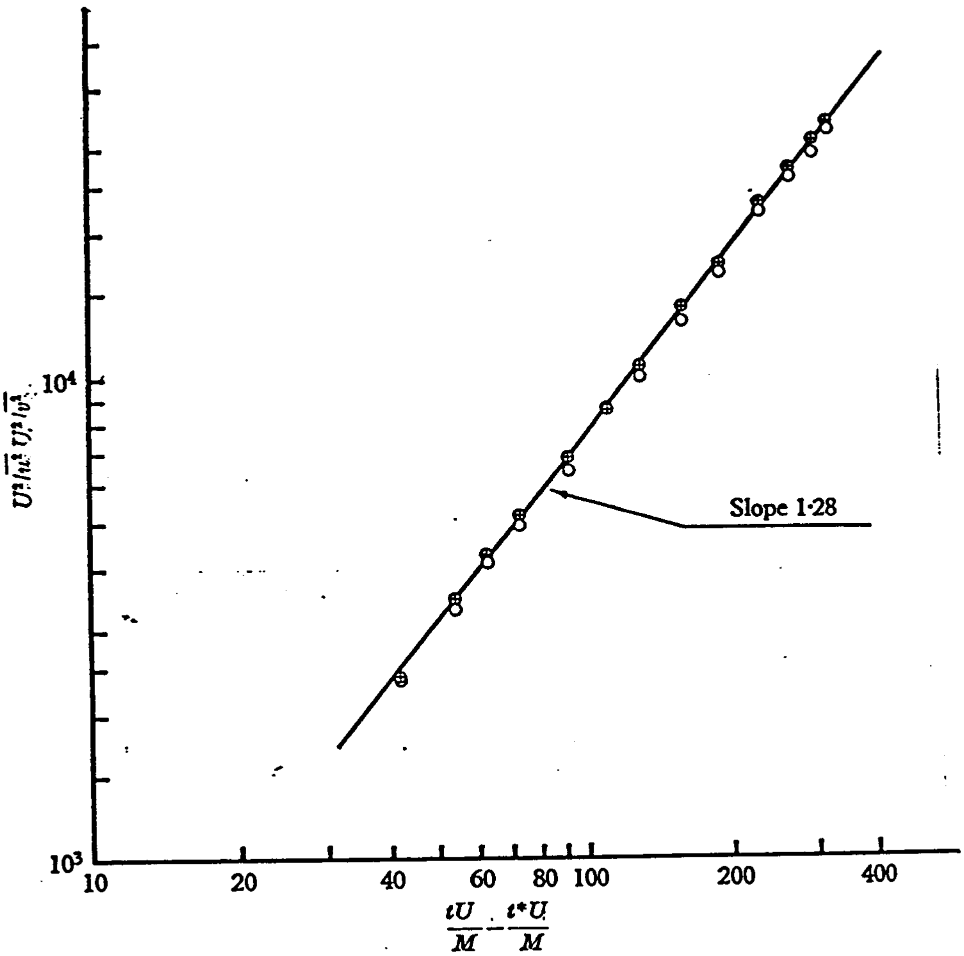

Figure 1 Turbulence intensity decay in a typical case after a contraction at 18M; square-rod grid with M=2.54 cm and U=20 m/sec. o, $U^2/\overline{u^2}$ with $t^*U/M=4$; +, $U^2/\overline{v^2}$ with $t^*U/M=4$ (from Comte Bellot and Corrsin 1966).

Thus in the limit R_M/R_λ^2 is a constant and uniquely determines the decay, and can at most be a function of the grid geometry. This relation is generally consistent with the experimental data of Comte-Bellot and Corrsin (1966), and was first used by Batchelor and Townsend (1947) on empirical grounds, but follows directly from the theoretical arguments presented herein.

From the discussions in Section 8, it is clear that whether $n \to -1$ as $R_M \to \infty$ is closely related to the question of the applicability of Kolmogorov's theory to grid turbulence. However, it has long been recognized that the Kolmogorov theory applies only beyond the Reynolds number range of most grid turbulence experiments (v. Stewart and Townsend 1951). In fact, only two experiments (Kistler and Vrebalovich 1966, Schedvin et al. 1974) were carried out at high enough grid Reynolds numbers to even begin to observe the $k^{-5/3}$ range expected for high Reynolds number turbulence. Both of these experiments reported best fit decay laws for which n=-1. Consistent with this is that there is some evidence (most notably the square bar data of Comte-Bellot and Corrsin 1966) that the decay exponent increases from n<-1 towards n=-1 with increasing grid Reynolds number. Figure 2 shows the variation of exponent with grid Reynolds number for the square bar grid data of Comte-Bellot and Corrsin (1966). Also plotted is the exponent determined by Kistler and Vrebalovich (1966) at very high Reynolds numbers using a round bar grid with the same solidity (σ = 0.34). Even without this latter point, the trend is clearly downward, and is especially apparent when each of the three grids used is considered independently.

11. The Taylor Microscale

Von Karman and Howarth (1938) showed for isotropic turbulence that if the turbulence decayed as a power law in time, then the Taylor microscale would increase as the square root of time. In view of the concensus reported above

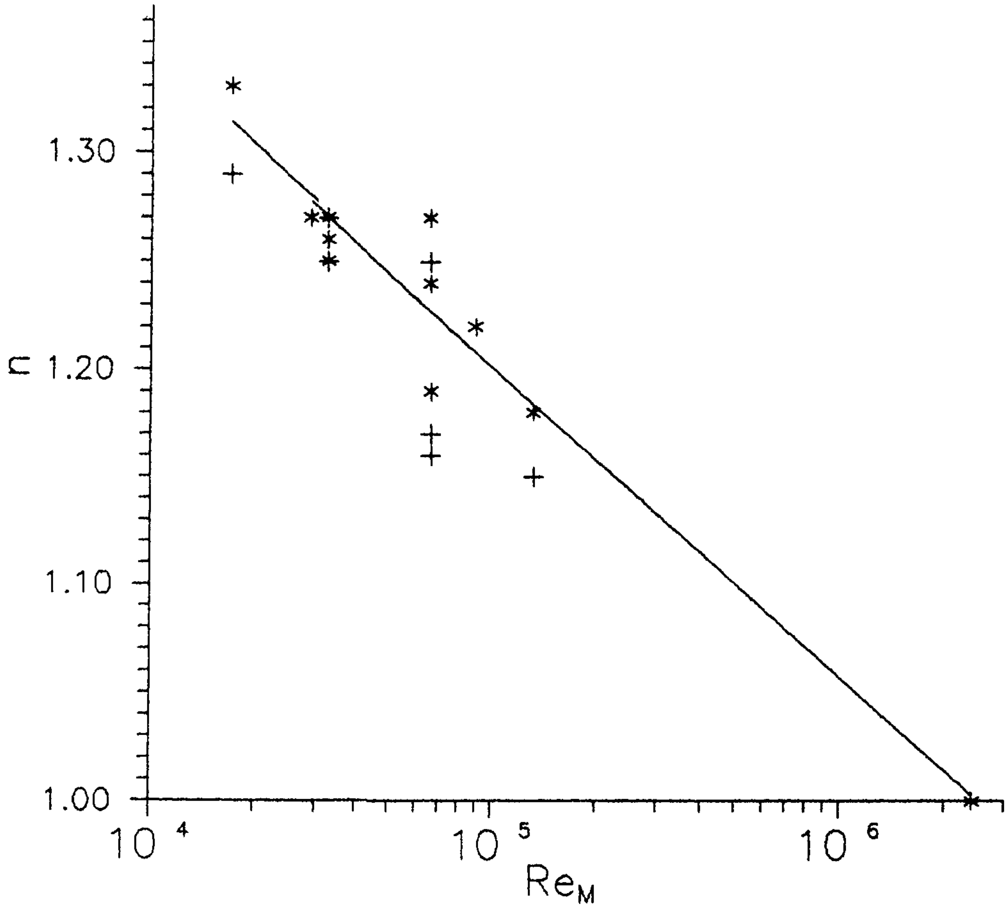

Figure 2. Variation of n with R_M, Square Bar Grid for $\overline{U^2/u^2}$ Data of Comte-Bellot and Corrsin 1966 (*, no contraction, + with contraction).

for the turbulence intensities, it is no surprise that all of the experiments beginning with that of Dryden (1943) are consistent with the expected dependence and equation (4.11b).

Figure (3) shows the variation of the square of the Taylor microscale with Ut/M for the two grids of Comte-Bellot and Corrsin (1971). Both show the expected dependence of $\lambda^2 \sim t$ as confirmed by the linear regression curve fits. From the tabulated data the ratio $\lambda^2/\nu t$ can be computed and found to be 8.24 and 8.20 for the 25.4 mm and 50.8 mm grids respectively. Since from equation (4.12) the coefficient should be -10/n, where n is the exponent for the energy decay, -n can be computed as 1.20 and 1.21 for the two grids respectively. These values are very close to the value of 1.25 suggested by the authors, the difference corresponding in part to the choices of virtual origin: $Ut_o/M=3.5$ for both grids by the authors from the turbulence intensity measurements, $Ut_o/M=8.95$ and 4.54 for the 25.4 mm and 50.8 mm grids respectively here. Note that the theory requires that the virtual origin for all statistical quantities be the same, which has not always been the practice of experimenters.

12. The Integral Scales

A more important test of the theory here is whether or not the integral scales are directly proportional to the Taylor microscale. In the experiments of Comte-Bellot and Corrsin (1971), the transverse integral scale, L_g, was computed by integrating the transverse velocity correlation to the point where it crossed the r-axis. The longitudinal integral scale was obtained from the measured one-dimensional spectrum in the limit of zero wavenumber. Neither of these procedures is particularly satisfactory, as pointed out by the authors, because of the high-pass filtering of the data by A.C. coupling, and the difficulties in carrying out the extrapolations. In addition, there must

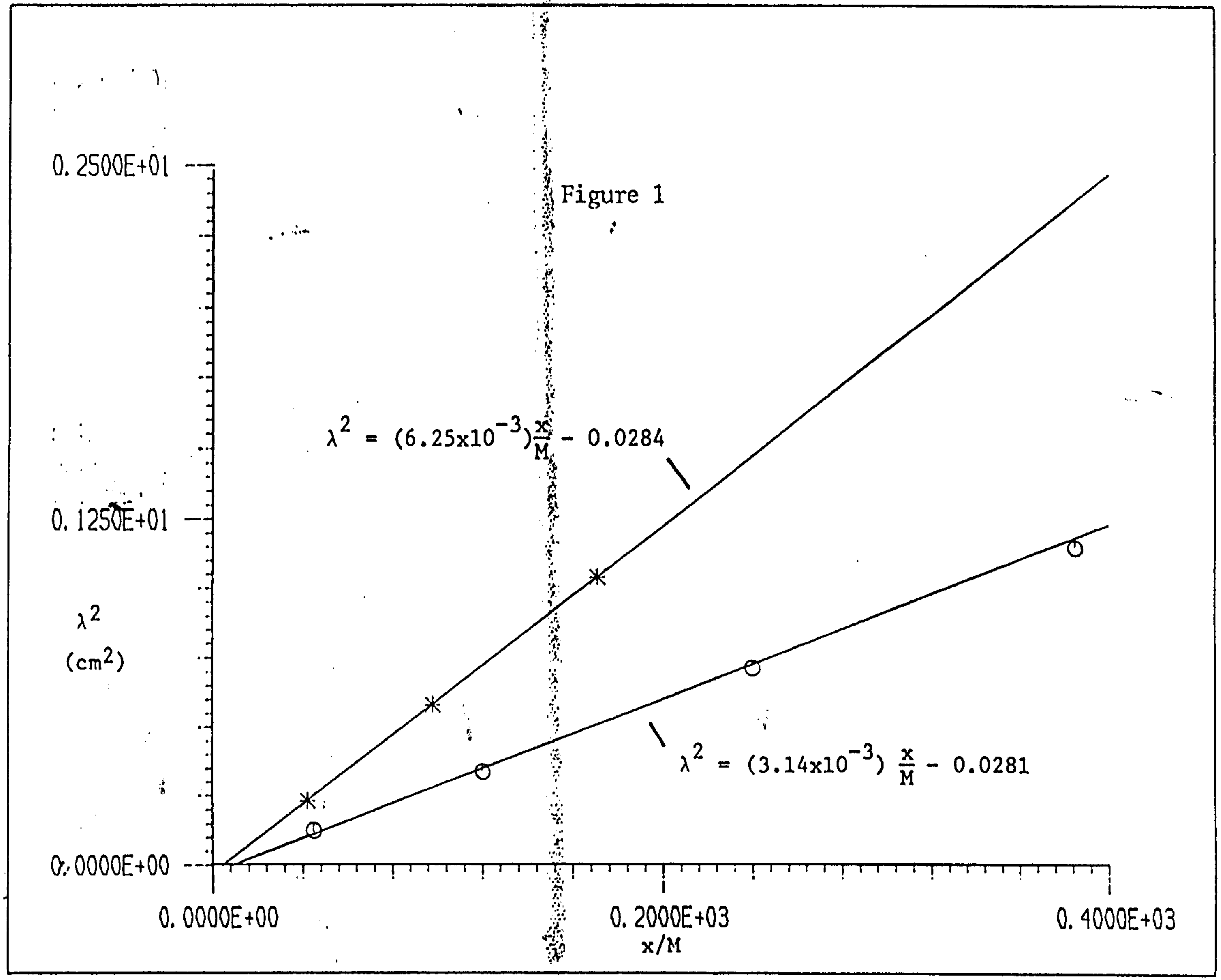
0.2500E+01
0.1250E+01
0.0000E+00
λ^2 (cm^2)
$\lambda^2 = (6.25\text{x}10^{-3})\frac{x}{M} - 0.0284$
$\lambda^2 = (3.14\text{x}10^{-3})\ \frac{x}{M} - 0.0281$
0.0000E+00
0.2000E+03
0.4000E+03
x/M

Figure 1

always be some concern as to whether the largest scales of motion reasonably approximate an isotropic turbulence in an infinite environment. The results are, however, in general agreement with prediction as illustrated in Figure 4 which plots the measured integral scales versus the measured Taylor microscale for the two grids at various x/M. It is suggested that the values at larger x/M are most suspectible to the problems mentioned above (since the spectrum progressively shifts to lower frequencies and larger scales), and are therefore the least reliable.

13. The Spectral Scaling

The principal difficulty in verifying the spectral predictions of the theory is that, in spite of the numerous experiments in grid turbulence over the past fifty years, there exist only a few sets of experimental data which are both well tabulated and over a wide range of downstream positions for fixed initial conditions. The exceptions to this are the experiments of Comte-Bellot and Corrsin (1971) for which the spectral and decay data have been conveniently tabulated. Therefore these experiments will be examined in detail first, then other experimental evidence will be considered.

Before carrying out this examination of the spectral data, however, it is useful to review what comparisons will be made and what kind of agreement (or disagreement) with other theories might be expected. Since the data is usually presented as a function of position behind the grid, x/M, it is interesting to note how the various scaling parameters vary with x/M. Three types of scaling are of interest: the Taylor scaling proposed here, the von Karman-Howarth scaling and the Kolmogorov scaling. The appropriate spectral scalings are summarized in Table I.

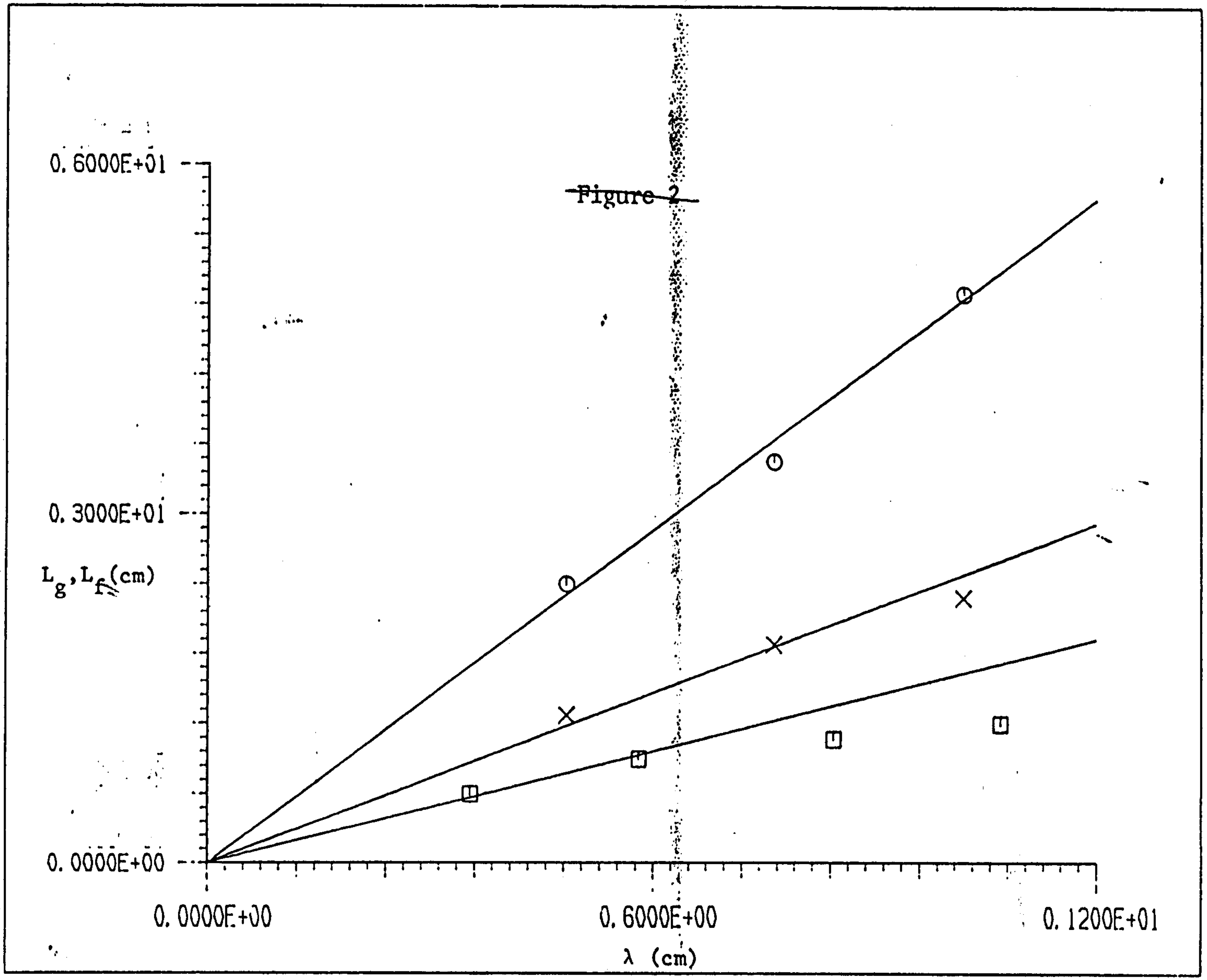

Figure 4. Integral scale versus Taylor microscale (data of Comte-Bellot and Corrsin 1971) (50.8 mm grid, x, lateral, o, longitudinal, 25.4 mm grid, ★ , lateral.

TABLE I

	Parameters	Spectrum
Taylor	u^2, λ	$E_{11}^{(1)}/u^2\lambda$ vs $k\lambda$
von Karman-Howarth	u^2, ℓ*	$E_{11}^{(1)}/u^2\ell$ vs $k\ell$
Kolmogorov	ϵ,ν	$E_{11}^{(1)}/\epsilon^{1/4}\nu^{5/4}$ vs $k\eta_K$**

* $\ell \equiv u^3/\epsilon$

** $\eta_K \equiv (\nu^3/\epsilon)^{1/4}$

Of particular interest will be the degree to which the spectral data at different x/M can be collapsed, and especially the range of wavenumbers for which the scaling appears to be valid. Since the von Karman-Howarth scaling is proposed for only the energy-containing range, it should not successfully collapse the high wavenumber region (since the Reynolds number also varies with distance from the grid). The Kolmogorov scaling, on the other hand should collapse only the high wavenumber region, and not the low since it is based on the local similarity of the smallest scales of motion. The Taylor scaling proposed here, on the other hand, must be expected to collapse the entire spectral range from lowest to highest wavenumbers if the theory is correct.

This uniformity of collapse over all scales for the proposed theory will be seen to be the most useful test of the proposed theory's validity. The reasons for this can be seen by examining how the various scaling parameters vary with distance from the grid. All measurements can be shown to be consistent with a power law decay for the energy and for a square root growth of the Taylor microscale, i.e.,

$$u^2 \propto \left[\frac{x-x_o}{M}\right]^n \tag{13.1}$$

$$\lambda \propto \left[(x-x_o)/M\right]^{1/2} \tag{13.2}$$

where for comparison with the theory $x-x_o=U(t-t_o)$, U being the free stream speed. The rate of dissipation can thus be shown to be proportional to

$$\epsilon \propto \frac{du^2}{dx} \propto \left[\frac{x-x_o}{M}\right]^{n-1} \tag{13.3}$$

It follows immediately that the length scale of the von Karman-Howarth theory is given by

$$\ell \equiv \frac{u^3}{\epsilon} \propto \left[\frac{x-x_o}{M}\right]^{\frac{1}{2}n} \tag{13.4}$$

and the Kolmogorov microscale is given by

$$\eta_K \equiv \left[\frac{\nu}{\epsilon}\right]^{1/4} \propto \left[\frac{x-x_o}{M}\right]^{(1-n)/4} \tag{13.5}$$

The magnitude of the differences which can be expected can be estimated by taking a nominal value for the energy decay exponent as $n= -1.2$. Then

$$\ell \sim \left[\frac{x-x_o}{M}\right]^{0.4} \tag{13.6}$$

$$\eta_K \sim \left[\frac{x-x_o}{M}\right]^{0.55} \tag{13.7}$$

which can be compared to

$$\lambda \sim \left[\frac{x-x_o}{M}\right]^{0.5} \tag{13.8}$$

The three spectral theories to be evaluated can now be reduced to (for $n = 1.2$), Taylor scaling:

$$E_{11}^{(1)} \left(\frac{x}{M}\right)^{0.7} \text{ vs. } k_1\left(\frac{x}{M}\right)^{0.5} \tag{13.9}$$

von Karman-Howarth Scaling:

$$E_{11}^{(1)} \left(\frac{x}{M}\right)^{0.8} \text{ vs. } k_1\left(\frac{x}{M}\right)^{0.4} \tag{13.10}$$

Kolmogorov scaling:

$$E_{11}^{(1)} \left(\frac{x}{M}\right)^{0.55} \text{ vs. } k_1\left(\frac{x}{M}\right)^{0.55} \tag{13.11}$$

Since x/M varies by less than a factor of 10 in all the experiments, the difference between the Taylor scaling and the others will be less than 25%. Thus it will be most difficult to sort out these differences from just the quality of the collapse. The <u>uniformity</u> of the collapse required by <u>only</u> the theory proposed here is therefore an important and distinguishing feature.

The two sets of experimental data to be considered from Comte-Bellot and Corrsin (1971) were obtained in a 1.0x1.3 m wind tunnel at an airspeed of 10 m/s. Two grids of 25.4 mm and 50.8 mm mesh size were used corresponding to grid Reynolds numbers of 17,000 and 34,000 respectively. The one-dimensional spectrum was measured for the 25.4 mm grid at downstream positions corresponding to Ut/M of 45, 120, 240, and 385, and for the 50.8 mm grid measurements were reported at 42, 98, and 171. (The spectral data are tabulated by the authors as $E_{11}^{(1)}$ vs k_1 in Table II and the relevant data on scales in Table 4 of their paper.)

Figures (5)-(7) show plots of the logarithm of the spectrum and the logarithm of the dissipation spectrum (k^2 times the spectrum) versus the logarithm of wavenumber using the three nondimensionalizations for the 25.4 mm grid (for which x/M variation is the greatest). The spectra plotted in von Karman-Howarth variables (Figure 5) collapse equally well over the entire

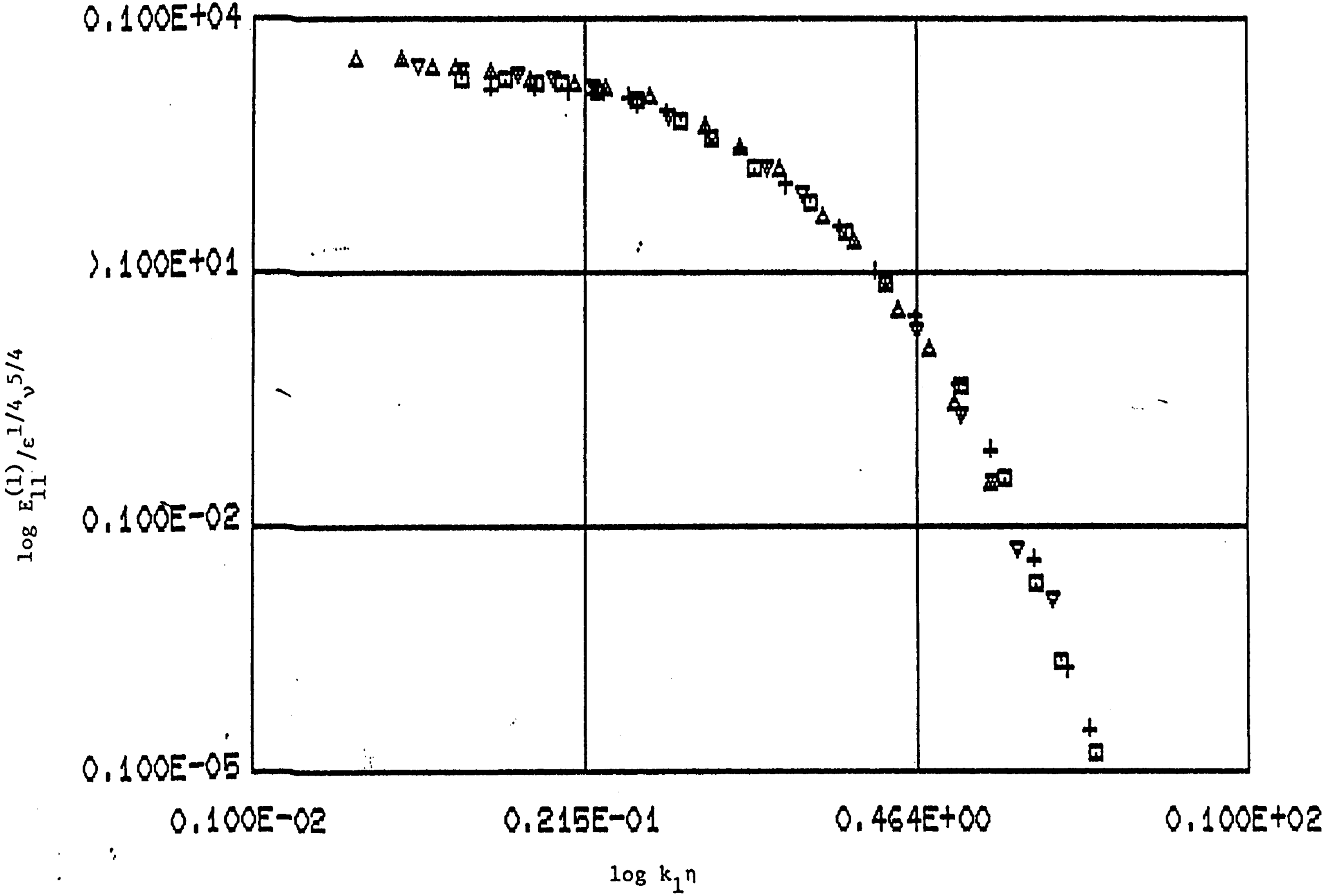

Figure 5(a) Spectra from 25.4 mm grid normalized in Kolomogorov variables. (data of Comte-Bellot and Corrsin 1971) (Δ – x/m = 45, ∇ – 120, – 240, + – 385).

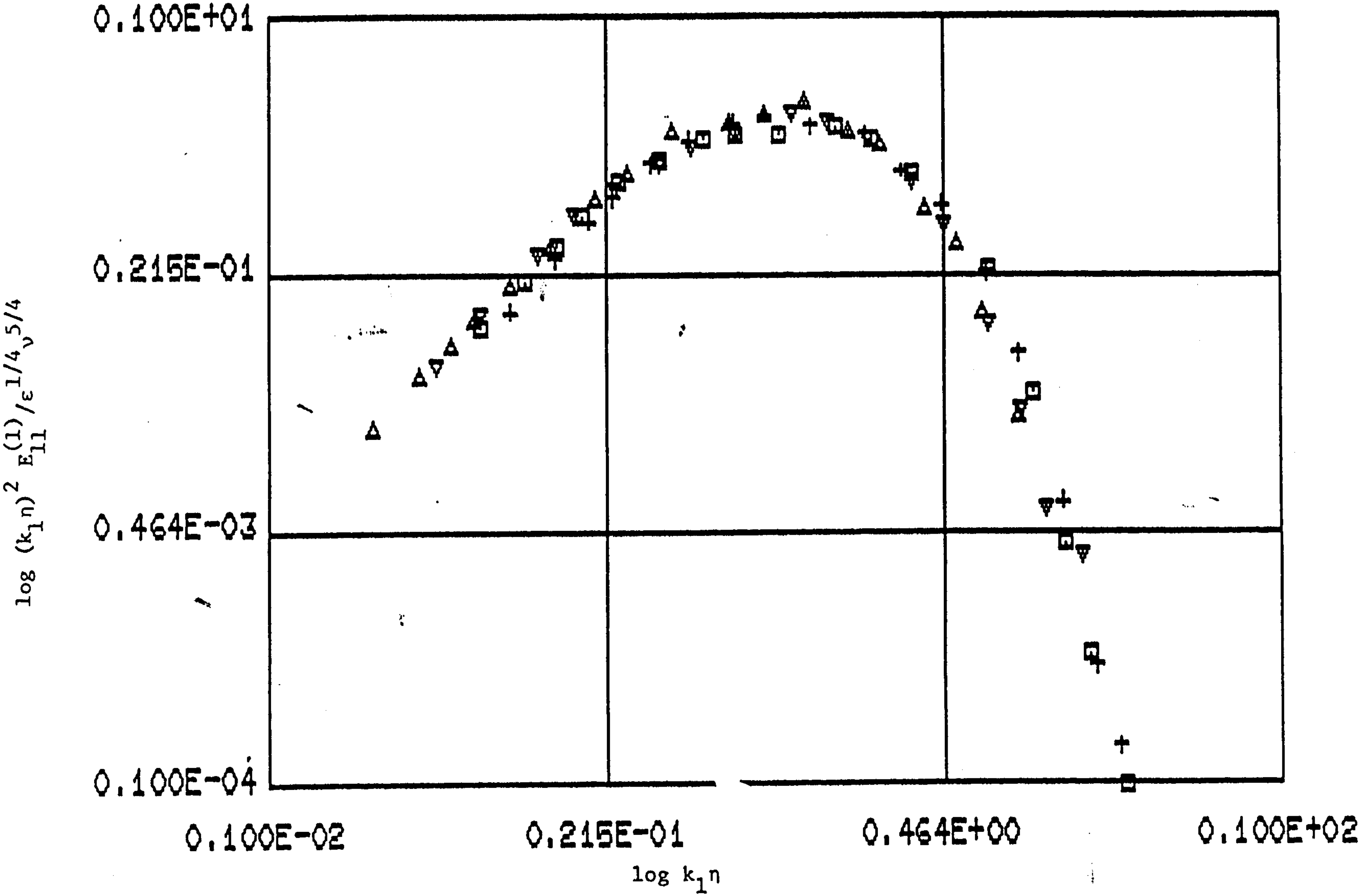

Figure 5 (b) Spectra from 25.4 mm grid normalized in Kolomogorov variables. (data of Comte-Bellot and Corrsin 1971) (Δ – x/m = 45, ∇ – 120, – 240, + – 385).

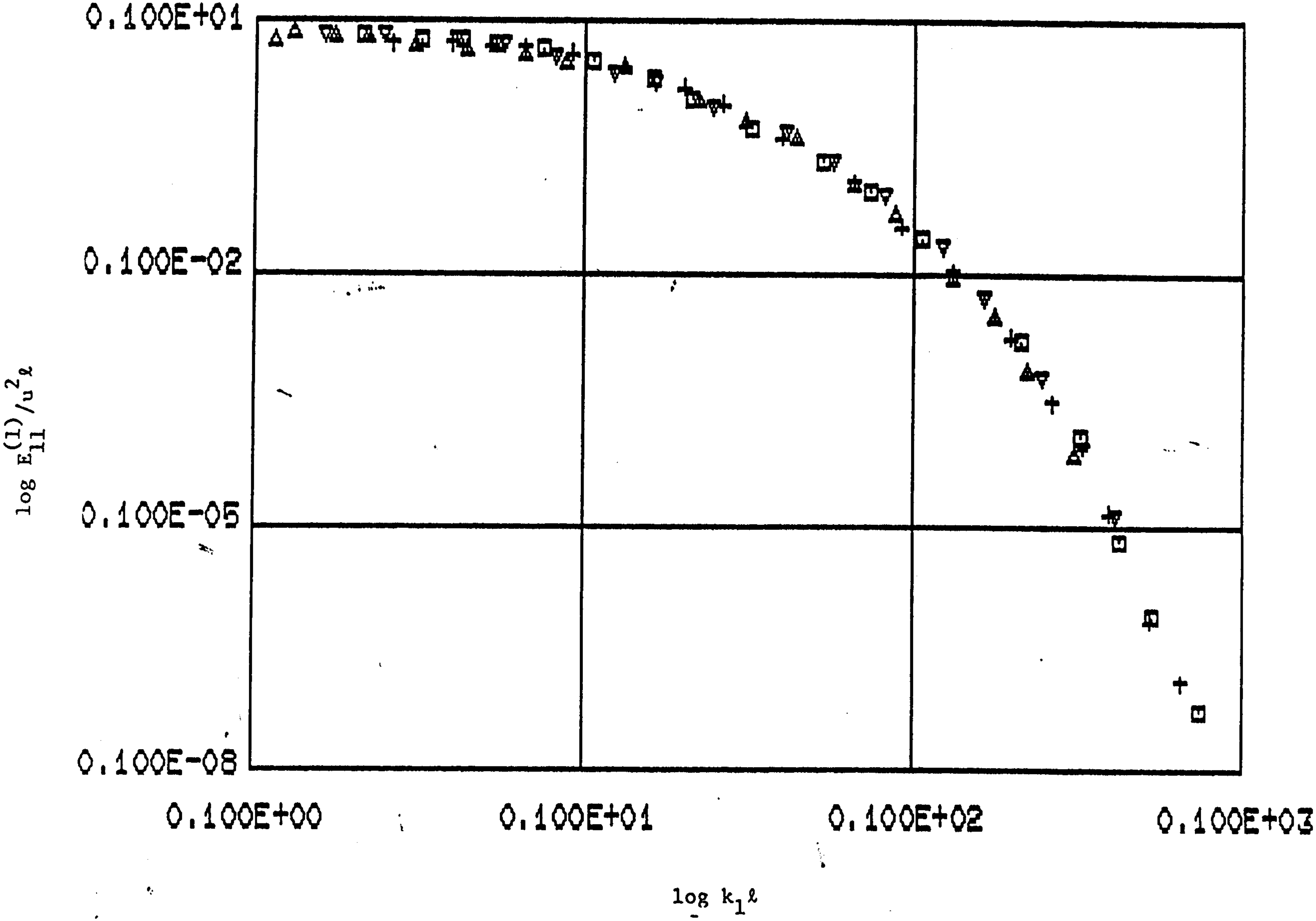

Figure 6 Spectra from 25.4 mm grid normalized in Von Karman–Horwarth (energy) variables (data of Comte-Bellot and Corrsin 1971) (Δ – x/m = 45, ∇ – 120, ⊡ – 240, + – 385).

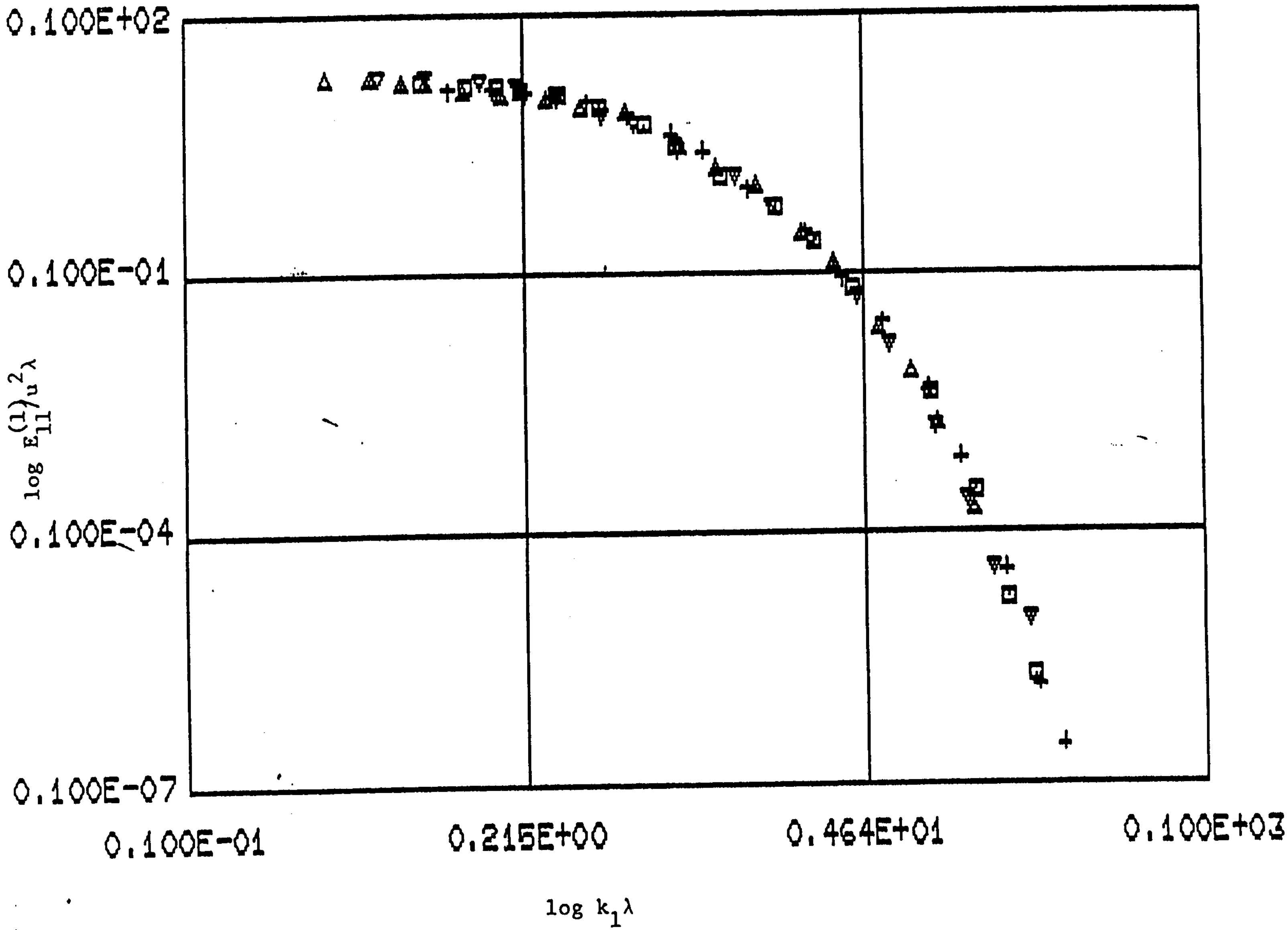

Figure 7(a) Spectra from 25.4 mm grid normalized in Taylor variables (data of Comte-Bellot and Corrsin 1971) (Δ – x/m = 45, ∇ – 120, ⊏ – 240, + – 385).

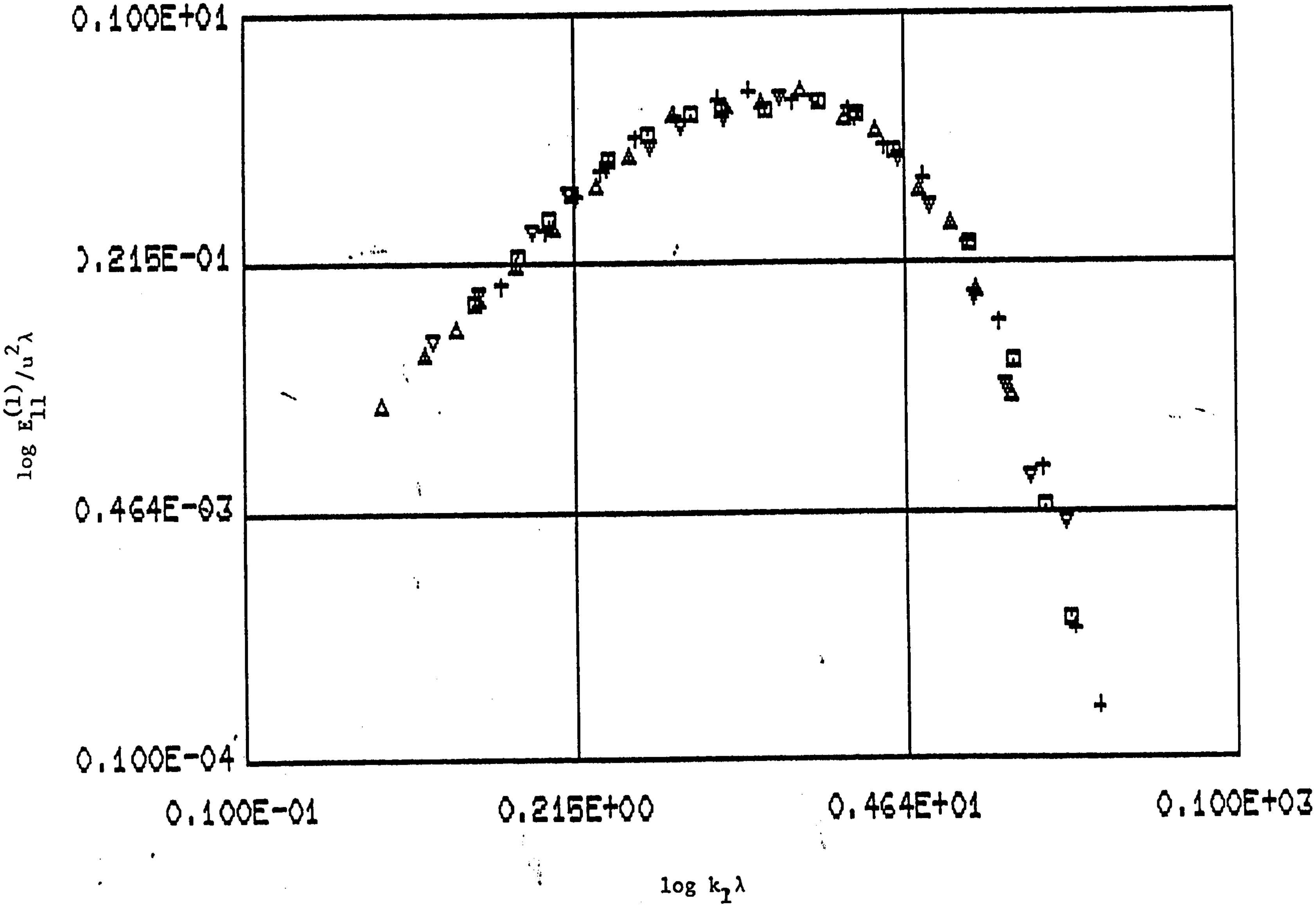

Figure 7(b) Spectra from 25.4 mm grid normalized in Taylor variables (data of Comte-Bellot and Corrsin 1971) (Δ – x/m = 45, ∇ – 120, ⊡ – 240, + – 385).

range of wavenumbers, and is, in fact, better at the high wavenumbers than is the Kolmogorov scaling (Figure 6). This is especially curious since the Kolmogorov scaling also works better at the lowest wavenumbers than at the highest. Both of these observations are contrary to previous expectations (see for example, Batchelor 1953).

The Taylor variable scaling (Figure 7) works well over the entire range of wavenumbers from well below ℓ^{-1} to above $3\eta^{-1}$. This is as predicted from the theory presented herein. Figure (8) shows the spectral data from the 50.8 mm grid plotted in Taylor variables, and the same collapse over all wavenumbers is indicated. Even without the relative comparisons among the three scalings, the collapse in Taylor variables over the entire spectral range of nearly nine decades and three decades in wavenumber is quite spectacular.

Figure (9) shows the spectra from both grids plotted together. It is clear that each experiment has its unique spectral shape. This is also consistent with the theoretical interpretation provided earlier which emphasized the dependence on initial conditions.

Prior to the experiments of Comte-Bellot and Corrsin, there were numerous efforts over the years to establish experimentally the self-preserving character of turbulence decay behind a grid. Particularly noteworthy were the results of Stewart and Townsend (1951) and Uberoi (1963). Monin and Yaglom (1975) provide an excellent review of these efforts. In spite of the lack of a theory of self-preservation which utilized the turbulence intensity and Taylor microscale (except for the final period of decay), these investigators did plot some of their spectral and two-point correlation results using u^2 and λ. Unfortunately, often the data from experiments at different initial conditions (especially grid mesh Reynolds number) were plotted on the same plot. As remarked earlier and demonstrated above, turbulence behind different

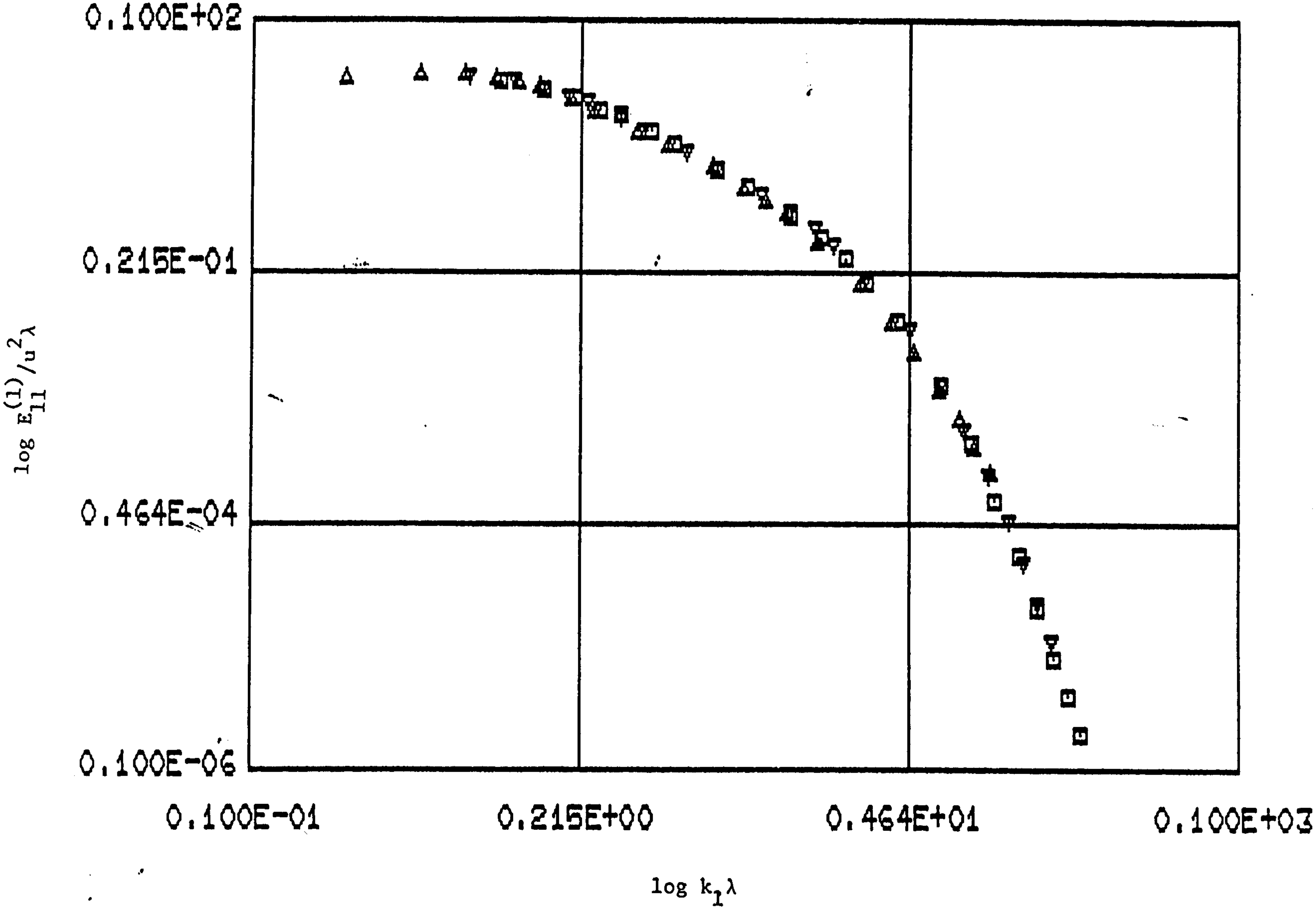

Figure 8. Spectra from 50.8 mm grid normalized in Taylor variables (data of Comte-Bellot and Corrsin 1971) (Δ – x/m = 42, ∇ – 98, $\square$– 171).

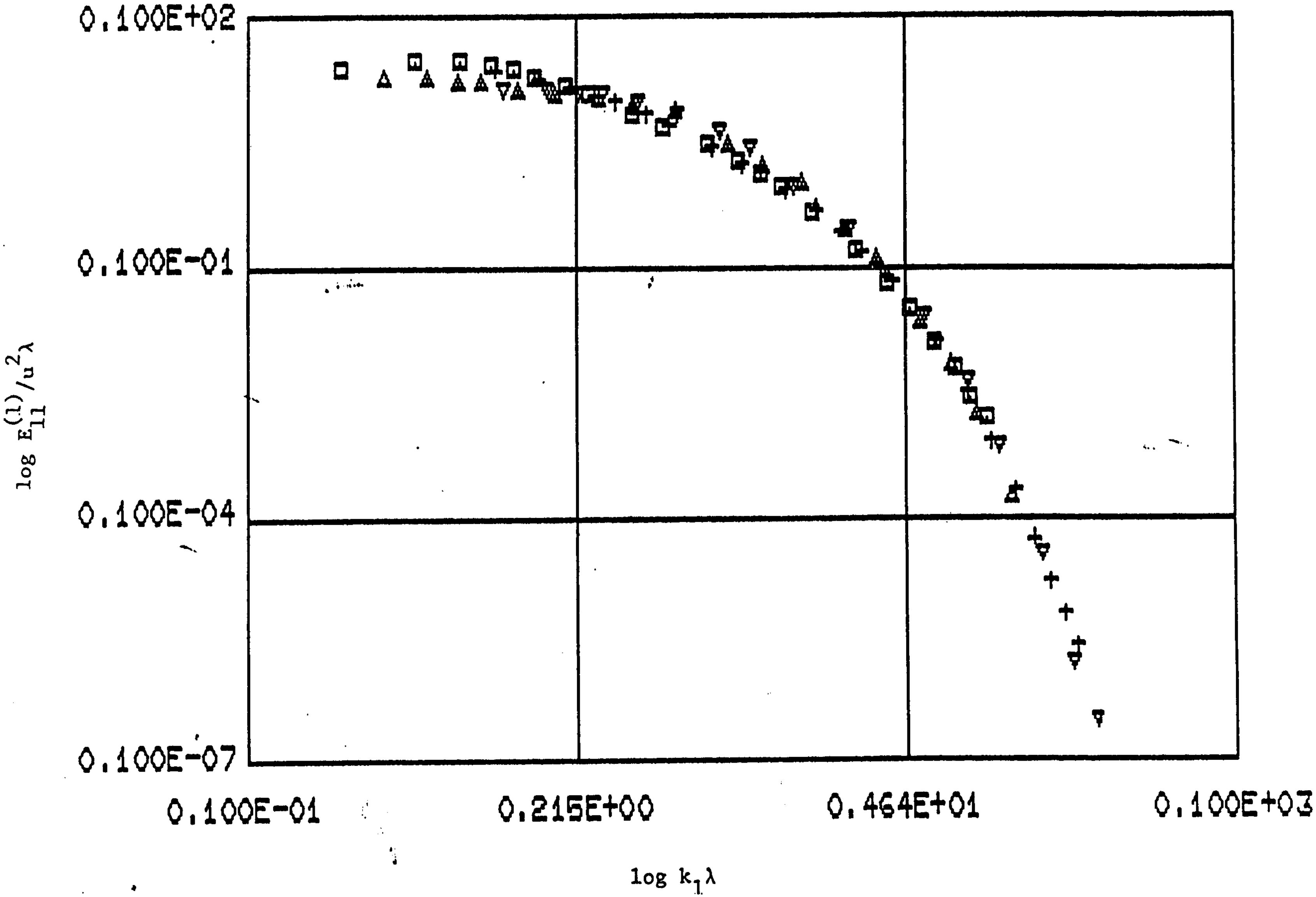

Figure 9. Spectra from both grids plotted in Taylor variables showing different spectral shape for each grid (data of Comte-Bellot and Corrsin 1971) (25.4 mm grid, Δ-x/m=45, ∇ – 385, 50.8 mm grid, □ – 42, + – 171).

grids or even similar grids at different Reynolds numbers is not universal. Therefore some of the scatter in these early experiments can be attributed to the differing shapes associated with different initial conditions.

Stewart and Townsend (1951) show plots of both the two point velocity correlation and the one-dimensional spectrum normalized by u^2 and λ. A cursory study of their results would appear to indicate the scaling to be less successful than for the later Comte-Bellot and Corrsin experiments. The reasons for this can largely be attributed to three causes: First, the plots include data from as close to the grid as x/M=10 whereas there is now general consensus that a self-preserving flow cannot be achieved before x/M=40. Second, the low turbulence intensities behind the grid and the broadband nature of the signals seriously stretched the limits of 1940-50's electronic technology, especially when the multiplication of signals was involved. When these factors are taken into account, the Stewart and Townsend measurements are not inconsistent with the proposed scaling, and had the present analysis been available might have been offered in support of it.

Uberoi (1963) presents measurements at three positions (x/M=48, 72, and 110) behind a grid at a single grid Reynolds number. The decay measurements are in relatively close agreement to those of Comte-Bellot and Corrsin. However, Uberoi provides extensive plots of the one-dimensional spectrum normalized by the turbulence intensity and the Taylor microscale calculated from the measured decay rate, and by the turbulence intensity and the integral scale (determined from the spectral intercept). Both of these scalings (and especially the former) are successful over the entire spectral range. This is, of course, the expected result since the integral scale and Taylor microscale should be (from the analysis presented herein) and are (as will be seen below) nearly proportional.

In summary, the proposed Taylor variable scaling is consistent with the measured spectral data. Aside from the inferences which can be made regarding the quality of the collapse at high and low wavenumbers, it will not be possible to further distinguish between the Kolmogorov and Taylor scalings until experiments are carried out in substantially longer wind tunnels.

14. The Velocity Derivative Skewness, the Spectral Transfer, and the Triple Velocity Correlation.

The first measurements of the velocity derivative skewness were due to Batchelor and Townsend (1947). Subsequently, there have been only a few attempts to document its variation with increasing distance behind a grid. There has been no shortage of theories, however.

The Kolmogorov (1941) theory for the universality of the small scale motions implies that the velocity derivative skewness in high Reynolds number turbulent flows should be constant. The modified Kolmogorov theory (Kolmogorov, 1963) which attempts to account for the internal intermittency of turbulent flows and the phenomenological theories of Corrsin (1962) and Tennekes (1968) imply that the skewness should increase with Reynolds number. While there is evidence that this is the case for turbulent shear flows, particularly in the atmosphere, there is no support for these theories from the grid turbulence experiments for fixed initial conditions. For example, [redacted]

Figure (10) summarizes the velocity-derivative skewness data as a plot of $S_{\partial u/\partial x}$ versus R_λ. (Note that this plot is an expanded version of that provided by Tavoularis et al. (1978) but with the data for fixed initial conditions identified as such.) The data of Bennett and Corrsin will be discussed later, but shows the same trend as the other experiments. While the

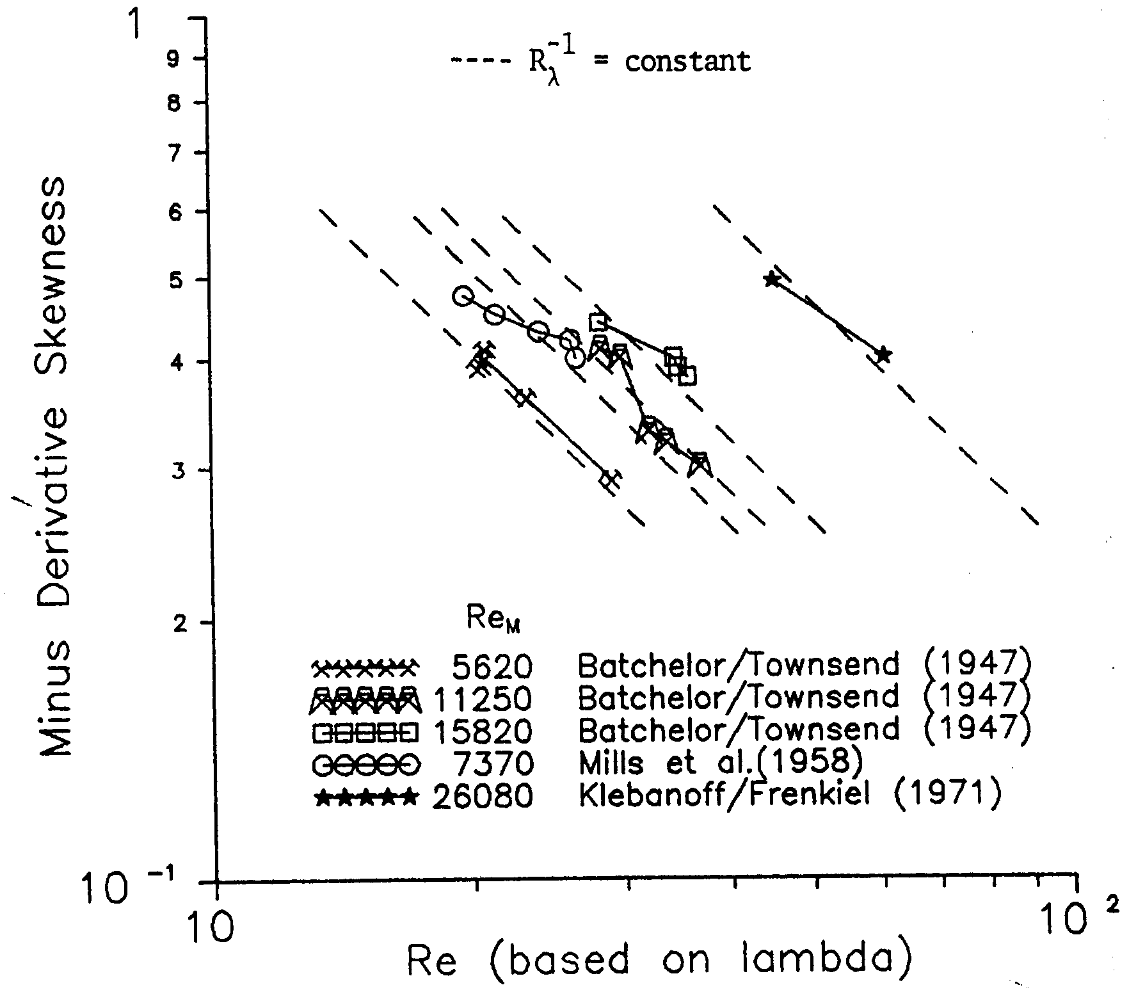

Figure 10. Velocity derviative skewness versus Reynolds number.

scatter is significant and the Reynolds number variation is limited (because of the limited distances from the grid as discussed earlier), the data are in reasonable agreement with the prediction of equation (6.4) that $S_{\partial u/\partial x} \sim R_\lambda^{-1}$. Also clearly evident is the dependence of the constant $R_\lambda S_{\partial u/\partial x}$ on the initial conditions. This was first noticed by Batchelor and Townsend (1947), but its significance was apparently not understood, perhaps because of the small variation of both R_λ and $S_{\partial u/\partial x}$ in their experiments.

Other features of the derivative skewness can also be seen in Figure (10). The first is that the derivative skewness has an apparent maximum value. Whether this is a consequence of the limited extent of the tunnel, or actually reflects a real limit (as suggested in Sections 8 and 17 below) needs further investigation. The second feature is the diminishing range of variation of both R_λ and $S_{\partial u/\partial x}$ for the higher Reynolds number experiments. This is consistent with the increase in the decay exponent toward $n = -1$ for which all of the data at the highest Reynolds numbers would be expected to lie within a cusp bounded at the top by the limiting value of derivative skewness given by equation (8.6).

The relation between the velocity derivative skewness and the spectral transfer has already been noted in equation (6.2). By substituting for $T(k,t)$ from equation (2.1), we can relate the velocity derivative skewness directly to the moments of the energy spectrum function by

$$S_{\partial u/\partial x} = S_1 + S_2 \tag{14.1}$$

where

$$S_1 = -\frac{3}{7}\frac{d}{dt}\int_0^\infty k_1^2 E_{11}(k_1,t)dk_1 \Big/ \left[\int_0^\infty k_1^2 E_{11}(k_1)dk_1\right]^{3/2} \tag{14.2}$$

and

$$S_2 = -2\nu\int_0^\infty k_1^4 E_{11}(k_1,t)dk_1 \Big/ \left[\int_0^\infty k_1^2 E_{11}(k_1)dk_1\right]^{3/2} \tag{14.3}$$

It is easy to see from equation (14.2) that S_1 is simply related to the time rate of change of the dissipation, i.e.

$$S_1 = -\frac{3}{7}\frac{d}{dt}\left[\frac{\epsilon}{15\nu}\right] \bigg/ \left[\frac{\epsilon}{15\nu}\right]^{3/2} \tag{14.4}$$

But for the power law decay of equation (10.1)

$$\epsilon = -\frac{3}{2}\frac{U^3}{M}\frac{d}{d(x/M)}\left\{\frac{u^2}{U^2}\right\} \tag{14.5}$$

since for the wind tunnel experiments d/dt = ud/dx. It follows after some algebra that

$$S_1 = \frac{30}{7}\left\{\frac{n-1}{n}\right\} R_\lambda^{-1} \tag{14.6}$$

which is exactly the first term of equation (6.6), thus the fact that the energy decay is described experimentally by a power law dictates that at least a part of the velocity derivative skewness scales in a manner consistent with the proposed self-preservation.

It is interesting to note that equation (14.1)–(14.3) can be written as

$$R_\lambda S = +\frac{30}{7}\left\{\frac{n-1}{n}\right\} - 2G \tag{14.7}$$

where

$$G = \lambda^4 \left.\frac{\partial^4 f}{\partial r^4}\right|_{r=o} \tag{14.8}$$

and where f is the longitudinal correlation coefficient. For n=–1, equation (14.7) reduces to the result of Batchelor and Townsend (1947). Figure 11 which is reproduced from their paper shows both the constancy of G and the dependence on the initial conditions. These results could be previously understood only by assuming that the decay exponent was given by n=–1, but are clearly consistent with the more general theory proposed here. Another immediate consequence of equation (10.4) and equation (8.6) is that SR_λ varies asymptotically as $R_M^{1/2}$, as suggested on empirical grounds by Batchelor and Townsend (1947). Table II summarizes their results and demonstrates that $SR_\lambda/R_M^{1/2}$ is constant at about 0.11 for their grid.

TABLE II

Variation of G and $R_\lambda S$ with R_M
(Data of Batchelor and Townsend 1947)

R_M	G	$R_\lambda S$	$SR_\lambda/R_M^{1/2}$
5444	8	7.4	0.10
10888	10	11.4	0.11
15392	11.5	14.4	0.12
30784	14	19.4	0.11

Table III shows the results of an attempt to use the grid data of Comte-Bellot and Corrsin (1971) to calculate S_2 from equation (14.3). While there is considerable scatter in the computed values of $S_{\partial u/\partial x}R_\lambda$, this is largely due to systematic errors in the spectral data at the very highest wavenumber which are perhaps attributable to the method for thermally-compensating the constant current hot-wires used. Thus even these data are consistent with the proposed scaling. It is, however, worth noting that the values of the velocity derivative skewness computed using the spectra are considerably higher than those reported using direct measurements. Unfortunately, there appears to be no results where both spectral and direct derivative measurements can be used to check the internal consistency of the data.

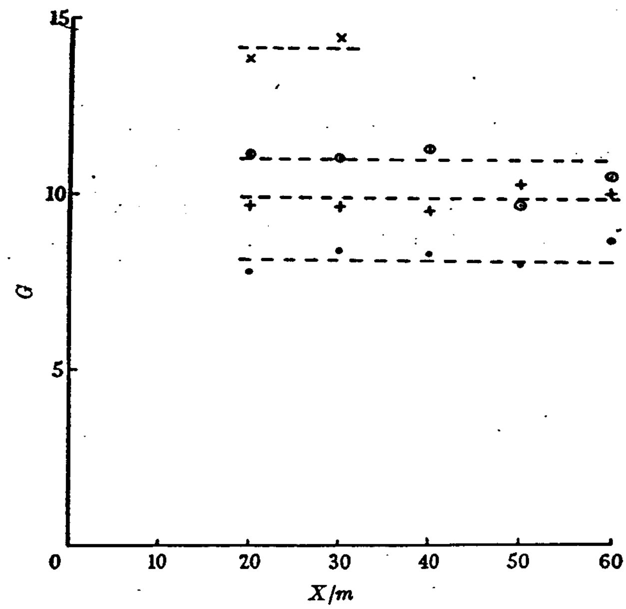

Figure 11. Variation of G with x/M for various R_M $\cdot$0-R_M = 5,444, + - 10,000, 0 - 15,392, x- 30,784) (from Batchelor and Townsend 1947.)

TABLE III

Calculated Velocity Derivative Skewness
(Comte-Bellot and Corrsin 1971), one-inch grid

x/M	R_λ	S_1R_λ	S_2R_λ	$S_{\partial u/\partial x}R_\lambda$	$S_{\partial u/\partial x}$
45	48.6	7.9	−36.2	−28.3	−0.58
120	41.1	7.9	−32.8	−24.9	−0.61
240	38.1	7.9	−43.6	−35.7	−0.93
385	36.6	7.9	−42.6	−34.7	−0.96

There have been only a few attempts to measure the spectral transfer (Uberoi, 1963, van Atta and Chen, 1969, and Helland et al. (1977). Only Uberoi presents data at several downstream distances so that scaling arguments can be evaluated. The spectral transfer function collapses reasonably well when normalized by u^3 and plotted as a function of wavenumber normalized by integral scale. This is not in accord with the theory presented here which requires normalization by $R_\lambda^{-1}u^3$ instead of u^3. However, for this experiment the ratio of u^3 and $R_\lambda^{-1}u^3$ is proportional to $(x/M)^{0.1}$. Therefore, the ratio (and R_λ itself) varies by only 9% over the entire range of the data. Thus it is not possible to distinguish between the two scalings, both collapsing the data to within the experimental error. It is interesting to note that Helland et al. (1977) using the data of Schedvin et al. (1974) introduced on empirical grounds a self-preserving form of the spectrum to compensate for a very limited range of downstream positions (x/M = 37-41). Their empirical scaling can be shown to be indistinguishable from that proposed here over the range of data considered.

The measurements of Van Atta and Chen (1969) of the one-dimensional spectra and of the one-dimensional triple spectra were made at only a single position (x/M=40) downstream of two geometrically similar grids operated at identical Reynolds numbers. Since only a single downstream location was used and since the Reynolds numbers (all of them) were the same for both grids, the results are not useful for verifying the self-preservation theory presented here. However, unlike the experiments of Comte-Bellot and Corrsin where identical grids at different Reynolds numbers gave spectra which were distinctly different, Van Atta and Chen's spectra collapsed perfectly. (Note that the choice of L_f, λ or η is irrelevant since the Reynolds number was the same.) This contrast highlights the fact that the decay is determined by the conditions as measured by geometry and Reynolds number.

Finally there have been several attempts to measure the triple correlation functions directly (Stewart and Townsend 1951, Mills et al. 1958, Van Atta and Chen 1969, Frenkiel and Klebanoff 1971). The earlier measurements appeared to suffer from high-pass filtering problems at the largest wavenumbers while the latter report insufficient downstream positions for a definitive test of the various scaling laws. Nonetheless, all of the data show an increase in the triple correlation with decreasing R_λ as predicted. Stewart and Townsend even show that approximate length scale is proportional to $x^{1/2}$, even though they are unable to relate it directly to λ. Table IV summarizes the value of the maximum from the data of Mills et al. which is consistent (to with experimental error) with the proposed scaling which requires $k_{max}R_\lambda$ = const.

TABLE IV

Maximum of Triple Velocity Correlation (Mills et al. 1958)

x/M	R_λ	k_{max}	$k_{max}R_\lambda$
17	26.5	0.056	1.48
32	26.1	0.062	1.63
69	20.9	0.067	1.41

In summary, in spite of the very limited range of R_λ for a given set of initial conditions, there is considerable reason to believe the proposed scaling of the non-linear terms to be correct. While there has been a tendency to dismiss the variations in the measured values of the velocity derivative skewness as scatter in the data, there have been clear and persistent warnings from the experimental community for the past 30 years that the trends in the data were not in accord with previous theories. Had the present theory been available when those measurements were made, it is reasonable to expect that all of the results would have been interpreted as confirming it.

15. Overall Assessment of Experimental Evidence

In spite of the wealth of experimental data obtained in grid turbulence experiments, there is surprisingly little which can be used to evaluate the theory proposed herein. And even the data which is relevant is taken over such a limited range of distance from the grid that it can not distinguish definitively between the competing theories. In view of this, the most that can be said is:

(i) [redacted]

(ii) [redacted]

(iii) [redacted]

It has long been recognized that experiments are most useful when designed to test between conflicting theories and hypotheses. At the very least the theory proposed herein would seem to provide a strong rationale for a new generation of experiments (or numerical simulations) especially designed to monitor the evolution of the turbulence at great distance from the grid. Such experiments should include measurements of the velocity derivative skewness and spectral transfer with careful attention to insuring consistency between the two. Only when these second (or third) generation experiments are performed will a more definitive verification be possible.

PART III – FURTHER CONSEQUENCES OF FULL SELF-PRESERVATION

16. Invariants of the Decay Process

There have been several attempts to establish theoretically the existence of integral invariants of the decay of homogeneous, isotropic turbulence. The first of these was due to Loitsiansky (1939) who argued that the fourth integral moment of the longitudinal velocity correlation should be an invariant of the motion, i.e.,

$$\int_0^\infty r^4 B_{LL}(r)dr = I_4 \tag{16.1}$$

Necessary conditions were the existence of the integral (which required that $B_{LL}(r)\to 0$ at least as fast as r^{-5} as $r\to\infty$) and that the two point triple correlation $B_{LL,L}(r)\to 0$ faster than r^{-4}.

Subsequently, Proudman and Reid (1954) and Batchelor and Proudman (1956) showed that these conditions were not satisfied in general (except in the final period of decay which will be discussed in the next section). In particular they showed that because of the influence of the pressure coupling terms in an incompressible flow, $B_{LL,L}(r)\to 0$ as r^{-4}, even when the turbulence was generated at the initial instant so that the velocity correlations rolled off exponentially with r. Saffman (1967a) considered a field of turbulence generated at an initial instant by a distribution of random impulsive forces. Such a distribution was shown to correspond at the initial instant to a turbulent field for which all vorticity correlations roll-off exponentially with r. By a series of arguments paralleling those of Batchelor and Proudman he was able to show that the second integral moment of the velocity correlation was an invariant of the motion, i.e.,

$$\int_0^\infty r^2 B_{LL}(r)dr = I_2 \tag{16.2}$$

The existence of integral invariants has a direct connection to the character of the velocity spectrum at small wavenumbers. If the Loitsiansky integral were an invariant, the spectrum at small wavenumbers can be represented as

$$E(k) = C_1 k^4 \tag{16.3}$$

where C_1 is also an invariant. If the Saffman integral represented an invariant, then

$$E(k) = C_2 k^2 \tag{16.4}$$

where C_2 is invariant. Either condition implies that the spectrum at small wavenumbers is invariant throughout the decay process, and leads to the notion of the 'permanence of the large eddies'.

The interrelation of self-preservation and the existence of an integral invariant have been realized from the beginning (Batchelor 1948, Saffman 1967b). Unfortunately, none of the measured decay rates (except in the final period) were consistent with both self-preservation and the proposed invariants. However, as pointed out by Monin and Yaglom (1975), there is no reason to believe that idealized models based on assumed behavior of turbulence in an infinite environment should be able to describe real turbulence, particularly that behind a grid in a wind tunnel.

It has already been established in the preceding section that the turbulence behind a grid is self-preserving at all scales. Moreover, this self-preservation has been shown to be consistent with the equations of motion for the flow. [redacted] Formally, the appropriate exponent m is sought for which

$$\int_0^\infty r^m B_{LL}(r)dr = I_m \tag{16.5}$$

where I_3 is presumed constant throughout the decay.

Since from self-preservation,

$$B_{LL}(r) = u^2\tilde{f}_L(\eta), \ \eta = r/\lambda \tag{16.6}$$

it follows that

$$\left[\lambda^{m+1}u^2\right] \int_0^\infty \eta^m \tilde{f}_L(\eta)d\eta = I_m \tag{16.7}$$

The integral is constant throughout the decay. Therefore,

$$\left[\lambda^{m+1}u^2\right] \sim \text{constant} \tag{16.8}$$

However, from equations (4.5) and (4.12),

$$u^2 \sim t^n \ ,$$

and

$$\lambda^2 \sim t^{1/2}$$

Therefore

$$t^{(m+1)/2} \quad t^n \sim \text{constant} \tag{16.9}$$

which implies that

$$m = -2n-1 \tag{16.10}$$

The experimental data of Comte-Bellot and Corrsin (1971) have been shown to be consistent with $n \approx -1.2$. Therefore for this data set $m \approx 1.4$ and

$$\int_0^\infty r^{1.4} B_{LL}(r)dr = I_m \tag{16.11}$$

represents an invariant of the decay. In view of the non-universality (i.e. dependence on initial conditions) demonstrated for the decay, there is no reason to believe that other data sets will exhibit this value. Note that a consequence of this fractional power is that the spectrum at the origin is not analytic.

On the other hand, the value of n = -1 appears to characterize the very high Reynolds number experiments of both Kistler and Vrebalovich (1966) and Schedvin et al. (1974). For n = -1, m = +1 so that the appropriate invariant is

$$\int_0^\infty r\, B_{LL}(r)dr = I_m \tag{16.12}$$

Whether or not this represents the asymptotic behavior of grid turbulence at high Reynolds number (see Section 17) must remain a matter for speculation pending further measurements.

It will be interesting to see whether an idealized model of the turbulence at the initial instant can be shown to be consistent with the fractional powers which characterize most experiments. Such a model would probably recognize the unique vortical character of grid-generated turbulence, and would most probably attribute the differences behind geometrically similar grids to the changes in these vortical structures with Reynolds number.

17. The Final Period of Decay (or Turbulence at Low Reynolds Number)

During the final period of decay or at low turbulence Reynolds numbers, an additional constraint can be added to the spectral equations considered earlier; in particular, the constancy of the Loitsiansky integral given by equation (16.1). Batchelor and Proudman (1956) argue that while the

Loitsiansky integral is not an invariant of the decay (as originally proposed), it is constant during the final period of decay. This is because viscosity forces B_{LL} to roll off sufficiently rapidly.

The final period analysis presented here uses the spectral equation analyzed earlier <u>including the spectral transfer term</u>, together with either equations (16.1) or its spectral equivalent, equation (16.3). Substituting the self-preserving forms of equations (3.1),(3.3) and (4.2) into equation (16.2) and using $L \sim \lambda$, yields

$$\left[u^2 \lambda^5\right] f(\eta) = C\eta^4 \tag{17.2}$$

Since C is a constant it immediately follows that

$$u^2 \lambda^5 \sim \text{constant} \tag{17.3}$$

The time dependence of λ and u^2 has already been determined by equations (3.11) and (4.5). These results can be satisfied together in the equation (17.3) only if

$$n + \frac{5}{2} = 0 \tag{17.4}$$

Thus for the final period of decay

$$u^2 \sim t^{-5/2} \tag{17.5}$$

and the decay exponent is uniquely determined.

The $t^{-5/2}$ decay law is not new having been previously derived theoretically and confirmed experimentally (Batchelor and Townsend 1948, Bennett and Corrsin 1978). (Batchelor (1948) even used the same arguments.) What is different here is that it has been derived <u>without assuming the negligibility of the spectral transfer terms</u> (or inertial terms). All previous analyses were based on linearized equations of motion. This is important since Bennett and Corrsin (1978), while confirming experimentally the predictions of the theory with regard to the -5/2 law and $\lambda^2 \sim \nu t$, showed that the inertial terms (as measured by the velocity derivative skewness) were certainly not negligible and in fact, increased in importance throughout the decay. Thus the analysis here would appear to have resolved this paradox by eliminating the necessity of neglecting the inertial terms.

There remains, however, another paradox. In arguing that the Loitsiansky integral was constant during the final period of decay, Batchelor and Proudman (1956) resorted to the linearized dynamical equations to argue that viscosity caused the correlation function to roll off exponentially, thereby insuring the convergence of the Loitsiansky integral. The apparent consistency of the theory presented here and its agreement with the experimental observations would argue for the constancy of the Loitsiansky integral during the final period of decay, independent of assumptions about the negligibility of the inertial terms. On the other hand, the fact that the constancy of the Loitsiansky integral enforces a -5/2 decay, lends considerable support to Batchelor and Proudman's argument that it is <u>not</u> constant throughout most of the decay process, since the data clearly do not show such a decay except during the final period. [redacted]. Further evidence for this need will be shown below in the consideration of the velocity derivative skewness.

The measurements of Tavoularis et al. (1978) in low Reynolds number grid-generated turbulence show that the velocity derivative skewness continues to increase with decreasing R_λ (as required for self-preservation) until $R_\lambda \simeq 2$, after which it decreases rapidly to zero. Thus for values of R_λ between about 2 and 30, the turbulence decays as $t^{-5/2}$, but with the <u>increasing</u> importance of the non-linear terms. Only for values of $R_\lambda < 2$ where viscosity dominates the problem does the linearized theory apply.

From the above it is clear that self-preserving solutions to the dynamical equations are possible long before the inertial terms are negligible. It is important to note, however, that the flow has evolved from one self-preserving form to another, from an initial region in which the energy decays as $\sim t^{-1.2}$ to one in which it decays as $t^{-5/2}$. In both regions, the Taylor microscale varies as the square root of t, but with different coefficients. Figures (12) and (13) taken from Bennett and Corrsin (1975) make clear this evolution from one period to another. Note especially the two linear regions in the plot of λ^2 versus x/M ($\sim$t) in Figure (13), the second corresponding to the coefficient of equation (4.12) modified to reflect $n = -5/2$. Also of interest is the plot of the longitudinal correlation versus $r\lambda$ in Figure (14) which shows a self-preserving shape distinctly different from the Gaussian shape which would be appropriate only where the linearized theory applies. As equation (6.4) makes clear, the spectrum (and its transformed counterpart, the correlation) should differ significantly from the initial period as well.

It is clear from the above that the final period of decay is not a single dynamical region as previously thought. Rather, [redacted] These might be distinguished as the 'Loitsiansky' period of decay, and the 'viscous' period respectively. It is the viscous period which has been the subject of the linearized analyses.

It is interesting to speculate whether there is a maximum value which the

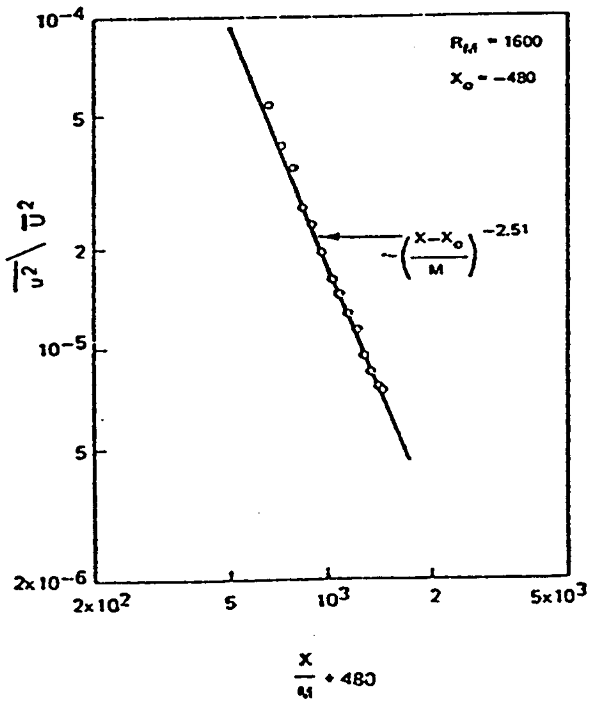

Figure 12. Turbulence intensity along centerline of tunnel (from Bennett and Corrsin 1978).

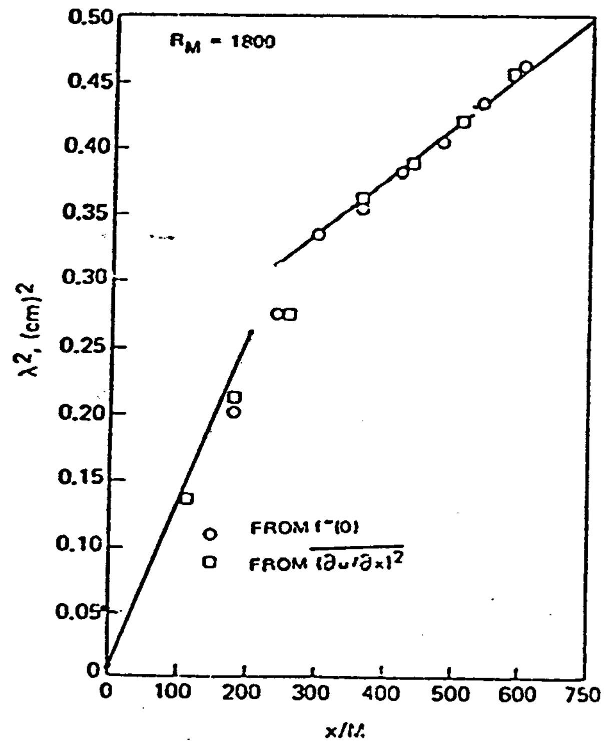

Figure 13. Taylor microscale variation (from Bennett and Corrsin 1978).

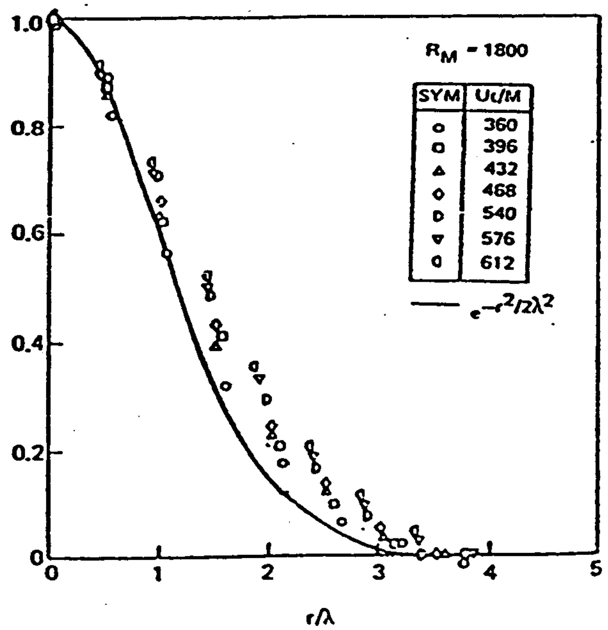

Figure 14. Longitudinal correlation coefficient for long decay times (from Bennett and Corrsin 1978).

velocity derivative skewness can obtain, and whether such a maximum can explain the transition from the initial period to the Loitsiansky period to the final 'final' period of decay. Certainly the breaking of waves on the sea is a direct consequence of the non-linear terms exceeding a critical value. Perhaps a similar critical value is reached by the decaying turbulence through the process of vorticity concentration in whatever structures are present.

19. Summary and Conclusions

A theory of self-preservation for decaying isotropic turbulence has been proposed in which the kinetic energy and the Taylor microscale are the appropriate scaling parameters for all scales of motion. The kinetic energy decays as a power law in time, the Taylor microscale grows as the square root of time, and the non-linear terms (as measured by the spectral transfer or the velocity derivative skewness) vary during decay as R_λ^{-1}. The coefficients and decay exponent are determined by the initial conditions. The theory is valid at finite Reynolds numbers, although different Reynolds number regimes appear to exist. The theory appears to be consistent with the wealth of experiment data.

The theory is used to reexamine the nature of integral invariants in decaying isotropic turbulence, and the appropriate invariant is seen to be governed by the decay exponent, and therefore by the initial conditions. Also the $t^{-5/2}$ energy decay usually associated with the final period of decay is shown to result directly from self-preservation and the assumed constancy of the Loitsiansky integral, with no assumptions required about the negligibility of the inertial terms. From this, two regions of $t^{-5/2}$ behavior were identified, the 'Loitsiansky period' where the inertial terms were important, and a 'viscous' period where they were negligible.

Finally, the relation of the proposed self-preservation of isotropic turbulence to the Kolmogorov theory for high Reynolds number turbulence was examined. The two theories were shown to be compatible in the limit of infinite Reynolds number, and a consequence of the latter is that the turbulence energy decays as t^{-1} in this limit.

It has been the custom since the introduction of Kolmogorov's ideas (of the statistical independence of the small scale motions) to view turbulence as being made up of large and small scales of motion which only weakly interact through an energy cascade. The results of this paper would certainly appear to call this view into question, at least at finite Reynolds numbers. For isotropic turbulence (at least), the local similarity theory of small scale turbulence (which may still describe many shear flows), gives way to a higher principle -- that of self-preservation at all scales. Only in the limit of infinite Reynolds number is Kolmogorov's theory recovered.

It is also clear from the arguments presented here that the large scale structures must play a crucial role since they determine the integral invariants which appear to control (or perhaps reflect?) how the decaying turbulence moves from one self-preserving regime to another. Kerr (1987) has suggested that the evolution of the large scale structures is itself the energy cascade. This is consistent with the concept of a self-preserving flow where large eddies evolve by vortex concentration and breakdown. Such a process has recently been demonstrated by Glauser and George (1987) for an axisymmetric jet mixing layer, and may manifest itself in different ways for different flows. The initial conditions come into play by dictating how this evolution begins, and the structures are then locked into this evolution until the physics dictate otherwise, initiating another self-preserving state of evolution and decay.

In fact, if one considers the lifetime of a coherent structure in decaying turbulence to be characterized by the time scale, ℓ/u, then the distance behind the grid for this life cycle to be completed is proportional to $U\ell/u$. Since ℓ is typically of order M and $u/U \sim 10^{-2}$, this distance can be measured in hundreds of mesh lengths from the grid. Thus, the decay of turbulence behind a grid may, in fact, represent just a random collection of single vortical structures (perhaps the generalized 'vortrons' suggested by Moffatt 1987) which simply evolve until diffused by viscosity. Such an evolution would appear to be quite consistent with the rather elegant idea of a flow in which all terms in the dynamical equations remain in balance. The role of viscosity in cutting vortex lines as the Reynolds number is reduced could explain the change in character of the invariants as the final period of decay (Loitsiansky period) is entered. The near factor of two increase in the energy decay exponent (from - 1.2 to -2.5) is intriguing, and perhaps indicates the breakdown of one structure into several smaller ones. If so, perhaps the first real links between coherent structures and the dynamics of turbulence will have been found.

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