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# THE STREAMWISE EVOLUTION OF COHERENT STRUCTURES IN THE AXISYMMETRIC JET MIXING LAYER[1]

by

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## ABSTRACT

The dynamical equations governing the streamwise evolution of coherent structures in the axisymmetric jet shear layer are derived. The method utilized is similar to that used by Aubry et al (1988) and Glauser et al (1989). This method consists of performing a Galerkin projection, using the basis functions obtained from application of the proper orthogonal decomposition (POD), onto the Navier Stokes equations. In this study, however, the so called random coefficients are written as function of the streamwise direction, and not time as in the previous studies. This results in a boundary value problem and not an initial value problem as was the case in the afore-mentioned work. This type of an approach is important from an experimentalist's point of view. The two-point correlation tensor (needed for application of the POD) can be measured much easier at one streamwise location as a function of time difference rather than at many streamwise separations. The basic idea is to measure the two-point correlation at one streamwise location and infer the evolution of the coherent structures in the streamwise direction from the dynamical equations. These equations and some ideas on how to solve them numerically will be discussed.

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# 1 Introduction

In recent years, two separate developments have altered the basic statistical framework of turbulence. There is an abundance of experimental evidence implying the existence of coherent structures and recent applications of dynamical systems theory to turbulence suggest that such flows reside on relatively low dimensional manifolds or attractors (v. Aubry et al 1988 and Bartuccelli et al 1989).

Aubry et al (1988) were the first to link low dimensional chaotic dynamics to a turbulent open flow system. They used the proper orthogonal decomposition (POD) to provide basis functions to obtain low dimensional sets of ordinary differential equations from the Navier–Stokes equations. Used in conjunction with Galerkin projection, the POD yields an optimal set of basis functions in the sense that the resulting truncated system of ordinary differential equations captures the maximum amount of kinetic energy among all possible truncations of the same order. Sirovich and Rodriguez (1987) have shown, for the Ginzburg–Landau system, that these basis functions are fairly robust and can be used over a wide range of the bifurcation parameter. Chambers et al. (1988) have seen similar trends for Burgers equation. The basis functions that Aubry et al (1988) used were those obtained experimentally by Herzog (1986) for the near wall region of a pipe flow. The solutions to the equations that they derived exhibited intermittent behavior, and were then analyzed using dynamical systems theory. The results to date show consistency between the behavior of these equations and events seen in experimental work.

The axisymmetric jet is also a good candidate for a similar approach because the series converges quickly. This was demonstrated by Glauser et al (1987), where the instantaneous signals were almost completely reconstructed with only 3 terms from the expansion. By assuming the flow to be approximately homogeneous in the streamwise direction, Glauser et al (1989) and Zheng and Glauser (1990) were able to use a similar approach (where the so called random coefficients are written as a function of time) and examined the sequence by which the streamwise wavenumbers and azimuthal mode numbers retained in the model contribute in time. Their initial results indicate that there is a transfer of energy between certain streamwise wavenumbers and azimuthal modes 4, 5 and 6, consistent with the mechanism for turbulence production suggested by Glauser and George (1987) for the jet shear layer.

In the study reported here the so called random coefficients, for which the equations are written, are functions of the streamwise direction and not time as in the previous studies. This results in a boundary value problem and not an initial value problem as was the case in the afore-mentioned work. This type of approach overcomes the need for assuming the flow to be homogeneous in the streamwise direction, and thus more closely models its spatially developing character. Also it more closely reflects the experiments (from which the basis functions are taken) since the two-point correlation tensor (needed for application of the POD) can be measured much more easily at one streamwise location as a function of time difference rather than at the many streamwise separations necessary to decompose a developing flow.

The axisymmetric jet is stationary so the appropriate decomposition in time is the Fourier transform (v. George 1988). In the azimuthal direction the flow is periodic, hence the discrete Fourier modes are appropriate. In the radial direction the flow is strongly inhomogeneous so that the eigenfunctions obtained from applying the POD are utilized. Galerkin projection is then used in conjunction with the POD to obtain a truncated system of ordinary differential equations. The modes neglected in the truncation are accounted for by a Heisenberg model as was done in the temporal studies of Aubry et al (1988) and Glauser et al (1989). Finally, the evolution of the coherent structures in the streamwise direction is inferred from the resulting dynamical equations. This work appears to be the first which utilizes low dimensional dynamics in conjunction with Galerkin projection and the POD to examine the *spatial* dynamics of coherent structures in a high Reynolds number axisymmetric jet free shear layer.

## 2 Proper Orthogonal Decomposition

In 1967 Lumley suggested that the coherent structure should be that structure which has the largest mean square projection on the velocity field. This process involves maximizing the mean square energy via the calculus of variations and leads to the following integral eigenvalue problem.

$$\lambda\phi_i(\vec{x}) = \int R_{ij}(\vec{x},\vec{x}')\phi_j(\vec{x}')d\vec{x}' \tag{1}$$

The symmetric kernel of this Fredholm integral equation is the two–point correlation tenser $R_{ij}$ defined by

$$R_{ij}(\vec{x},\vec{x}') = \overline{u_i(\vec{x})u_j(\vec{x}')}, \tag{2}$$

where $\vec{\phi}$ is the candidate structure and $\vec{x}$ and $\vec{x}'$ represent different spatial points in the inhomogeneous directions and different times if the flow is non–stationary.

From the Hilbert–Schmidt theory it can be shown that the solution of a Fredholm integral equation of the first kind for a symmetric kernel and a finite energy domain (i.e., statistically inhomogeneous) is a discrete set, hence equation (1) can be written as

$$\lambda^n\phi_i^n(\vec{x}) = \int R_{ij}(\vec{x},\vec{x}')\phi_j^n(\vec{x}')d\vec{x}' \tag{3}$$

The eigenfunctions of the Fredholm equation are orthogonal over the interval and

$$\int \phi_i^n(\vec{x})\phi_i^m(\vec{x})d\vec{x} = \delta_{nm} \tag{4}$$

for normalized eigenfunctions. The eigenvalues of the Fredholm equation with a real symmetric kernel are all real and uncorrelated

$$\overline{a^n a^m} = \lambda^n\delta_{nm} \tag{5}$$

and the fluctuating random field $\tilde{u}_i$ can be reconstructed from the eigenfunctions in the following way

$$\tilde{u}_i(\vec{x}) = \sum_{n=0}^{\infty} a^n \phi_i^n(\vec{x}). \tag{6}$$

The random coefficients are calculated from

$$a^n = \int \tilde{u}_i(\vec{x}) \phi_i^{n*}(\vec{x}) d(\vec{x}) \tag{7}$$

where the $\phi_i^n$ are the eigenfunctions obtained from equation (3). The turbulent kinetic energy is the sum over n of the eigenvalues $\lambda^n$, and each structure makes an independent contribution to the kinetic energy and Reynolds stress.

If the random field is homogeneous or periodic in one or more directions or stationary in time, the eigenfunctions become Fourier modes, so that the POD reduces to the harmonic orthogonal decomposition in these directions. In this study we will treat the jet flow as periodic in the azimuthal direction and stationary in time. We will not transform over the streamwise direction in this case because we are writing the random coefficients as a function of the streamwise variable $z$. Under these conditions the spectral tensor may be defined by

$$S_{ij}(r, r', m, f, \overline{z}) = \int R_{ij}(r, r', \theta, \tau, \overline{z}) e^{-i(2\pi f + m\theta)} d\tau d\theta \tag{8}$$

where $r$ and $r'$ represent different spatial locations in the radial direction ( the strongly inhomogeneous direction), $\tau$ and $\theta$ are the separations in time and the azimuthal direction respectively, $f$ is the frequency, $m$ is the azimuthal mode number and $\overline{z}$ denotes the streamwise location where the correlation tensor is measured. Equation (3) now becomes

$$\lambda^n(m, f) \phi_i^n(r, m, f, \overline{z})$$

$$= \int_\Omega S_{ij}(r, r', m, f, \overline{z}) \phi_j^n(r', m, f, \overline{z}) r' dr'. \tag{9}$$

This equation can be solved numerically using the measured values of $S_{ij}(r, r', m, f, \overline{z})$ obtained by Glauser and George (1987) which were obtained at 3 diameters downstream in the jet shear layer. It should be noted that in this case the eigenvalues and eigenfunctions are now a function of azimuthal mode number $m$ and frequency $f$.

## 3 The Dynamical Equations

The momentum equations for an incompressible flow in cylindrical coordinates are

$$\rho\left(\frac{Du_r}{Dt} - \frac{u_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left(\nabla^2 u_r - \frac{u_r}{r^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\right) + F_r \quad (10)$$

$$\rho\left(\frac{Du_\theta}{Dt} + \frac{u_\theta u_r}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left(\nabla^2 u_\theta - \frac{u_\theta}{r^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta}\right) + F_\theta \quad (11)$$

$$\rho\left(\frac{Du_z}{Dt}\right) = -\frac{\partial p}{\partial z} + \mu \nabla^2 u_z + F_z \quad (12)$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u_r\frac{\partial}{\partial r} + \frac{u_\theta}{r}\frac{\partial}{\partial \theta} + u_z\frac{\partial}{\partial z}$$

and

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

We assume that the $\rho$ and $\mu$ are constant and $F_r = F_\theta = F_z = 0$.

By decomposing the instantaneous dependent variables into a fluctuation and an average over the azimuthal direction and time, the equation for the fluctuating velocity can be obtained as

$$\frac{\partial}{\partial t}\tilde{u}_i + M\tilde{u}_i + \left(-\frac{\tilde{u}_\theta^2}{r} + \overline{U}_z\frac{\partial \tilde{u}_r}{\partial z} + \overline{U}_r\frac{\partial \tilde{u}_r}{\partial r} + \tilde{u}_r\frac{\partial \overline{U}_r}{\partial r}\right)\delta_{ir}$$

$$+\left(\frac{\tilde{u}_r\tilde{u}_\theta}{r} + \overline{U}_z\frac{\partial \tilde{u}_\theta}{\partial z} + \overline{U}_r\frac{\partial \tilde{u}_\theta}{\partial r}\right)\delta_{i\theta}$$

$$+\left(\overline{U}_z\frac{\partial \tilde{u}_z}{\partial z} + \overline{U}_r\frac{\partial \tilde{u}_z}{\partial r} + \tilde{u}_r\frac{\partial \overline{U}_z}{\partial r} + \tilde{u}_z\frac{\partial \overline{U}_z}{\partial z}\right)\delta_{iz}$$

$$-\overline{M\tilde{u}_i} + \frac{\overline{\tilde{u}_\theta^2}}{r}\delta_{ir} + \frac{\overline{\tilde{u}_r\tilde{u}_\theta}}{r}\delta_{i\theta}$$

$$= -\frac{r(\delta_{ir}+\delta_{iz})+\delta_{i\theta}}{r\rho}\frac{\partial\tilde{p}}{\partial i}$$

$$+\nu[\nabla^2\tilde{u}_i - (\frac{\tilde{u}_r}{r^2} + \frac{2}{r^2}\frac{\partial\tilde{u}_\theta}{\partial\theta})\delta_{ir} + (-\frac{\tilde{u}_\theta}{r^2} + \frac{2}{r^2}\frac{\partial\tilde{u}_r}{\partial\theta})\delta_{i\theta}] \quad (13)$$

where $M = \tilde{u}_r\frac{\partial}{\partial r} + \frac{\tilde{u}_\theta}{r}\frac{\partial}{\partial\theta} + \tilde{u}_z\frac{\partial}{\partial z}$ , $i = r, \theta, z$, $\delta$ is the Kronecker delta, $\tilde{p}$ is the fluctuating pressure, $\tilde{u}_r$, $\tilde{u}_\theta$ and $\tilde{u}_z$ are the fluctuating velocities in the radial, azimuthal, and streamwise directions respectively, and $\overline{U}_z$ and $\overline{U}_r$ are the mean streamwise and radial velocity respectively. These mean velocities must also satisfy the averaged continuity equation. It should be noted that the terms involving $\overline{U}_r$ are kept in the *spatially* evolving problem considered here whereas Zheng and Glauser (1990) neglected these terms. In the high Reynolds number jet studied here we can neglect the viscous terms in equation (13) and this will be done in the following section when the Galerkin projection is performed.

We seek a relationship which relates the mean velocities $\overline{U}_z$ and $\overline{U}_r$ to the Reynold's stress since the actual measured mean velocity profiles will be incorrect for the truncated system of equations to be studied. The equation which relates these (to second order) can be derived from the $z$ momentum equation as

$$\frac{1}{r}\frac{\partial}{\partial r}(r\overline{\tilde{u}_r\tilde{u}_z}) = -(\overline{U}_z\frac{\partial}{\partial z}\overline{U}_z + \overline{U}_r\frac{\partial}{\partial r}\overline{U}_z) \quad (14)$$

where the mean pressure term has been eliminated from the streamwise equation by integrating the averaged radial equation (v. Tennekes and Lumley 1972). Equation (14) indicates that the Reynold's stress is balanced by the mean convection and provides communication between the mean flow and the Reynold's stress for the *evolving* shear layer. This result is quite different than the that of Zheng and Glauser (1990) who treat the flow as homogeneous in $z$, resulting in the Reynold's stress being balanced by the viscous terms. This modifies the character of the ODEs as will be seen in the next section.

## 4 Galerkin projection

The Galerkin method is well known and has been used extensively to study turbulence and the instability of various fluid flows (v. Lin et al (1987) and references therein). The essential idea of the method is to expand the dependent variables in terms of a finite series of independent basis functions. The basis functions form a complete basis for the relevant class of functions, and they satisfy the relevant boundary conditions. In this work we use a Galerkin projection in conjunction with the POD (to supply the basis functions) which minimizes the error due to the truncation and yields a set of ordinary differential equations for the coefficients (v. equation 7).

The Galerkin projection is performed on the Fourier Transform (over time and in the azimuthal direction) of the Navier Stokes equations so that is is useful to define the following equations:

$$\tilde{u}_i(z,\theta,r,t) = \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-2\pi j(tf+m\theta)} \hat{u}_i(f,m,r,z) df \tag{15}$$

$$\hat{u}_i(f,m,r,z) = \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} e^{2\pi j(tf+m\theta)} \tilde{u}_i(z,\theta,r,t) dt d\theta \tag{16}$$

We expand $\hat{u}_i(f,m,r,z)$ in terms of the coefficients and eigenfunctions in the following manner

$$\hat{u}_i(f,m,r,z) = \sum_n a^n_{f,m}(z) \phi^n_{i,f,m}(r). \tag{17}$$

Note the $z$ dependence of the random coefficients. We then substitute these equations into the Fourier Transform of equation (13) *after eliminating the viscous terms* and perform the Galerkin projection. This can be written as

$$(\phi^l, N) = \int_0^R N_i(r,z) \phi_i^{l*}(r) dr = 0 \tag{18}$$

where $N_i(t,z)$ represents the Fourier Transform of the Navier–Stokes equations (i.e. equation 13 with the viscous terms neglected).

Finally, after performing the above and utilizing the orthogonality conditions (v. equation 4) we obtain the following set of coupled ordinary differential equations for the coefficients:

$$\begin{aligned}
0 &= \sum_n a^n_{f,m}[-2\pi j f \delta_{nl}] \\
&+ \sum_{f'm'}\sum_{pq} a^p_{f',m'}\{a^q_{f-f',m-m'}\int_0^R [\phi^p_{r,f',m'}\frac{d}{dr}\phi^q_{i,f-f',m-m'} \\
&\quad + \frac{-j(m-m')}{r}\phi^p_{\theta,f',m'}\phi^q_{i,f-f',m-m'} - (\frac{1}{r}\phi^p_{\theta,f',m'} \\
&\quad \phi^q_{\theta,f-f',m-m'})\delta_{ir} + (\frac{1}{r}\phi^p_{r,f',m'}\phi^q_{\theta,f-f',m-m'})\delta_{i\theta}]\cdot\phi^{l*}_{i,f,m}dr \\
&\quad + b^q_{f-f',m-m'}\int_0^R [\phi^p_{z,f',m'}\phi^q_{i,f-f',m-m'}]\cdot\phi^{l*}_{i,f,m}dr\} \\
&+ MeanFlowFluctuationInteractionTerms \\
&- \int_0^R [\frac{r(\delta_{ir}+\delta_{iz})+\delta_{i\theta}}{\rho r}\frac{\partial\hat{p}_{f,m}}{\partial i}\cdot\phi^{l*}_{i,f,m}]dr \qquad (19)
\end{aligned}$$

where

$$b^l_{f-f',m-m'} = \frac{da^l_{f-f',m-m'}}{dz}. \qquad (20)$$

In our ordinary differential equation

$$\begin{aligned}0 = Bl + Cq_1 + Dq_2 + MeanFlowTerms \\ +PressureTerm \qquad (21)\end{aligned}$$

there are 5 parts. The first term on the right hand side (RHS) is a linear term that comes from the time derivative of the Navier–Stokes equations. The second and third terms on the RHS are quadratic terms that are a consequence of the fluctuation–fluctuation interactions and exhibits the energy transfer between the different eigenfunctions(from the POD) and the Fourier modes and B, C, and D, are all matrices. The fourth term, not shown here due to lack of space, is a result of the mean flow, fluctuation terms in equation (13). It should be noted that the third and fourth terms contain the spatial derivative of our coefficient $a$ with respect to $z$. The final term is the pressure term which will vanish if the integration covers the whole domain. In the jet shear layer we cover most of the domain in our integration (unlike Aubry et al 1988 who studied the near wall region and not the whole domain of the turbulent boundary layer), hence we will neglect this pressure term. This equation, unlike the temporal equations derived by Glauser et al (1989), does not have a cubic term. This is a direct result of how we are handling the relationship between the mean flow and the Reynold's stress for the *evolving* shear layer. Instead of eliminating the mean flow terms from the system of ODEs as was done by Glauser et al (1989), here we propose to solve equation (14), along with the mean continuity equation, simultaneously with the system of ODEs. Perhaps it would be easier to decompose the flow initially by using the mean and fluctuating components instead of only the fluctuating. However, looking at the fluctuating part by itself, is more instructive for now since it highlights the differences from Glauser et al (1989) and Aubry et al (1988).

An interesting point to note is that the $b$ terms (i.e., $\frac{da^l_{f,m}}{dz}$) can be eliminated from equations (19) through (21) by utilizing the equation of continuity for the fluctuations, which, in terms of the random coefficients can be written as

$$\frac{da^l_{f,m}}{dz}\phi^l_{z,f,m} = -a^l_{f,m}[\frac{d}{dr}\phi^l_{r,f,m} + \frac{1}{r}\phi^l_{r,f,m} + \frac{im}{r}\phi^l_{\theta,f,m}]. \qquad (22)$$

This reduces the ODEs to a system of coupled algebraic equations which is an intriguing result.

## 5 The Energy Transfer Model

When we truncate at some cutoff point we need to account for the energy transfer between the included and neglected modes. The effect of the neglected modes will be accounted for by utilizing a Heisenberg model(v. Aubry et al. (1988)). The assumption is that the neglected modes withdraw energy from the modes that our kept, as if a certain turbulence viscosity were present.

The equation for $\nu_T$, our turbulence viscosity, can be shown to be (v. Zheng (1990))

$$\nu_T = \frac{\alpha \sum_{f,m,n} \lambda^n_{f,m}}{R \sum_{f,m,n} \lambda^n_{f,m} \overline{S}} \tag{23}$$

where

$$\overline{S} = \overline{\int_0^R [\frac{d\phi^n_{i,f,m}}{dr}\frac{d\phi^{n*}_{i,f,m}}{dr} - \frac{1}{r^2}4\pi^2 m^2 \phi^n_{i,f,m}\phi^{n*}_{i,f,m}]dr - 4\pi^2 f^2\}}$$

$\alpha$ is a dimensionless parameter and the sums are over the first neglected modes. We now substitute $\nu_T$ into our ordinary differential equations. The equations then have the form

$$0 = B_1 l_1 + \nu_T B_2 l_2 + C q_1 + D q_2$$

$$+ MeanValueFluctuationTerms \tag{24}$$

where $l_1$ is the linear term from the time derivative of the Navier–Stokes equations as was shown earlier in equation (21) and $l_2$ is a linear term introduced because of the assumption of the neglected modes acting as a viscosity on the modes that are kept.

The effect of the energy transfer model is to introduce the parameter $\alpha$ which becomes the bifurcation parameter in our system of ODEs. The larger $\alpha$ the more energy that the neglected modes take from our system so that the system should be stable. As $\alpha$ decreases less energy is extracted so that we expect our system to become unstable.

## 6 Truncations and Discussion

A truncated version of equation (19) must be developed. A basic guide in the selection of which terms to keep in the model is to retain a minimum number of terms but yet keep as much energy in the system as is necessary to retain the essential dynamics of the flow phenomena (v. Aubry et al 1988 and Glauser et al 1989). Figure 1 shows the dominant eigenvalue obtained from the application of the POD in the jet shear layer at 3 diameters downstream, plotted versus frequency and azimuthal mode number. This plot indicates that there is an exchange of energy between the various frequencies and azimuthal mode numbers, and in particular, that there is a maximum energy ridge (remember that the eigenvalues are energy integrated across the jet shear layer) between the two peaks for azimuthal modes 0 and 1 and the peak at azimuthal mode 5. The peak in the frequency direction for azimuthal mode 0 corresponds to the preferred frequency of the jet. The peak in the azimuthal mode number direction is approximately at mode 5. A possible combination of frequencies and azimuthal modes that could be used as an initial truncation are shown in figure 2. These were selected to try and capture the maximum energy ridge shown in figure 1. As an initial step, only the dominant eigenfunction will be necessary in the inhomogeneous direction. This corresponds to setting $l = n = p = q = r = 1$ in equation (19). This is justified because the dominant eigenvalue is significantly larger than the next smallest eigenvalue and in fact, typically contains 50 percent or more of the energy. All of the higher azimuthal modes exhibit this same dominance of the first eigenvalue (v. Glauser and George 1987). This particular truncation will result in a set of 18 complex or 36 real, ordinary differential equations.

The integrations needed to obtain the coefficient matrices in the truncated version of equation (19) can be obtained using the trapezoidal rule. A fourth order Runge Kutta scheme could then be used to integrate the truncated version of equation (19) with the necessary boundary conditions. As a first step in the selection of the boundary conditions, we can argue that at $z = 0$ the value of our coefficient $a$ is zero, since the fluctuations are small at this position.

## 7 Conclusions

The dynamical equations have been derived for the spatially evolving random coefficients in the axisymmetric jet shear layer. An initial truncation has been proposed which would involve a system of 36 ODEs. Instead of eliminating the mean flow terms from the system of ODEs as was done by Glauser et al (1989), here we propose to solve for the mean quantities, $\overline{U}_z$ and $\overline{U}_r$, simultaneously with the system of ODEs. Hence, the resulting ODEs do not exhibit the cubic character as described by Aubry et al (1988) and Glauser et al (1989).

An interesting extension of this work would be to write the random coefficients as a function of time and the streamwise direction (i.e., $a = a(z,t)$). This would result in a system of partial differential equations in $z$ and $t$. This type of an approach would allow for the temporal and spatial problems to be studied simultaneously.

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REFERENCES

Aubry, N., Holmes, P., Lumley, J.L. and Stone, E. (1988) The Dynamics of Coherent Structures in the Wall Region of a Turbulent Boundary Layer, *JFM vol.192, pp.115-173.*

Bartuccelli, M., Constantin, P., Doering, C.R., Gibbon, J.D., and Gisselfalt, M. (1989), Hard Turbulence in a Finite Dimensional Dynamical System?, *Physics Letters A-Nonlinear Science*

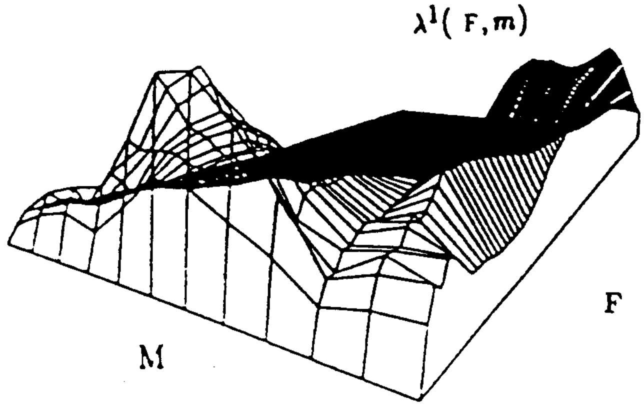


Figure 1: Dominant eigenvalue obtained from the application of the POD in the jet shear layer at 3 diameters downstream.

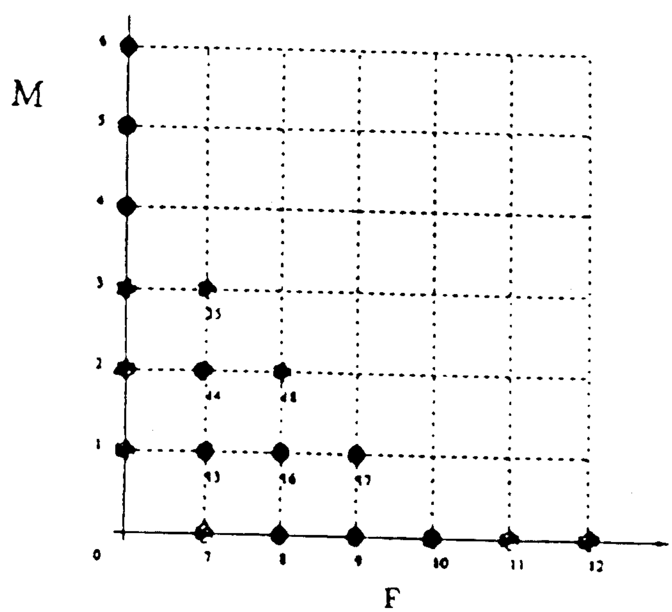


Figure 2: Possible combination of frequencies and azimuthal modes that could be used as an initial truncation.