OUTLINE OF A UNIFIED SIMILARITY THEORY FOR TURBULENT CONVECTION NEXT TO VERTICAL SURFACES

William K. George, Jr.

Department of Mechanical Engineering
State University of New York at Buffalo
Buffalo, New York 14214

ABSTRACT

A similarity theory is presented for the fully developed turbulent boundary layer next to vertical surfaces where heat transfer effects are important. The theory is applicable over the entire domain from pure forced to pure natural convection. Particular limiting cases considered are: (i) pure forced convection, (ii) mixed but primarily forced convection and (iii) pure natural convection.

The unifying characteristic of these flows is seen to be the existence of a constant heat flux layer near the wall. In the absence of externally imposed pressure gradients other than the hydrostatic gradients, the inner part of the flow is shown to be completely characterized by α/u_*, the conduction length scale, ν/u_*, the viscous length scale, k, a roughness height and a "Monin-Obukov type" length scale L_1 defined by $L_1 = u_*^3/g\beta F_o$ where u_* is the friction velocity, $g\beta$ is the buoyancy parameter, and F_o is the wall heat flux, q_w, divided by the thermal capacity per unit volume, ρC_p.

For distances from the wall such that yu_*/α, yu_*/ν, and y/k are sufficiently greater than unity, the mean velocity and temperature excess (from infinity) are given by

$$\frac{dU}{dy} = \frac{u_*}{y} \; f_1(y/L_1)$$

and

$$\frac{dT}{dy} = \frac{T_*}{y} \; g_1(y/L_1)$$

where $T_* \equiv F_o/u_*$, and f_1, g_1 are universal functions.

In the limit of forced convection or for adiabatic surfaces where $g\beta F_o \to 0$, $L_1 \to \infty$, $y/L_1 \to 0$, the functions f_1 and g_1 can be expanded about zero to yield log-linear profiles for the velocity and temperature. In the limit of pure natural convection where $y/L_1 \to \infty$, it can be shown that $f_1 \sim (y/L_1)^{1/3}$ and $g_1 \sim (y/L_1)^{-1/3}$.

An attempt is made to evaluate the universal constants from the available data. Finally, the significance of these results to experimenters and numerical modelers is assessed.

NOMENCLATURE

c_{fs}	= Prandtl number dependent coefficient (eq. 13) for smooth walls
c_{fr}	= defined by eq. (14)
C_p	= specific heat at constant pressure
F_o	= defined by eq. (6)
g	= gravitational acceleration
f_o	= dimensionless velocity gradient in boundary layer
f_1	= universal velocity gradient in intermediate layer
f_2	= universal velocity gradient in free convection limit
g_o	= dimensionless temperature gradient in boundary layer
g_1	= universal velocity temperature gradient in intermediate layer
g_2	= universal temperature gradient in free convection limit
k	= roughness height
L_1	= convection length scale defined by eq. (9)
q_w	= wall heat flux
T	= local mean temperature
t	= fluctuating temperature
T_∞	= temperature at infinite distance from the wall
T_*	= temperature scale defined by eq. (12)
$\overline{U},\overline{V}$	= mean velocity components
u,v	= fluctuating velocity components
$\overline{U}_\infty$	= velocity at infinite distance from the wall
u_*	= friction velocity defined by eq. (5)
x	= coordinate measured along wall
y	= coordinate measured perpendicular to wall
α	= thermal diffusivity
β	= coefficient of thermal expansion
δ	= outer length scale similar to boundary layer thickness
ΔT	= local mean temperature minus T_∞

ΔT_w = difference between temperature at wall and at infinite distance from the wall

η_v = viscous length scale defined by eq. (7)

η_T = thermal length scale defined by eq. (8)

ν = kinematic viscosity

ρ = density

τ_w = wall shear stress

ζ = dimensionless coordinate $\zeta = y/L_1$

INTRODUCTION

The problem of the turbulent convection of heat from vertical surfaces is one of the important problems of heat transfer because of its widespread occurrence, both in engineering practice and in nature. Turbulent convection can be loosely grouped into three groups: natural, mixed, and forced. Natural convection is entirely driven by the buoyancy arising from differentially heated parcels of fluid while forced convection is entirely driven by an imposed flow. In mixed convection, both a forcing flow and the buoyancy of the fluid are important.

In spite of the importance and general nature of the problem of turbulent mixed convection next to vertical surfaces, there has been little progress toward understanding the dynamics of these flows. Progress has been made, however, in understanding the problems of pure forced convection (c.f. Monin and Yaglom (1)) and pure natural convection next to vertical surfaces (George and Capp (2)).

The problem of forced convection next to smooth and rough surfaces has been the subject of numerous theoretical and experimental investigations. These have been discussed in detail by Monin and Yaglom (1). By hypothesis, the buoyancy has no influence on the dynamics of the motion. As a Reynolds number and a Peclet number characteristic of the turbulence become very large, it is easy to show that the equations of motion become independent of the viscous and conduction terms over most of the flow. The no-slip and thermal boundary conditions at the wall, however, can be met only if there is a region of the flow near the wall in which these terms are retained. These considerations lead immediately to a picture of a turbulent boundary flow in which two primary regions are recognized: an outer region in which the mean convection of momentum and heat is balanced only by the turbulent Reynolds stress and turbulent heat flux respectively; and, an inner layer in which mean convection terms are negligible and the turbulence terms are balanced by the viscous and conductive terms. Of particular importance to theoreticians and experimentalists alike is the existence of the inertial or logarithamic sublayer which joins these regions at high Reynolds numbers.

It is the purpose of this paper to present a unified similarity analysis for turbulent convection next to vertical surfaces which encompasses the entire range of conditions from the pure natural convection boundary layer (no forcing free stream velocity) to the pure forced convection boundary layer (no buoyancy). The important problem of a natural convection flow next to an adiabatic wall will also be seen to be a special case of the general formulation. Particular attention will be paid to the region of the boundary layer at distances sufficiently far from the wall for the mean equations to be uninfluenced directly by viscosity, thermal diffusivity, or roughness but close enough to the wall to be independent of mean convection or mean inertia effects. This region corresponds to the logarithmic or inertial sublayer in forced flows and the buoyant sublayer in natural convection flow and will be referred to hereafter as the intermediate layer.

EQUATIONS OF MOTION

We restrict ourselves to a two-dimensional mean motion and choose the upward vertical direction to correspond to the x-axis and the distance from the plate to correspond to the y-axis (see Fig. 1).

We define the pressure P to be the difference between the local static pressure and the hydrostatic pressure and assume that $\partial P/\partial x$ is identically zero at large distance from the plate†. We assume that a velocity $\overline{U}_\infty$ is imposed at large distance from the plate and that either a heat flux, q_w, or a temperature difference (from free stream), ΔT_w, is imposed at the wall. We further assume that the spatial variation of $\overline{U}_\infty$ and q_w or ΔT_w is sufficiently small that the turbulent flow is in local equilibrium; that is, all profiles will be functions only of local parameters.

It is straightforward to show by simple scaling arguments that as turbulent Reynolds and Peclet numbers become large, the equations of motion for the main part of the turbulent boundary layer to within the Boussinesq approximation are given by (c.f. references (1) and (2))

$$\overline{U}\frac{\partial \overline{U}}{\partial x} + \overline{V}\frac{\partial \overline{U}}{\partial y} = \frac{\partial}{\partial y}(-\overline{uv}) + \{g\beta(T-T_\infty)\} \qquad (1)$$

$$\overline{U}\frac{\partial T}{\partial x} + \overline{V}\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}(-\overline{vt}) \qquad (2)$$

The pressure has been eliminated using the y-momentum equation and the incompressible continuity equation has not been written. The buoyancy term has been enclosed in brackets since it will be absent for forced convection.

Since these equations obviously cannot be used near the wall because the viscous and conduction terms are missing, it is necessary to seek an alternate set of equations for the near wall region. It is easy to show that the appropriate set is (c.f. references (1) and (2)).

$$0 = \frac{\partial}{\partial y}\left[-\overline{uv} + \nu\frac{\partial \overline{U}}{\partial y}\right] + \{g\beta(T-T_\infty)\} \qquad (3)$$

$$0 = \frac{\partial}{\partial y}\left[-\overline{vt} + \alpha\frac{\partial T}{\partial y}\right] \qquad (4)$$

†This restriction can be relaxed at some increase in complexity.

As before, we have included the buoyancy term in brackets.

We can now use this inner set of equations to distinguish the fundamental differences between forced flows and those in which buoyancy plays a role. Equation (3) can be integrated to yield

$$-\overline{uv} + \nu \frac{\partial \overline{U}}{\partial y} + \{ \int_0^y g\beta(T-T_\infty)dy' \} = \tau_w/\rho \equiv u_*^2 \qquad (5)$$

where τ_w/ρ is the wall shear stress and u_* is defined by the equation. Similarly, equation (4) can be integrated to yield

$$-\overline{vt} + \alpha \frac{\partial T}{\partial y} = -\frac{q_w}{\rho C_p} \equiv -F_o \qquad (6)$$

where q_w is the wall heat flux, C_p is the specific heat at constant pressure, and F_o is defined by the equation.

It is clear that all vertical flat plate flows are characterized by a layer next to the wall in which the total heat flux away from the wall (turbulent transport and conduction) is constant. Thus F_o must be considered a fundamental parameter of the motion since it represents the heat provided to <u>both</u> the inner layer <u>and</u> the main part of the boundary layer.

Unlike the temperature equation, the inner momentum equation is directly dependent on the degree to which buoyancy is important. We can best understand the nature of this equation if we first examine the forced convection limit where the buoyancy term is completely absent. It is clear that in this case the inner layer is also a <u>constant stress layer</u>; that is, the total stress (turbulent and viscous) is unchanged across the layer. Because of this, the wall shear stress, τ_w/ρ, or the friction velocity, u_*, is a measure of the degree to which the inner layer is forced by the outer layer and the flow external to the boundary layer. Thus, in the case of forced flow, u_* measures both an input to the inner layer from the outer layer and a boundary condition on the outer layer. Therefore, u_* or τ_w must be considered an independent parameter.

The momentum equation for pure natural convection can be contrasted with that discussed above. Since there is no external forcing of the inner layer and since all motion is uniquely determined by the heat flux at the wall, the wall shear stress is dependent only on parameters specified in the inner layer; namely, $g\beta$, F_o, ν and α. Since u_* contains no new information, it is entirely a dependent parameter and its inclusion in a dimensional analysis would be redundant. This fact is the fundamental difference between the analysis of forced and natural convection boundary layers.

The temperature equation also provides an interesting contrast between forced and natural convection. For natural convection we have already seen that inner layer is uniquely determined when $g\beta$, F_o, α and ν are specified. For forced convection, however, we have the parameter u_* instead of $g\beta$. In addition, we note that the temperature has no direct effect on the momentum equation. Therefore, we can expect the momentum field in the inner layer to be entirely specified by u_* and ν while F_o and α must be added only to specify the temperature field. In a sense, this is the reverse of the natural convection case where the heat flux determined both equations; here, u_* enters both equations. Clearly, the differences are entirely due to which parameter measures the forcing of the flow.

UNIVERSAL VELOCITY AND TEMPERATURE PROFILES FOR MIXED CONVECTION

For mixed convection it is clear that there are two "sources" of motion. In addition to that motion resulting from buoyancy there is also an "input" from the outer flow. The parameter F_o is still characteristic of the inner layer because of the fact that the heat flux is constant across it. The wall shear stress, on the other hand, is a parameter which includes both the effect of the outer flow and the buoyancy. Thus, while not an ideal parameter because of this mixed origin, it is the only one available which accounts for the forcing of the outer flow.

If we hypothesize that the velocity and temperature profiles at any given cross-section of the boundary layer depend only on local parameters, the functional form of those profiles depends only on the following list: $g\beta$, F_o, u_*, δ, α, ν, k and y, where δ is an outer length scale (e.g., the boundary layer thickness) and k is a roughness height (c.f. Schlicting (<u>3</u>)). From this list five independent dimensionless parameters can be formed; we chose

$$y/L_1,\ \eta_v/L_1,\ \delta/\eta_v,\ \eta_T/L_1,\ k/L_1$$

where

$$\eta_v \equiv \nu/u_*, \qquad (7)$$

$$\eta_T \equiv \alpha/u_*, \qquad (8)$$

and

$$L_1 \equiv u_*^3/g\beta F_o \qquad (9)$$

The length scales η_v and η_T are the familiar viscous and conduction scales and L_1 is similar to the Monin-Obukov length introduced in the atmospheric surface layer analysis (c.f. Monin and Yaglom (<u>1</u>), Chapter 7).

The most general functional form for the velocity and temperature gradients can now be written as

$$\frac{d\overline{U}}{dy} = \frac{u_*}{y} f_o(y/L_1,\ \eta_v/L,\ \eta_T/L_1,\ \delta/\eta_v,\ k/L_1) \qquad (10)$$

and

$$\frac{d\overline{T}}{dy} = \frac{T_*}{y} g_o(y/L_1,\ \eta_v/L_1,\ \eta_T/L_1,\ \delta/\eta_v,\ k/L_1) \qquad (11)$$

where T_* is a temperature scale defined by

$$T_* \equiv F_o/u_* \tag{12}$$

THE CONVECTION LENGTH SCALE L_1

It is appropriate before proceeding further to explore the nature of the length scale L_1. For reasons which will be clear later, we dub this the convection length scale.

We adopt the following sign conventions: $L_1 > 0$ when buoyancy acts in the same direction as the forcing flow and $L_1 < 0$ when the buoyancy and the forcing flow oppose each other. For example, $L_1 > 0$ when U_∞ is upward and heat is being added to the fluid. Thus, y/L_1 can take all values in the interval $(-\infty,\infty)$ depending on the sign of the heat flux for a given direction of the forcing flow.

By observing the consequences of letting $g\beta F_o \to 0$ it is clear that the ratio y/L_1 is a measure of the relative importance of buoyancy and external forcing. Thus $y/L_1 \to 0$ corresponds to $u_* >> (g\beta F_o y)^{1/3}$. This limit could be interpreted as the pure convection limit although it could also be achieved for a buoyant flow next to an adiabatic wall.

George and Capp (2) have shown that for natural convection on smooth walls

$$u_*^2 = c_{fs} \cdot (g\beta F_o \alpha)^{1/2} \tag{13}$$

where c_{fs} is Prandtl number dependent, while for rough walls

$$u_*^2 = c_{fr} \cdot (g\beta F_o k)^{2/3} \tag{14}$$

where c_{fr} depends on the roughness H-number, $g\beta F_o{}^4/\alpha^3$, and Prandtl number. As anticipated, u_* is no longer an independent parameter. Substitution of these values into the definition of L_1 yields for pure natural convection

$$L_1 = c_{fs}{}^{3/2} \left(\frac{\alpha^3}{g\beta F_o}\right)^{1/4} = c_{fs}{}^{3/2}\ \eta_F \tag{15}$$

or

$$L_1 = c_{fr}{}^{3/2} \cdot k \tag{16}$$

Thus $y/L_1 \propto y/\eta_F$ or y/k, whichever is appropriate. If we restrict our attention to regions of the flow for which $y >> \eta_F$, k then the natural convection limit is seen to correspond to $y/L_1 \to +\infty$.

VELOCITY AND TEMPERATURE PROFILES IN AN INTERMEDIATE LAYER

We now confine our attention to what we shall call the intermediate layer; namely, a region of the flow sufficiently removed from the wall to be outside the direct influence of the wall parameters, k, ν, α and yet close enough to the wall to be independent of the outer scales. Thus we are talking about a region in which η_v, η_T, $k << y << \delta$. It follows immediately from our previous arguments that the velocity and temperature gradients can depend only on $g\beta$, F_o, u_* and y. Thus, we must have, defining $\zeta = y/L_1$,

$$\frac{d\bar{U}}{dy} = \frac{u_*}{y} f_1(\zeta) \tag{17}$$

and

$$\frac{d\Delta T}{dy} = \frac{T_*}{y} g_1(\zeta) \tag{18}$$

These represent universal velocity and temperature gradient profiles in an intermediate layer. The only restrictions are that appropriately defined turbulent Reynolds and Peclet numbers must be sufficiently high and there be no imposed pressure gradient.

In the preceding section we saw that as $\zeta = y/L_1 \to 0$, we could approach the forced convection limit. If we assume the functions f_1 and g_1 to be well-behaved in this limit, we can write a Taylor expansion for them about the point $\zeta = 0$ in the following manner

$$f_1(\zeta) \doteq f_1(0) + \zeta \cdot f_1'(0) + \frac{1}{2}\zeta^2\ f_1''(0) + \ldots \tag{19}$$

$$g_1(\zeta) \doteq g_1(0) + \zeta \cdot g_1'(0) + \frac{1}{2}\zeta^2\ g_1''(0) + \ldots \tag{20}$$

where primes denote differentiation. Substituting these into equations (17) and (18) we obtain

$$\frac{d\bar{U}}{dy} = \frac{u_*}{y} \cdot \{f_1(0) + \zeta \cdot f_1'(0) + \frac{1}{2}\zeta^2\ f_1''(0) + \ldots\} \tag{21}$$

$$\frac{d\Delta T}{dy} = \frac{T_*}{y} \cdot \{g_1(0) + \zeta \cdot g_1'(0) + \frac{1}{2}\zeta^2\ g_1''(0) + \ldots\} \tag{22}$$

Equations (21) and (22) can be readily integrated to yield

$$\frac{\bar{U}}{u_*} = \{\frac{1}{\kappa}\ \ell ny + \text{const}\} + f_5'(0) \cdot \frac{y}{L_1} + f_5''(0) \cdot \frac{y^2}{4L_1{}^2} + \ldots \tag{23}$$

$$\frac{\Delta T}{T_*} = \{\frac{1}{\kappa\alpha_t}\ \ell ny + \text{const}\} + g_5'(0) \cdot \frac{y}{L_1} + g_5''(0) \cdot \frac{y^2}{4L_1{}^2} + \ldots \tag{24}$$

where we have defined

$$f_1(0) = \frac{1}{\kappa} \tag{25}$$

and

$$g_1(0) = \frac{1}{\kappa\alpha_t} \tag{26}$$

The terms in brackets in the preceding expressions are readily recognized as the logarithmic profiles of the forced convection solution, and the constants κ and α_t are the well known von Karman constant and turbulent Prandtl number respectively (c.f. Monin and Yaglom (1)). It is clear that as $y/L_1 \to 0$, the higher order terms vanish and the velocity and temperature profiles reduce to the pure forced convection solutions.

If we include only the terms of order y/L_1, the profiles are log-linear. The existence of this type of profile has long been known for the atmospheric boundary layer and similar flows where the direction of the gravitational force is perpendicular to the mean motion (c.f. Monin and Yaglom). This is believed to be the first time that this type of profile has been suggested for mixed convection flows next to vertical surfaces.

From section (2) we recall that for the pure natural convection limit, we must eliminate u_* from consideration in a dimensional analysis since it is not an independent parameter. Accordingly, appropriate forms for the velocity and temperature gradients in the intermediate layer near the pure natural convection limit are

$$\frac{d\overline{U}}{dy} = \frac{(g\beta F_o)^{1/3}}{y^{2/3}} f_2(\zeta) \tag{27}$$

and

$$\frac{d\Delta T}{dy} = \frac{F_o^{2/3}}{(g\beta)^{1/3} y^{4/3}} g_2(\zeta) \tag{28}$$

In the natural convection limit, we saw earlier that L_1 was proportional to η_T or k whichever is appropriate. Since we have already assumed $y >> \eta_T$, η_v or k it follows that f_2 and g_2 must approach constant values, say c_1 and c_2 respectively.

It is easy to show that we can also express this limit using the formulation of equations (17) and (18). We find

$$\lim_{y/L_1 \to \infty} f_1(\zeta) = c_1 \zeta^{1/3} \tag{29}$$

and

$$\lim_{y/L_1 \to \infty} g_1(\zeta) = c_2 \zeta^{-1/3} \tag{30}$$

EVALUATION OF CONSTANTS FROM THE EXPERIMENTAL DATA

Our ability to evaluate the constants in equations (23) through (38) is severely limited by an almost complete lack of experimental information on turbulent mixed convection boundary layers. Therefore, we shall have to be content to evaluating those which have been established for forced and natural convection and leaving for subsequent experimenters the completion of this work.

For forced convection it is well-known (c.f. Monin and Yaglom (1) that the velocity and temperature profiles in the inertial sublayer (the intermediate layer) can be written for smooth walls as

$$\frac{\overline{U}}{u_*} = \frac{1}{\kappa} \ln (y/\eta_v) + B \tag{31}$$

$$\frac{\Delta T}{T_*} = \frac{1}{\kappa\alpha_t} \ln (y/\eta_v) + A(Pr) \tag{32}$$

κ is referred to as the von Karman constant and α_t is a type of turbulent Prandtl number. κ is very close to 0.4 and α_t is generally agreed to be about unity. The constant B is independent of Prandtl number and is usually taken to be about 5. A(Pr) has been calculated from assumed turbulence models; these are reviewed in reference (1), Section 5.7.

For rough walls the velocity and temperature profiles can be written as

$$\frac{\overline{U}}{u_*} = \frac{1}{\kappa} \ln (y/k) + A_1 \left(\frac{u_* k}{\nu}\right) \tag{33}$$

and

$$\frac{\Delta T}{T_*} = \frac{1}{\kappa\alpha_t} \ln (y/k) + B_1 \left(\frac{u_* k}{\nu}, Pr\right) \tag{34}$$

The constants κ and α_T are unchanged but the functions A_1 and B_1 depend not only on the roughness height, k, but also on the type of roughness.

George and Capp (2) have shown for natural convection next to smooth surfaces that the velocity and temperature profiles in the buoyant sublayer (the intermediate layer) are given by

$$\frac{\overline{U}}{(g\beta F_o \alpha)^{1/4}} = K_1 \left(\frac{y}{\eta_F}\right)^{1/3} + A_2(Pr) \tag{35}$$

and

$$\frac{\Delta T (g\beta\alpha)^{1/4}}{F_o^{3/4}} = K_2 \left(\frac{y}{\eta_F}\right)^{-1/3} + B_2(Pr) \tag{36}$$

The constants K_1 and K_2 were evaluated as 27 and 5.6 respectively. Not enough information was available to evaluate $A_2(Pr)$ and $B_2(Pr)$.

For natural convection next to rough surfaces the velocity and temperature profiles in the buoyant sublayer were given by

$$\frac{\overline{U}}{(g\beta F_o k)^{1/3}} = K_1 (y/k)^{1/3} + B_3 (H_k^*, Pr) \tag{37}$$

and

$$\frac{\Delta T (g\beta)^{1/3}}{F_o^{2/3} k^{1/3}} = K_2 (y/k)^{-1/3} + A_3 (H_k^*, Pr) \tag{38}$$

where

$$H_k^* = \frac{g\beta F_o k^4}{\nu^3} \qquad (39)$$

is a modified roughness H-number. Although no experimental data is available, the values of K_1 and K_2 should be the same as for the smooth wall.

From the information and results listed above, it is possible to obtain

$$f_1(0) = \frac{1}{\kappa} = 2.5 \qquad (40)$$

$$g_1(0) = \frac{1}{\kappa\alpha_t} = 2.5 \qquad (41)$$

$$\lim_{\zeta\to\infty} f_1(\zeta) = C_1(\zeta)^{1/3} \doteq 9(\zeta)^{1/3} \qquad (42)$$

$$\lim_{\zeta\to\infty} g_1(\zeta) = C_2(\zeta)^{-1/3} \doteq -1.87(\zeta)^{-1/3} \qquad (43)$$

This appears to be the extent of our present ability to evaluate the coefficients in equations (23) through (30). Clearly information on at least $f_1'(0)$ and $g_1'(0)$ would be very useful.

APPLICABILITY TO EXPERIMENTAL EFFORTS

The logarithmic or inertial sublayer profiles have long been used by experimentalists to determine the friction velocity (and hence the wall shear stress) in forced flows where determination by other means was difficult or inconvenient. This has especially been true for meterologists who have had no other option. If one is willing to accept the value of α_t (eq. 32) as known, the logarithmic temperature profile can similarly be used to determine the heat flux.

The universal profiles for mixed convection proposed here should provide the experimentalist and engineer with further important tools for rapid determination of wall shear stress and heat flux. As for forced flow, only a few velocity and temperature measurements need be taken in the intermediate layer to completely specify the inputs. Such utilization must, of course, first be preceded by a series of careful basic experiments to determine $f_1(\zeta)$ and $g_1(\zeta)$.

APPLICABILITY TO NUMERICAL MODELLING EFFORTS

Aside from their value in contributing to organization of experimental results and understanding the physics fo the flow, a possibility for immediate application of these results is their incorporation into numerical modelling programs for turbulent flow. The most successful turbulent closure models to date (c.f. Reynolds (4)) are useful only at high turbulent Reynolds numbers. As the wall is approached, the turbulent Reynolds number approaches zero and the models must be modified by a series of ad-hoc assumptions. Examples of this approach are detailed in Hanjalic and Launder (5).

An alternate approach for forced flow has been to use the logarithmic layer as an inner boundary condition to which the outer flow must conform (c.f. Launder and Spalding (6)). This approach has the advantage that a turbulence model need be constructed only for regions outside the influence of viscosity; that is, at high turbulent Reynolds number. The same approach can be used for natural convection flows using the aforementioned buoyant sublayer profiles.

It would appear that for the difficult problem of calculating turbulent boundary layers in mixed convection the universal profiles of velocity and temperature in the intermediate layer could be used to great advantage as inner boundary conditions for a turbulent model which calculates the outer flow.

SUMMARY AND CONCLUSIONS

The outline of a similarity theory for fully developed turbulent boundary layer flow next to vertical surfaces has been given. Particular attention has been paid to the intermediate layer where only turbulent transport and buoyancy are important.

Scaling laws for the profiles of temperature and velocity in this region have been proposed and explicit functional forms have been derived which are valid near the limits of pure natural convection and pure forced convection and reduce to the correct functional behavior in the limits.

ACKNOWLEDGEMENTS

The author is grateful to Professor D. Taulbee and Mr. S. Capp for helpful discussions of this work. The work was supported by the National Science Foundation, Atmospheric Sciences Division.

REFERENCES

1. Monin, A.S. and Yaglom, A.M., Statistical Fluid Mechanics: Mechanics of Turbulence, Vol. 1, MIT Press, Cambridge, MA, 1971.
2. George, W.K. and Capp, S.P. "Theory for Natural Convection Turbulent Boundary Layers Next to Heated Vertical Surfaces", J. Heat Mass Trans. 22, pp. 813-826.
3. Schlicting, H., Boundary Layer Theory, McGraw-Hill, NY, 1968.
4. Reynolds, W.C., Annual Reviews of Fluid Mechanics, Vol. 8, p. 183-208, Academic Press, NY, 1968.
5. Hanjalic, E. and Launder, Journal Fluid Mech., 74, 4, pp. 593-610.
6. Launder, B.E. and Spalding D.B., Mathematical Models of Turbulence, Academic Press, NY, 1972.

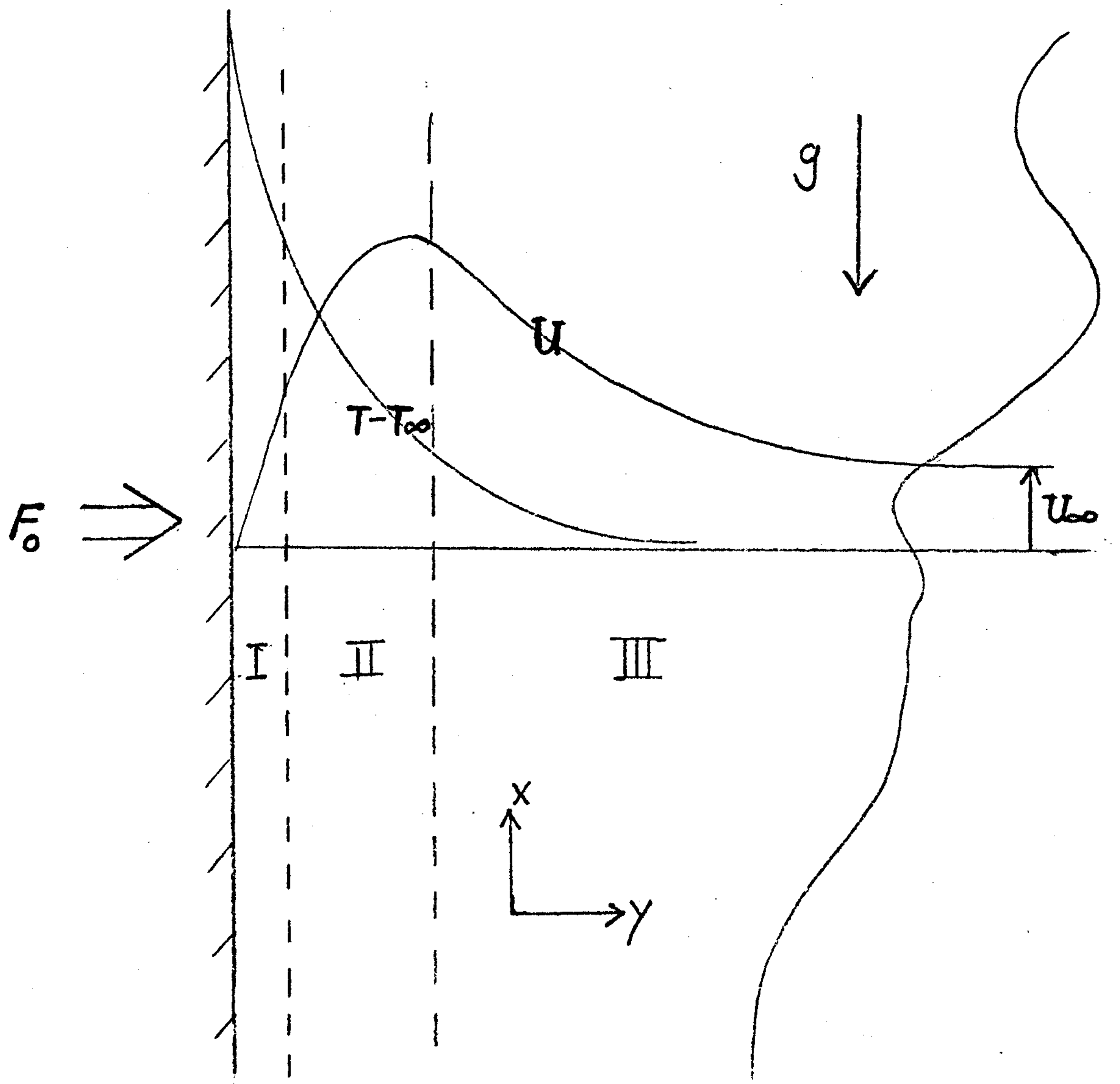

Figure 1. The mixed convection boundary layer.
(Region I: viscous conductive region;
Region II: intermediate layer;
Region III: outerflow.)