THE SELF-PRESERVATION OF HOMOGENEOUS SHEAR FLOW TURBULENCE

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ABSTRACT

An analysis of the equations governing homogeneous shear flow shows the possibility of solutions which are self-preserving at all scales of motion, and that these solutions are dependent on the initial conditions. The appropriate velocity scale is the one obtained from the kinetic energy, while the length scale is the Taylor microscale. Two different cases of self-preserving flow were identified: one corresponding to a constant mean shear, and the other to a mean shear which is inversely proportional to time. For the first case (the only one considered in detail) the principal results of the postulated similarity are that the Taylor microscale is constant, as is the ratio of the production to dissipation rates, and that the kinetic energy varies exponentially with time. It also is shown that the velocity spectra scale over all wavenumbers with the turbulence kinetic energy and Taylor microscale, and have a spectral shape which is determined by the initial conditions. The experimental evidence is generally consistent with the theory.

Part I: THEORETICAL DEVELOPMENT

1. Introduction

The simplest type of turbulent shear flow is one in which unbounded homogeneous turbulence is maintained by a uniform gradient of mean velocity. Turbulent transport of the turbulence moments is formally zero, the turbulence energy is generated by interaction with the mean shear, distributed between components by interactions with the fluctuating part of the pressure, and dissipated through the action of viscosity. Many of the basic features of inhomogeneous turbulence are thus retained, while the absence of diffusion simplifies theoretical treatment. Measurements in flows of this type have been widely used to study the coupling between the turbulence and the mean field, and to validate mathematical models of the intercomponent energy transfer mechanism.

Von Karman (1937) appears to have been the first to draw attention to the importance of homogeneous shear flow in basic turbulence research. Corrsin (1963) suggested how it might be set up in a wind tunnel by using a non-uniform grid, and encouraged Rose (1966) to perform the first experiment. Later attempts to generate this type of moving equilibrium have been made by Champagne et al. (1970), Mulhearn and Luxton (1975), Harris et al. (1977), Tavoularis and Corrsin (1981), Karnik and Tavoularis (1983), Gibson and Kanellopoulos (1987), and Rohr et al. (1988). The latter provide a useful survey of the experiments and include a careful analysis of the governing equations.

One of the principal difficulties in interpreting the experiments has been to determine whether or not the shear flow reaches full development to a postulated asymptotic state which is determined by the mean shear, and not by the distance to the shear generator. A particular problem in this regard has been identifying what the asymptotic state should be. The earliest studies appeared to expect a state in which the energy reached a constant value, while the consensus of the more recent work is that it continues to increase monotonically. Rohr et al. argue convincingly that the wind tunnel experiments reasonably simulate a homogeneous shear flow evolving in time in the convected frame of reference, and provide a qualified endorsement for the semi-empirical argument of Tavoularis (1985) that the turbulence moments increase exponentially with time (or distance downstream).

It is the purpose of this paper to present a new theory of homogeneous shear flows in which the flows achieve an asymptotic state which is both self-preserving and determined by the initial conditions. The present theory is an outcome of recent studies on the nature of self-preservation in turbulent flow. George (1988) has recently re-examined the conditions for similar solutions of the equations of motion for inhomogeneous shear flows, with results that highlight the importance of the initial conditions in determining the asymptotic state of the developing flow. In a separate study of the decay of homogeneous isotropic turbulence, the same author (George 1987a,1987b) found self-preserving solutions to the spectral energy equation which were valid at all scales of motion. In this case the velocity scale was obtained from the turbulence kinetic energy, and the characteristic length scale was shown to be the Taylor microscale. From the conditions for self-preservation, it was possible to show that the energy decay followed a power law in time, while the Taylor microscale increased with the square root of time. It was also shown that a universal law for the energy decay cannot exist because the exponent and constants must depend on the initial conditions.

The evolution of homogeneous shear flow is now to be treated in a similar manner. As in George (1987a,b), the starting point is the dynamical equation for the energy spectrum (Hinze 1975), this time with additional terms arising from the presence of a mean shear rate and anisotropy. Self-preserving solutions are sought for which all terms in the equation retain the same relative magnitude as the flow evolves. As in the earlier analysis of isotropic turbulence, the characteristic velocity and length scales will be shown to be those determined from the energy and the Taylor microscale.

The analysis shows that two types of self-preserving solutions are possible: The first appears to be of academic interest only, at present, since it follows from the similarity constraints that the mean rate of shear must vary inversely as time (or distance), a condition that has not yet been realised experimentally. The second type of solution corresponds closely to actual experimental conditions since self-preservation requires that both the Taylor microscale and the mean shear must be constant. Full self-preservation also requires that the energy spectra collapse to a single curve, unique to each set of initial conditions, and that the integral scale must be constant.

The equations and theory are developed in the next two sections of the paper. The analysis is followed by a discussion of the development of real flows in space (as distinct from the development of the theoretical flow in time) and the difficulty of achieving exact homogeneity in the former. The experimental evidence is then examined. It is found that the predictions of the theory are consistent with the experimental data. In particular, the Taylor microscale is indeed constant in all developed flows, and the measured one-dimensional energy spectra collapse well, especially considering the accuracy of the data and the degree to which the experiments simulate a homogeneous shear flow.

2. The Spectral Equations

The two-point Reynolds stress equations under the assumption of homogeneous flow can be derived as (Hinze 1975),

$$\frac{\partial}{\partial t}C_{i,k} + (U_j' - U_j)\frac{\partial}{\partial r_j}C_{i,k}$$
$$= P_{i,k} + T_{i,k} - \left[C_{j,k}\frac{\partial U_i}{\partial x_j} + C_{i,j}\frac{\partial U_k'}{\partial x_j'}\right] + 2\nu\frac{\partial^2}{\partial r_j \partial r_j}C_{i,k} \qquad (1)$$

where $C_{i,k}$ is the two point Reynolds stress tensor defined as

$$C_{i,k} = C_{i,k}(\underline{r}) = \overline{u_i(\underline{x})u_k(\underline{x}+\underline{r})} \qquad (2)$$

and $\underline{r}$ is the separation vector defined as

$$\underline{r} = \underline{x}' - \underline{x} \tag{3}$$

The mean velocity is space dependent, $\underline{U}'$ denoting the velocity at the point $\underline{x}'$ and $\underline{U}$ the velocity at point $\underline{x}$.

The term $P_{i,k}$ represents the pressure-velocity interaction term defined by

$$P_{i,k} = P_{i,k}(\underline{r}) = \left[\frac{\partial}{\partial r_k} C_{i,p} - \frac{\partial}{\partial r_i} C_{p,k}\right] \tag{4}$$

where

$$C_{i,p} = C_{i,p}(\underline{r}) = \frac{1}{\rho}\,\overline{u_i(\underline{x})\,p(\underline{x}+\underline{r})} \tag{5}$$

and

$$C_{p,k} = C_{p,k}(\underline{r}) = \frac{1}{\rho}\,\overline{p(\underline{x})\,u_k(\underline{x}+\underline{r})} \tag{6}$$

The term $T_{i,k}$ represents the divergence of the triple velocity correlations and is defined by

$$T_{i,k} = T_{i,k}(\underline{r}) = \frac{\partial}{\partial r_j}[C_{i,jk} - C_{ij,k}] \tag{7}$$

where

$$C_{i,jk} = C_{i,jk}(\underline{r}) = \overline{u_i(\underline{x})\,u_j(\underline{x}+\underline{r})\,u_k(\underline{x}+\underline{r})} \tag{8}$$

and

$$C_{ij,k} = C_{ij,k}(\underline{r}) = \overline{u_i(\underline{x})\,u_j(\underline{x})\,u_k(\underline{x}+\underline{r})} \tag{9}$$

Thus the terms on the right-hand side of equation (1) represent respectively the contributions of pressure, inertia, gradient production, and viscous stresses to the rate of change of the two-point Reynolds stress tensor. The presence of third-order and the pressure-velocity correlations are manifestations of the essential non-linearity of the turbulence equations and the lack of closure it introduces into the averaged equations. The second term on the left hand side arises from the non-uniform mean velocity.

Hereafter we shall assume the mean velocity to be in the x_1 direction only and to vary linearly in the x_2 direction. Thus, if K is the constant value of the velocity gradient, U_i can be represented as

$$U_i = Kx_2\delta_{i1} \tag{10}$$

Equations (1) now reduce to

$$\frac{\partial}{\partial t}C_{i,k} + Kr_2\frac{\partial}{\partial r_1}C_{i,k} = P_{i,k} + T_{i,k} - K[C_{2,k}\delta_{i1} + C_{i,2}\delta_{k1}] + 2\nu\frac{\partial^2}{\partial r_j \partial r_j}C_{i,k} \tag{11}$$

Since the turbulence field is assumed to be homogeneous, equations governing the spectral tensor of the two-point Reynolds stress can be obtained by taking the three-dimensional Fourier transforms of equations (11). We define spectral tensors corresponding to each of the two-point correlations as follows

$$\Phi_{i,k}(\underline{k}) = \iiint_{-\infty}^{\infty} e^{i\underline{k}\cdot\underline{r}} C_{i,k}(\underline{r})\, d\underline{r} \tag{12}$$

$$\Pi_{i,k}(\underline{k}) = \iiint_{-\infty}^{\infty} e^{i\underline{k}\cdot\underline{r}} P_{i,k}(\underline{r})\, d\underline{r} \tag{13}$$

and

$$\Gamma_{i,k}(\underline{k}) = \iiint_{-\infty}^{\infty} e^{i\underline{k}\cdot\underline{r}} T_{i,k}(\underline{r})\, d\underline{r} \tag{14}$$

Using these the two-point Reynolds stress equations for linear mean velocity become

$$\frac{\partial}{\partial t}\Phi_{i,k} - Kk_1\frac{\partial}{\partial k_2}\Phi_{i,k}$$
$$= \Pi_{i,k} + \Gamma_{i,k} - K[\Phi_{2,k}\delta_{i1} + \Phi_{i,2}\delta_{k1}] + 2\nu k_j k_j \Phi_{i,k} \tag{15}$$

where the second term has been obtained by integrating by parts.

This equation can be cast into the more familiar form of an equation for the three-dimensional energy spectrum function E(k). E(k) is defined as the average over spherical shells of radius k, of the contracted three-dimensional spectrum. Thus

$$E(k) = \frac{1}{2}\iint_{k=|\underline{k}|} \Phi_{i,i}(\underline{k})\, d\sigma(\underline{k}) \tag{16}$$

where $\Phi_{i,i}$ represents the contraction of $\Phi_{i,k}$. It follows from the definition of $\Phi_{i,k}$ that the integral of E(k) over all wavenumbers yields the kinetic energy per unit mass, i.e.

$$\frac{1}{2}q^2 = \frac{1}{2}\overline{u_i u_i} = \int_0^{\infty} E(k)\, dk \tag{17}$$

Integration of equation (15) over spherical shells of radius k yields

$$\frac{\partial}{\partial t}E(k,t) - K\iint_{k=|\underline{k}|} k_1 \frac{\partial}{\partial k_2}\Phi_{i,i}(\underline{k})\, d\sigma(\underline{k})$$
$$= \pi(k) + \gamma(k) - 2K\iint_{k=|\underline{k}|} Co_{1,2}(\underline{k})\, d\sigma(\underline{k}) - 2\nu k^2 E(k) \quad (18)$$

where

$$\pi(k) = \iint_{k=|\underline{k}|} \Pi_{i,i}(\underline{k})\, d\sigma(\underline{k}) \quad (19)$$

and

$$\gamma(k) = \iint_{k=|\underline{k}|} \Gamma_{i,i}(\underline{k})\, d\sigma(\underline{k}) \quad (20)$$

The cospectrum, $Co_{1,2}$ is defined as the real part of $\Phi_{1,2}$, and arises in this context from the fact that

$$\Phi_{i,j}(\underline{k}) + \Phi_{j,i}(\underline{k}) = \Phi_{i,j}(\underline{k}) + \Phi^{*}_{i,j}(\underline{k}) = 2Co_{i,j}(\underline{k}) \quad (21)$$

where * denotes the complex conjugate.

The second term on the left side of equation (18) has been identified by Lumley (1964) as giving rise to a spectral scrambling due to the mean shear. The third term on the right side is the production of turbulence kinetic energy by the working of Reynolds stresses against the mean velocity gradient. The remaining terms on the right side of equation (18) arise in homogeneous flows in the absence of shear and have been discussed in detail by Batchelor (1953); They represent respectively the spectral energy transfer to the wavenumber k from all other wavenumbers due to the nonlinear interactions, the spectral transfer due to the pressure-velocity interactions, and the viscous dissipation.

3. Self-Preserving Solutions of the Spectral Equations

We seek self-preserving solutions of the energy spectral equations for which all terms in the equation remain in relative balance. For now we examine only equation (18) obtained by averaging over spherical shells of radius k. We begin by assuming solutions of the form

$$E(k,t) = E_s(t)\ f_1(\eta) \tag{22}$$

$$\gamma(k,t) = \gamma_s(t)\ g(\eta) \tag{23}$$

$$\pi(k,t) = \pi_s(t)\ h(\eta) \tag{24}$$

$$-\iint_{k=|\underline{k}|} k_1 \frac{\partial}{\partial k_2} \Phi_{1,1}(\underline{k})\, d\sigma(\underline{k}) = Q_s(t)\ f_2(\eta) \tag{25}$$

$$-2\iint_{k=|\underline{k}|} Co_{1,2}(\underline{k})\, d\sigma(\underline{k}) = R_s(t)\ f_3(\eta) \tag{26}$$

where

$$\eta = kL \tag{27}$$

and

$$L = L(t) \tag{28}$$

These can be substituted directly into equation (18) so that the transformed equation becomes

$$[\dot{E}_s]f_1 + [E_s\frac{\dot{L}}{L}]\eta f_1' + [KQ_s]f_2$$
$$= [\pi_s]h + [\gamma_s]g + [KR_s]2f_3 - [\nu E_s L^{-2}]2\eta^2 f_1 \qquad (29)$$

where ' denotes differentiation with respect to η.

For self-preservation, all of the bracketed terms must have the same time-dependence since they depend explicity on time. It is convenient to divide by the last term in brackets so that the equation reduces to

$$[\frac{\dot{E}_s L^2}{\nu E_s}]f_1 + [\frac{L\dot{L}}{\nu}]\eta f_1' + [\frac{KQ_s L^2}{\nu E_s}]f_2$$
$$= [\frac{\pi_s L^2}{\nu E_s}]h + [\frac{\gamma_s L^2}{\nu E_s}]g + [\frac{KR_s L^2}{\nu E_s}]2f_3 - [1]2\eta^2 f_1 \qquad (30)$$

It is clear from the presence of the constant coefficient multiplying the last term that self-preserving solutions are possible only if the other bracketed terms are also constant. Therefore we must have,

(a) $[\frac{\dot{E}_s L^2}{\nu E_s}] = \text{const}$ (b) $[\frac{L\dot{L}}{\nu}] = \text{const}$

(c) $[\frac{KQ_s L^2}{\nu E_s}] = \text{const}$ (d) $[\frac{\pi_s L^2}{\nu E_s}] = \text{const}$ (31)

(e) $[\frac{\gamma_s L^2}{\nu E_s}] = \text{const}$ (f) $[\frac{K R_s L^2}{\nu E_s}] = \text{const}$

Conditions (31d) and (31e) assure us that π_s and γ_s must be proportional, and can be chosen equal with a possible constant factor absorbed into either $h(\eta)$ or $g(\eta)$. Thus,

$$\pi_s \sim \gamma_s \tag{32}$$

It is easy to see from the definitions that Q_s, R_s and E_s are all related directly to the spectral tensor Φ_{ik}. Since for self-preservation each of the components of Φ_{ik} reach equilibrium with respect to each other so that they grow or decay together, it follows that

$$R_s \sim Q_s \sim E_s \tag{33}$$

so that a single scale can be chosen for all the second order quantities.

With the above considerations, the conditions for the self-preservation of a homogeneous shear flow reduce to

$$[\frac{\dot{E}_s L^2}{\nu E_s}] = \text{const} \tag{34a}$$

$$[\frac{L\dot{L}}{\nu}] = \text{const} \tag{34b}$$

$$[\frac{K L^2}{\nu}] = \text{const} \tag{34c}$$

$$[\frac{\gamma_s L^2}{\nu E_s}] = \text{const} \tag{34d}$$

The function E_s and the length scale L can be related directly to physical quantities by considering the energy and dissipation integrals. Consider first the kinetic energy defined by

$$\frac{q^2}{2} = \int_0^\infty E(k)\,dk = [E_s L^{-1}] \int_0^\infty f_1(\eta)\,d\eta \tag{35}$$

It follows immediately from equation (35) that we can take

$$E_s = q^2 \cdot L \tag{36}$$

without loss of generality by absorbing a constant into f_1.

The rate of dissipation of turbulent energy is given by

$$\epsilon = 2\nu \int_0^\infty k^2 E(k)\,dk = [\nu E_s L^{-3}]\, 2 \int_0^\infty \eta^2 f_1(\eta)\,d\eta \tag{37}$$

Thus, from equations (36) and (37) it follows that

$$\epsilon \sim \nu q^2 L^{-2} \tag{38}$$

But the Taylor microscale, λ, is defined as

$$\overline{\left[\frac{\partial u_1}{\partial x_1}\right]^2} = \frac{\overline{u_1^2}}{\lambda^2} \tag{39}$$

and the dissipation can be written as

$$\epsilon = 15 C \nu \overline{\left[\frac{\partial u_1}{\partial x_1}\right]^2} = 5 C \nu \frac{q^2}{\lambda^2} \tag{40}$$

where for isotropic turbulence C = 1. Thus we can take

$$L = \lambda \tag{41}$$

without loss of generality, since the turbulence quantities are assumed to be in equilibrium with respect to each other. Therefore, the length scale, L, can be identified as the Taylor microscale, λ, and

$$E_s = q^2 \lambda \tag{42}$$

The Reynolds shear stress can be obtained by integrating equation (26) so that

$$\begin{aligned} -\overline{u_1 u_2} &= \int_0^\infty \left\{ -2 \iint_{k=|\underline{k}|} Co_{1,2}(\underline{k})\, d\sigma(\underline{k}) \right\} dk \\ &= [R_s \ \lambda^{-1}] \int_0^\infty f_3(\eta)\, d\eta \end{aligned} \tag{43}$$

From equations (33) and (42) it follows immediately that

$$-\overline{u_1 u_2} \sim q^2$$

or

$$-\overline{u_1 u_2} = D \ q^2 \tag{44}$$

where D is a constant.

The rate of production of turbulence kinetic energy is then given by

$$P = -\overline{u_1 u_2} \ K = D \ q^2 \ K \tag{45}$$

The ratio of production to dissipation can now be obtained from equations (40), (45), and (34c) as

$$P/\epsilon = \frac{D}{5C} \left[\frac{K\lambda^2}{\nu}\right] = \text{constant} \qquad (46)$$

Integration of equation (34b) yields the time dependence of L (or λ) as

$$\lambda^2 = A\nu t + B \qquad (47)$$

where A and B are constants. Two cases can be identified: one in which the length scale is constant, the other in which it increases with the square root of time; i.e.,

Case (i), $A = 0$: $\quad \lambda^2 = B = \lambda_0^2 \qquad$ (48a)

Case (ii), $A \neq 0$: $\quad \lambda^2 = A\nu(t'-t_0) = A\nu t \qquad$ (48b)

(Note that in case (ii) we have taken $-A\nu t_0 = \lambda_0^2$ without loss of generality, and have shifted the time origin to account for a non-zero value of λ_0^2.) To this point in the analysis, the results apply equally to the self-preserving development of shear-free homogeneous turbulence. When $K = 0$, case (ii) corresponds to the decay of homogeneous turbulence as discussed by George (1987a,b).

When K is not equal to zero, its time variation is constrained by equation (34c). For self-preserving solutions to exist we must then have

$$K/\nu \propto \lambda^{-2} = (At + B)^{-1} \qquad (49)$$

Consequently, cases (i) and (ii) above can be identified with a specific time dependence of the mean flow field as follows:

Case (i), $\lambda = \text{constant} = \lambda_0$, $K = \text{constant}$ (50a)

Case (ii), $\lambda = [A\nu(t-t_0)]^{1/2}$, $K \propto [A(t-t_0)]^{-1}$ (50b)

Since all experiments to-date have been carried out at constant mean shear, only case (i) will be considered further here. We do, however, note the possible existence of a second class of similarity solutions for homogeneous shear flows where the mean shear varies inversely with time, the Taylor microscale increases as the inverse square root of time, and the second moments vary according to a power law in time.

For constant mean shear, it follows from equations (34a) and (50a) that

$$\frac{\dot{E}_s}{E_s} = \beta\frac{\nu}{\lambda_0^2} \tag{51}$$

where β is the constant of equation (34a) and is determined by the initial conditions. Therefore

$$E_s(t) = E_s(0)\ \exp\{\beta\nu t/\lambda_0^2\} \tag{52}$$

where $E_s(0)$ is the energy spectrum at some reference time chosen as t=0.

Equation (42) can be used together with equation (50a) to yield the turbulence kinetic energy as

$$\begin{aligned} q^2 &= [\lambda]^{-1}(t)E_s(t) \\ &= \lambda_0^{-1}E_s(0)\exp\{\beta\nu t/\lambda_0^2\} \\ &= q^2(0)\ \exp\{\beta\nu t/\lambda_0^2\} \end{aligned} \tag{53}$$

where $q^2(0)$ is the initial value of q^2. We can define a more convenient rate constant, β', using equation (34c) as

$$\beta' = \beta\left[\frac{\nu}{\lambda_0^2 K}\right] \tag{54}$$

Note that β', like β, is also dependent on the initial conditions. Now if time is non-dimensionalized by the mean shear rate so that

$$\tau = Kt, \tag{55}$$

then equation (53) can be rewritten as

$$q^2 = q^2(0)\exp\{\beta' Kt\} \tag{56}$$

Thus for self-preservation of a homogeneous turbulent flow with constant mean shear, the turbulent energy increases (or decreases) exponentially at a rate determined by the initial conditions and the mean shear rate.

The growth rate constant of the turbulence kinetic energy, β', can be expressed in terms of $-\overline{u_1u_2}/q^2$ and P/ϵ using the turbulence kinetic energy equation. By evaluating equation (1) at $\underline{r} = 0$ (or by integrating equation (18) over all wavenumbers) we obtain

$$\frac{d}{dt}\left(\frac{1}{2}q^2\right) = P - \epsilon$$

$$= Kq^2 \left[\frac{-\overline{u_1u_2}}{q^2}\right] [1 - \epsilon/P] \qquad (57)$$

From equations (44), (46), (55) and (56), it follows that

$$\beta' = 2D[1 - \epsilon/P] \qquad (58)$$

which is the result obtained from equation (57) by Tavoularis (1985). Equation (58) shows that the (constant) value of P/ϵ will determine whether q^2 will increase or decrease. Tavoularis assumed that $\overline{u_1u_2}/q^2$ and P/ϵ would be constants in homogeneous shear flow. It has now been shown that this is a necessary condition for self-preservation. It follows that the shear stress, the energy components, and the energy production and dissipation rates all grow exponentially at the same rate. The principal results of the theory for self-preserving development are summarised in Table I.

TABLE I

Self-Preservation Constraints for Constant Mean Shear

Mean Shear	$K\ (=dU/dy)$	$=$ constant
Length Scale	$L\ (=\lambda)$	$= \lambda_0 =$ constant
Energy Spectrum	$E_s(t)$	$= E_s(0)\exp\{\beta' Kt\}$
Spectral Transfer	γ_s	$\propto \nu E_s/K\lambda_0^{\ 2}$
Kinetic Energy	q^2	$= q^2(0)\ \exp\{\beta' Kt\}$
Shear Stress/Kinetic Energy	$-\overline{u_1u_2}/q^2$	$= D =$constant
Production/Dissipation	P/ϵ	$= \frac{D}{5C}\left[\frac{K\lambda_0^{\ 2}}{\nu}\right]$
Growth Rate	β'	$= 2D[1 - \epsilon/P]$

Part II: EXPERIMENTAL VERIFICATION

4. Spatial Development and the Experiments

Of the two possible cases of self-preserving flow identified in the analysis, only the homogeneous flow with a constant mean shear has been studied experimentally (as far as we have been able to determine). The experiments have been carried out in steady flows in wind tunnels (and in one case a water channel) so that the rates of change with respect to time that appear in the governing equations considered above are replaced by space derivatives in the equations describing the experimental flows. This substitution raises fundamental questions about the way in which the steady flow developing in space models the time-dependent flow of the analysis.

The conditions sought in the experiments were a steady mean flow with a single uniform mean shear, and turbulence homogeneous in planes perpendicular to the mean flow direction. At first it seems to have been expected that the natural state would be one in which turbulence energy production would be balanced by dissipation, the convection by the mean flow as well as the turbulence transport being negligible by comparison. There is, in fact, no reason to expect such a balance. As the analysis shows, exponential energy growth (or decay) will result from all but the most special choice of initial conditions.

If uniform mean shear could be associated with exact transverse homogeneity, the rate of change of turbulent energy is given by equation (57) as the difference between production and dissipation. As Harris et al. (1977) and Tavoularis (1985) have pointed out, homogeneous conditions have not been exactly produced in the experiments. The difficulty arises because the kinetic energy equation for a laboratory flow which is steady in the mean and homogeneous in the transverse directions is given by

$$U_1 \frac{d}{dx_1}\left(\frac{q^2}{2}\right) = P - \epsilon \tag{59}$$

Now, since $U_1 = U_1(x_2)$, $U_2 = U_3 = 0$ and dU_1/dx_2 is a constant, the left-hand side of equation (59) depends on x_2 while the right-hand side does not, on account of the postulated homogeneity. Harris et al. (1977) concluded that "a steady rectilinear flow with constant velocity gradient plus transverse homogeneity of turbulence moments is impossible. The stationary flow cannot be homogeneous, not even transversely."

Thus, strictly speaking it is necessary to consider all the terms in the turbulence kinetic energy equation. With justifiable neglect of viscous transport, the full equation is.

$$U_1 \frac{\partial}{\partial x_1}\left(\frac{q^2}{2}\right) = -\overline{u_1 u_2}\frac{dU_1}{dx_2} - \epsilon - \left\{\frac{\partial}{\partial x_2}\left(\frac{\overline{q^2 u_2}}{2} + \overline{pu_2}\right) + \frac{\partial}{\partial x_1}\left(\frac{\overline{q^2 u_1}}{2} + \overline{pu_1}\right)\right\} \tag{60}$$

If, as is usually assumed, the shear stress and velocity gradient are uniform across the flow, the dependence of mean flow transport on x_2 must be accounted for by the cross-stream variation of the turbulent transport terms in equation (60), or possibly by a non-uniform rate of dissipation. All of the evidence suggests that turbulent transport in the streamwise direction is negligible, while the cross-stream turbulent transport in highly sheared flow has been estimated by Harris et al. (1977) to be only about 3% of the mean flow convection. There was also some evidence of transverse inhomogeneity of the dissipation rate in this experiment and in the very similar experiment of Tavoularis and Corrsin (1981), where the Taylor microscale increased slightly in the direction of increasing mean velocity. The overall effect on the energy balance of these departures from the ideal is therefore small as may be seen by rewriting equation (60) as,

$$U_0\frac{\partial}{\partial x_1}\left(\frac{q^2}{2}\right) = -\overline{u_1u_2}\frac{dU_1}{dx_2} - \epsilon + \left\{(U_0 - U_1)\frac{\partial}{\partial x_1}\left(\frac{q^2}{2}\right) - \frac{\partial}{\partial x_2}\left(\frac{\overline{q^2u_2}}{2} + \overline{pu_2}\right) - \frac{\partial}{\partial x_1}\left(\frac{\overline{q^2u_1}}{2} + \overline{pu_1}\right)\right\} \qquad (61)$$

where U_0 is a constant velocity, say the value on the tunnel axis. It will be appreciated that even if the individual terms in the brackets are not negligible, their difference probably is.

Hinze (1975) has also pointed out that the basic assumptions of homogeneous turbulence with constant mean shear are inconsistent since the generation of turbulence from the mean shear must simultaneously decrease the kinetic energy of the mean flow and thereby decrease the mean velocity gradient. In a real flow, this reduction in the mean shear causes a mean velocity gradient to develop in the streamwise direction which can be balanced in the mean continuity equation only by the corresponding development of a cross-stream component of the mean velocity. This effect has been observed only in the water tunnel experiment of Rohr et al. (1988) where the degradation of the mean gradient was rather small, but it may have been large enough to contribute to the decline in the ratio of production to dissipation, an unusual feature of this experiment.

It is no doubt possible, then, to devise a wind tunnel experiment that will satisfy equation (57) to sufficient accuracy to test the theory, at least for scales of motion smaller than the characteristic dimensions of the facility. The remaining problem is that of ensuring that the shear flow has sufficient length to develop. In all the experiments the upstream part of the flow is dominated by the decay of the initial turbulence caused by the shear generator, and only far downstream has most of the turbulence been produced by interaction with the mean shear. For flows with moderate-to-high mean shear rates, the dissipation rate passes through a minimum and eventually the ratio of production to dissipation rates (P/ϵ) becomes a constant (except in the flow of Rohr et al. (1988) where it was found to continue to decrease). Fully-developed conditions apparently occur when the dimensionless distance (time) given by

$$\tau = Kt = \frac{x_1}{U_0} \frac{dU_1}{dx_2} \tag{62}$$

is greater than about six, where x_1 is measured from the shear generator.

In the first experiments, made before 1977, the rates of shear were too low and the development lengths were too short for maximum values of τ to exceed 3.5. Thus only the later work, starting from the highly sheared flow of Harris et al. (1977), is relevant here. In this experiment, which was repeated and extended by Tavoularis and Corrsin (1981), the maximum value of τ was raised to 13 by the use of a high shear rate. An unexpected side-effect was an increase in the ratio P/ϵ from values near unity in the earlier weak shear flows to approximately 1.7. Karnik and Tavoularis (1983) subsequently made measurements in three flows with τ values as high as 29, and remarkably, P/ϵ values as low as 1.4. Gibson and Kanellopoulos (1987) tried to obtain a developed flow with a low shear rate, not too far from equilibrium. They were not entirely successful in this, but their flow with P/ϵ of 1.3 extended to $\tau = 8$. Finally, although some of the water-channel measurements of Rohr et al. (1988) were taken at distances up to $\tau = 30$, their P/ϵ ratios were not constant, as in the other experiments.

These five experiments then form the data base for testing the theoretical predictions. Note that no experiments for P/ϵ less than unity have been considered nor have any been reported.

5. The Conditions for Self-Preservation

The theory presented in Part I is directly verified if it can be shown that, when scaled with the turbulence intensity and Taylor microscale, the measured energy spectra are self-similar over *all* scales for which the flow can be assumed to be reasonably homogeneous. In addition, the measured microscales must achieve a constant value. The appropriate scaling for the measured one-dimensional spectra is then

$$F^1_{1,1}(k_1) = \overline{u_1^2}\ \lambda\ \tilde{F}^1_{1,1}(k_1\lambda) \qquad (63)$$

and

$$F^1_{2,2}(k_1) = \overline{u_2^2}\ \lambda\ \tilde{F}^1_{2,2}(k_1\lambda) \qquad (64)$$

where $F^1_{1,1}$ and $F^1_{2,2}$ are longitudinal wavenumber spectra for the 1- and 2- components of velocity respectively, and λ is the Taylor microscale defined by equation (39).

It follows from the considerations of Part I that self-preserving flow will have fixed ratios of the component energies, and of $-\overline{u_1u_2}/q^2$. The turbulence energy, its components, and the components of the Reynolds stresses will increase exponentially with τ, and the ratio P/ϵ will also be constant. The integral scales, obtained either from integration of the autocorrelation coefficient or from the zero-frequency intercept of the one-dimensional spectrum, must be proportional to λ, and must therefore be constant. On the other hand, the turbulence length scale L given by

$$L = \frac{(q^2)^{3/2}}{\epsilon} \tag{65}$$

and the mixing length $(-\overline{u_1 u_2})^{1/2}/(dU_1/dx_2)$ will both increase exponentially as $(q^2)^{1/2} \sim \exp\{\beta' \tau/2\}$. The Kolmogorov microscale defined by

$$\eta = (\nu^3/\epsilon)^{1/4} \tag{66}$$

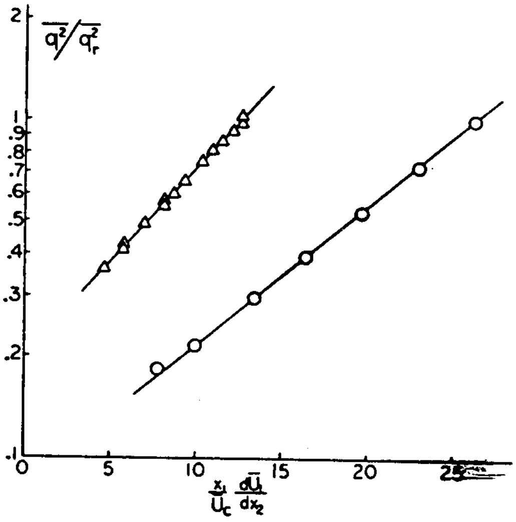
can easily be shown to decrease exponentially as $\exp\{-\beta' \tau/4\}$. The turbulence Reynolds number R_λ defined by

$$R_\lambda = \left(\frac{q^2}{3}\right)^{1/2}\left(\frac{\lambda}{\nu}\right) \tag{67}$$

increases with τ as $\exp\{\beta' \tau/2\}$.

6. The Turbulence Energy, Production and Dissipation Rates

All of the wind-tunnel experiments confirm that P/ϵ approaches a constant value as a homogeneous shear flow evolves, with *both* P and ϵ increasing as q^2. The dissipation rates have been variously obtained in the different experiments by assuming local isotropy and integrating velocity derivative spectra, from measured mean square velocity derivatives and the use of equation (40) with C = 1 for isotropic turbulence (v. Hinze), and by balance of the kinetic energy equation for the turbulence, equation (59). The only exception to the finding that $P \sim \epsilon$ is from the water-channel experiment of Rohr



△ $d\bar{U}_1/dx_2 = 46.8\ \text{sec}^{-1}$, $\bar{U}_c = 12.4\ \text{msec}^{-1}$, ○ $d\bar{U}_1/dx_2 = 84\ \text{sec}^{-1}$, $\bar{U}_c = 13.0\ \text{msec}^{-1}$; in both cases, the reference conditions are taken near the wind-tunnel exit.

Figure 1. Downstream development of turbulence kinetic energy (adapted from Tavoularis 1985).

et al. (1988) in which both P and ϵ increased nearly linearly, but at different rates to asymptotic values of P/ϵ of about 1.0 and 1.7 for the low and high shear rates respectively. This result appears to be inconsistent with weak exponential growth of the turbulence energy, and may be associated with the progressive degradation of the mean shear in this experiment. On the other hand, this is the only one of the experiments in which the Reynolds shear stress was not directly measured, so that the production had to be obtained from the energy balance, instead of the dissipation as was usually the case. As a consequence, the usual redundancy for the dissipation measurement was not available. In view of this, and the otherwise excellent agreement of the Rohr et al. experiment with the other experiments and with the predictions of the theory, perhaps this deviation should not be given too much weight.

According to equation (56) q^2 will vary exponentially for self-preservation. Tavoularis (1985) has demonstrated that this is the case for the two highly-sheared flows of Tavoularis and Corrsin (1981) and Karnik and Tavoularis (1983), where the rate constant of equation (58) was about 0.23 and 0.31-0.34 respectively. Figure (1) is reproduced from Tavoularis' paper. Tavoularis and Corrsin (1981) had assumed parabolic growth, but this is indistinguishable from weak exponential growth. In the weakly sheared flow of Gibson and Kanellopoulos (1987), the calculated rate constant of 0.0425 was so low that it was impossible to distinguish between exponential and linear growth of q^2 in the developed region. This was also the case for the data of Rohr et al. (1988) who showed both weakly exponential and linear growth to be consistent with the variation of the turbulence kinetic energy in their experiment.

7. The Taylor Microscale

In all of the developed homogeneous shear flows, the Taylor microscale is nearly independent of distance downstream, as predicted by the theory. Figure (2) plots the ratio $\lambda^2 K/\nu$ (which must be constant according to equation 34c) as a function of τ for several of the experiments. Immediately apparent is the fact that the ratio is reasonably constant and dependent on the initial conditions, as predicted. Because the intermediate scales respond more quickly to the imposition of the mean shear, the constant value of λ is attained upstream of the positions where the stress ratios reach constant values, and far upstream of full development of the integral scales (see, for example, Figure 4 of Harris et al. 1977 and Figure 7 of Gibson and Kanellopoulos 1987). The present theory is not invalidated by the slight transverse variation of λ found, for example, by Tavoularis and Corrsin (1981), which Tavalouris (1985) associates with the transverse variation of ϵ needed to balance the turbulence kinetic energy equation (as also discussed in Section 4 earlier).

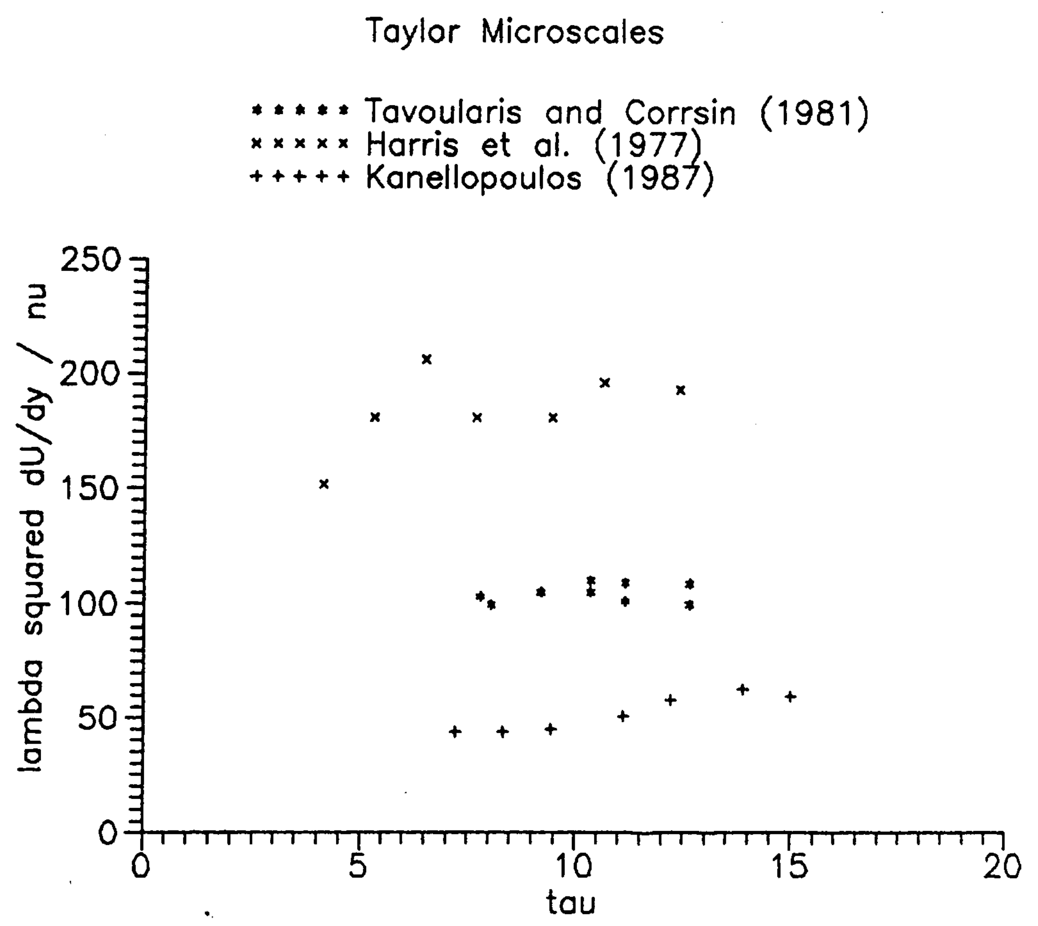


Figure 2. Variation of Taylor microscale with dimensionless time.

8. The Energy Spectra

One-dimensional energy spectra have been presented by Harris et al. (1977), by Kanellopoulos (1988), and by Rohr et al. (1988). Figure 3 shows the functions $F^{1}_{11}(k)$ scaled from Figure 14 of Harris et al. (1977) and replotted when normalized with $\overline{u^2}$ and λ. The curves correspond to measurements made at distances of x_1/h = 7.5, 9.5, and 11.0 where h is the tunnel height. The uniformity of the collapse of the spectra is remarkable in the data from the two downstream stations, and reasonably good overall if the divergence of the x_1 = 7.5 profile at low wavenumbers is ascribed to the slower adjustment of the large scales to the imposed uniform shear, or to the limits to the experimental model imposed by the tunnel walls. (Both of these points are discussed in more detail below.) The same comments apply to the Kanellopoulos (1988) spectra scaled with $\overline{u^2}$ and λ and plotted in Figure 4. At x_1/h = 14 and 15, the spectra collapse on a single curve, while the upstream data from x_1/h = 12 depart slightly at the lower wavenumbers.

Normalized spectra from these two experiments are plotted together in Figure 5. (Note that Tavoularis and Corrsin plot spectra down to frequencies as low as 1 Hz, while a lower limit of about 8 Hz was imposed by the instrumentation of Kanellopoulos.) It is clear from Figure 5 that each set of measurements has a unique spectral shape, consistent with the emphasis laid by the self-preservation analysis on the initial conditions. The comparison provides support for the theoretical

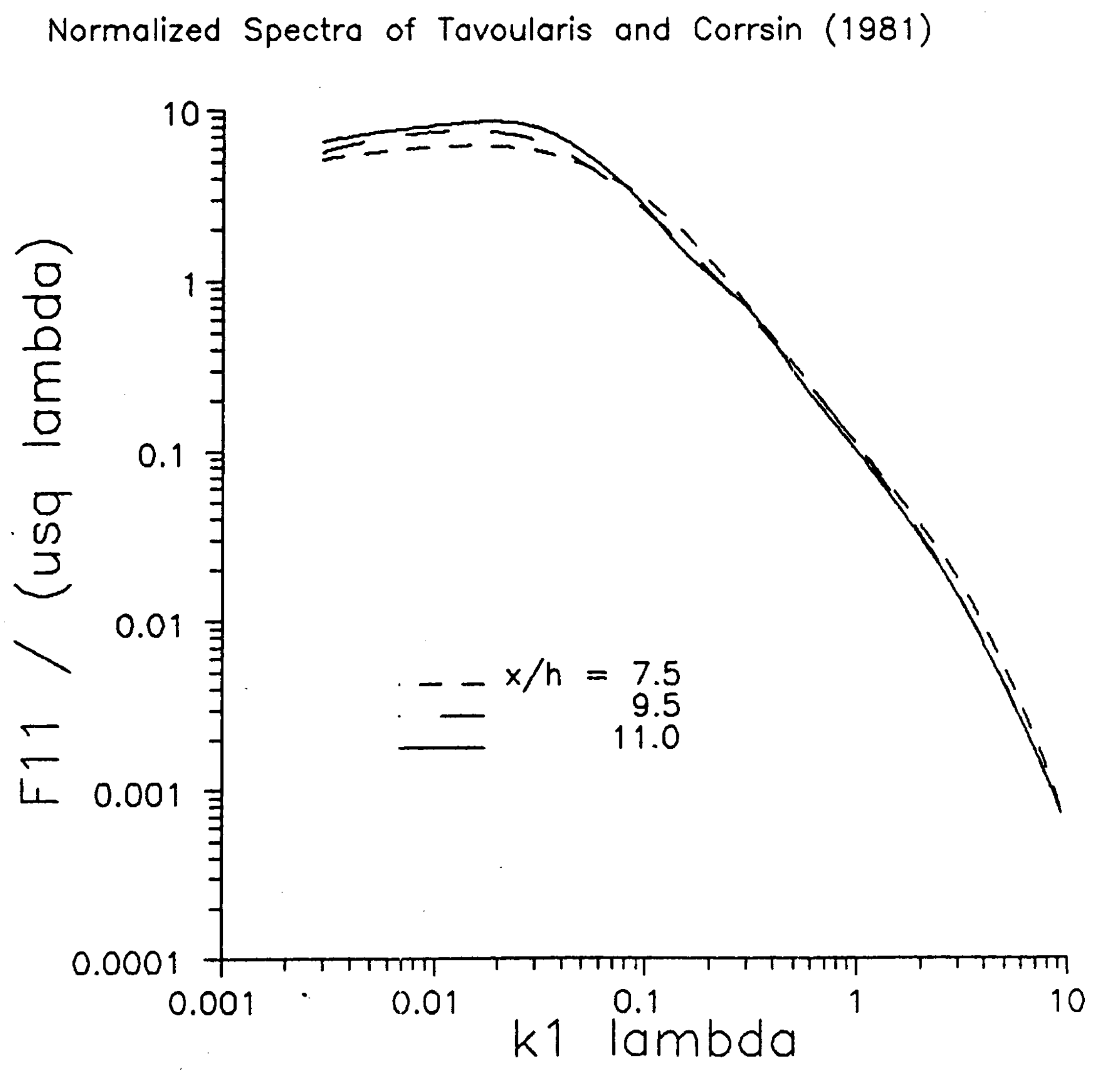


Figure 3. Normalized spectra of Tavoularis and Corrsin (1981).

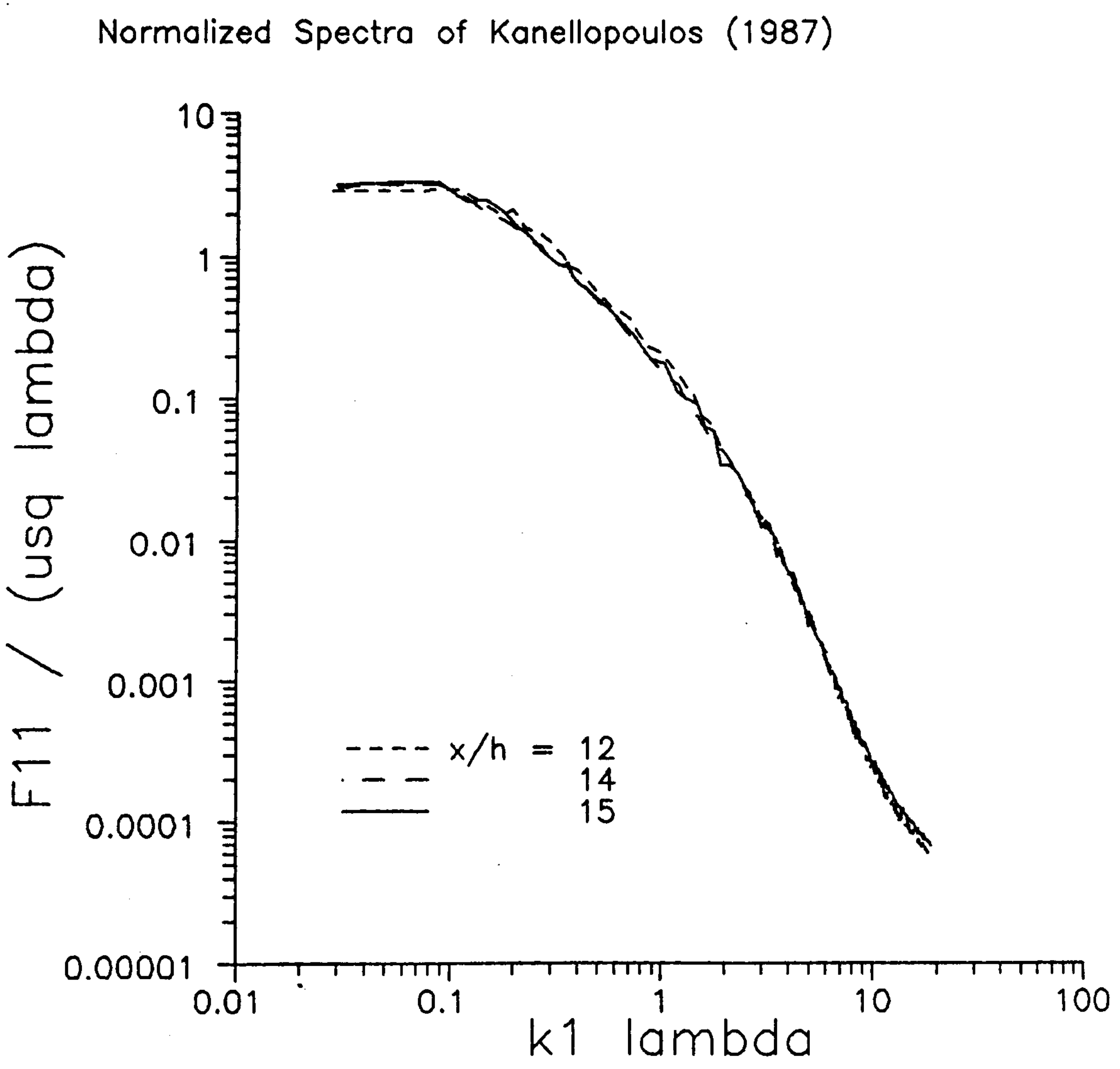


Figure 4. Normalized spectra of Kanellopoulos (1987).

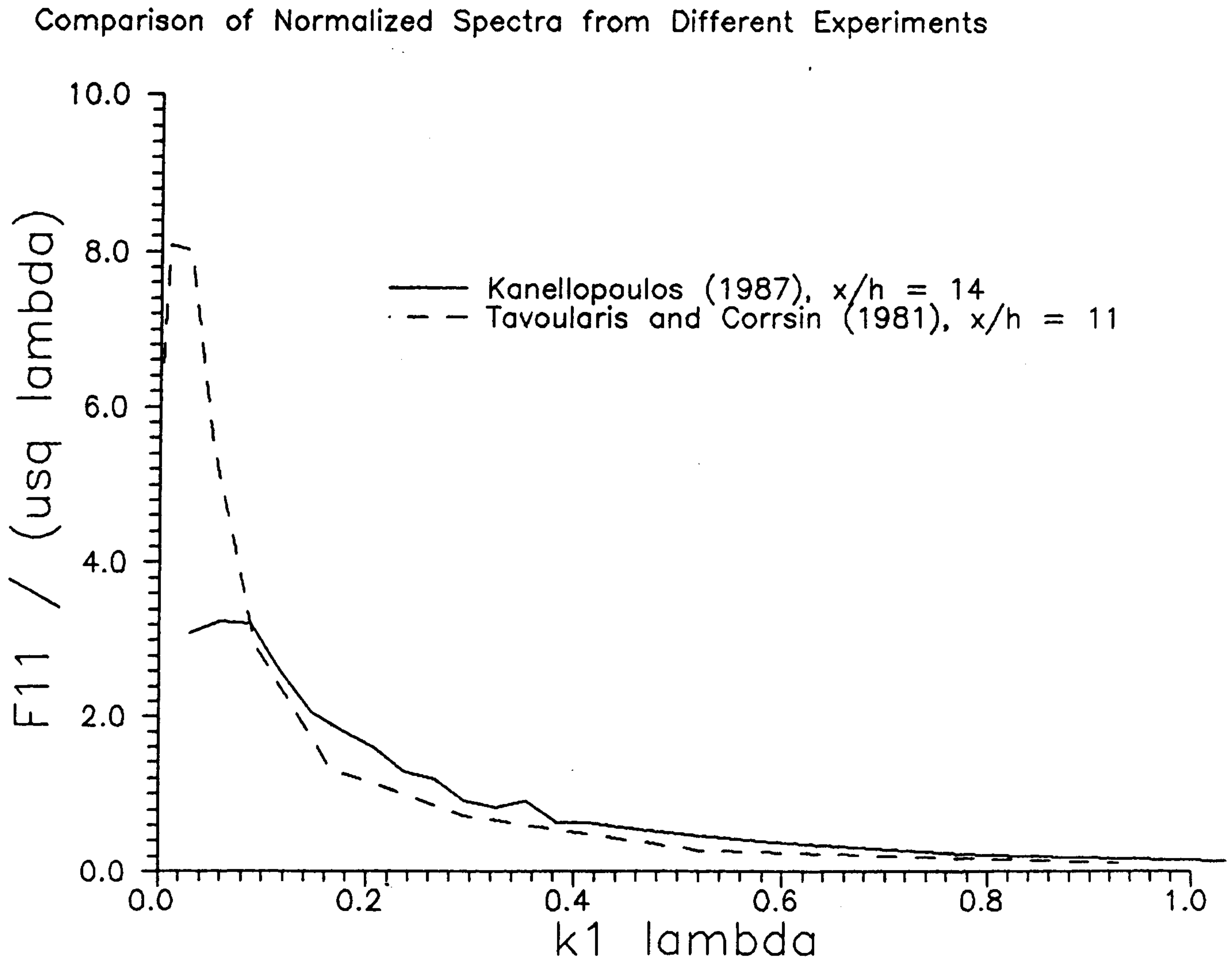


Figure 5. Comparison of normalized spectra for different experiments.

finding that although each flow is self-preserving, no universal spectrum for homogeneous flow exists.

The spectra of both streamwise and cross-stream velocities obtained by Rohr et al. (1988) from hot-film measurements in water have been rescaled in Taylor variables and plotted in Figures 6 and 7. Again, the spectra tend to collapse to a single curve, the exceptions being associated with the lowest wavenumbers of the data at the farthest upstream position. It is unfortunate that these authors discontinued their spectral measurements about half-way along their water channel.

While the lack of collapse of all the spectra at the lowest wavenumbers might in part be attributed to the slower adjustment of the larger scales to the imposed shear, there is also reason to question whether the flow at these scales reasonably models homogeneous shear flow in the absence of boundaries. All of the tunnels (wind and water) were square channels of 30 cm in extent, except for that of Kanellopoulos (1988) which was 45 cm. Thus, the lowest wavenumber of any possible physical significance for evaluating a model based on homogeneous flow would be $k_1 = 2\pi/30\ cm^{-1}$. (In fact, probably 4 - 5 times greater than this.) Since $\lambda \sim 1$ cm for all of the experiments, this would indicate that the data below about $k_1\lambda \sim 0.08 - 0.1$ is not representative of homogeneous flow in the absence of boundaries and should not be considered here, in which case the discrepancies at low wavenumber should be ignored. Figures 8 and 9 show the spectra of Rohr et al. plotted on a linear plot which

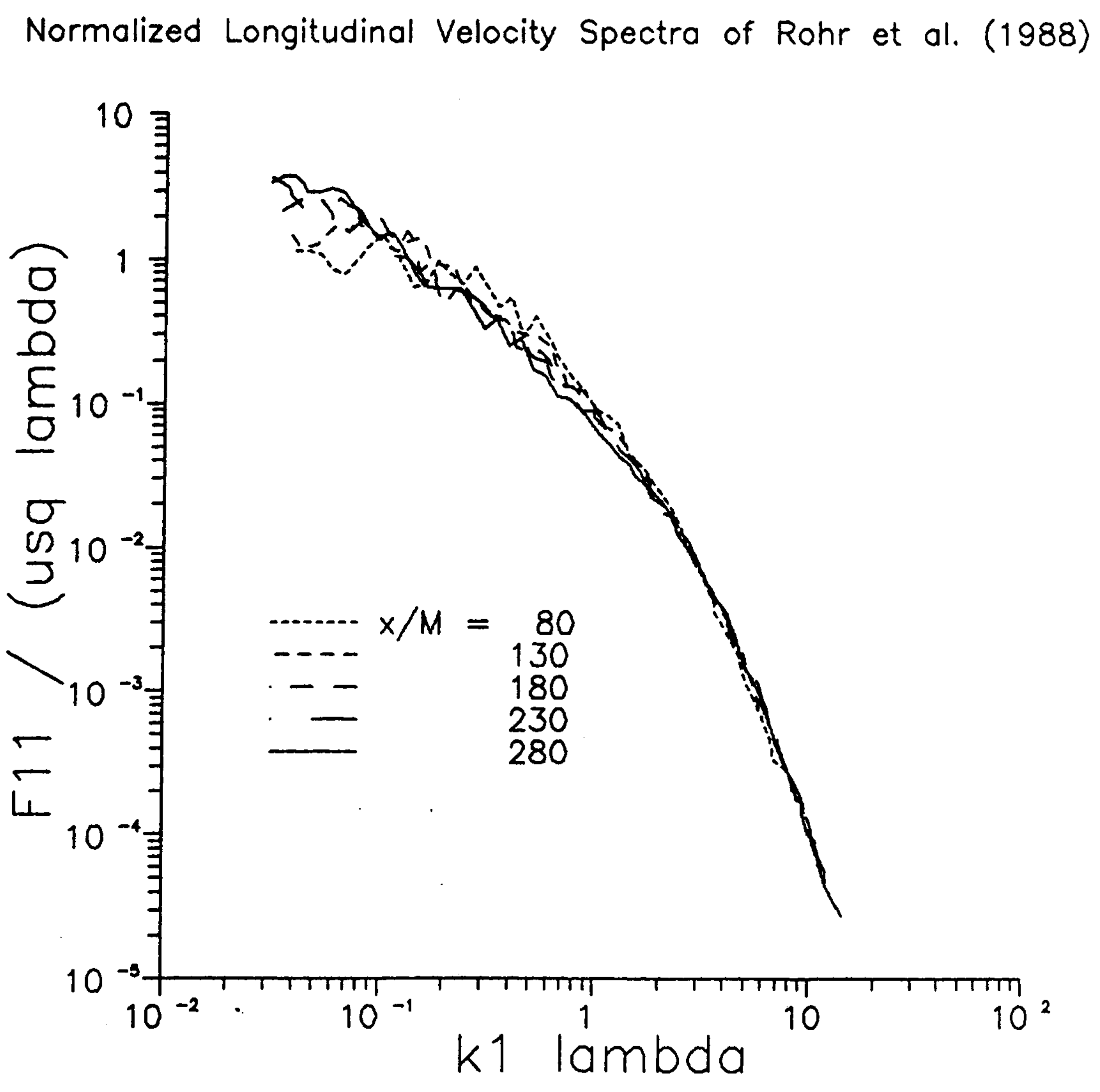


Figure 6. Normalized longitudinal velocity spectra of Rohr et al. (1988).

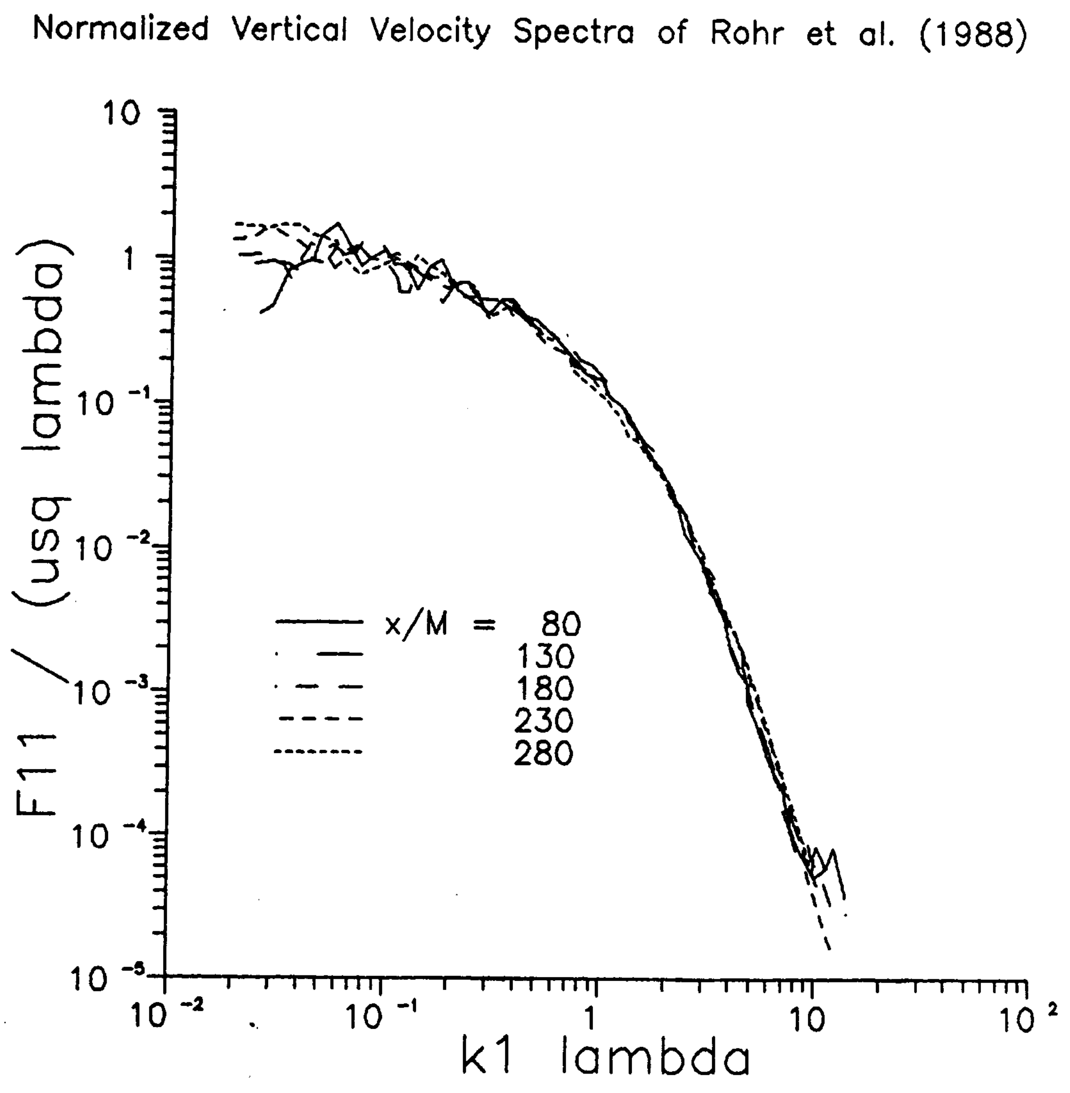


Figure 7. Normalized vertical velocity spectra of Rohr et al. (1988).

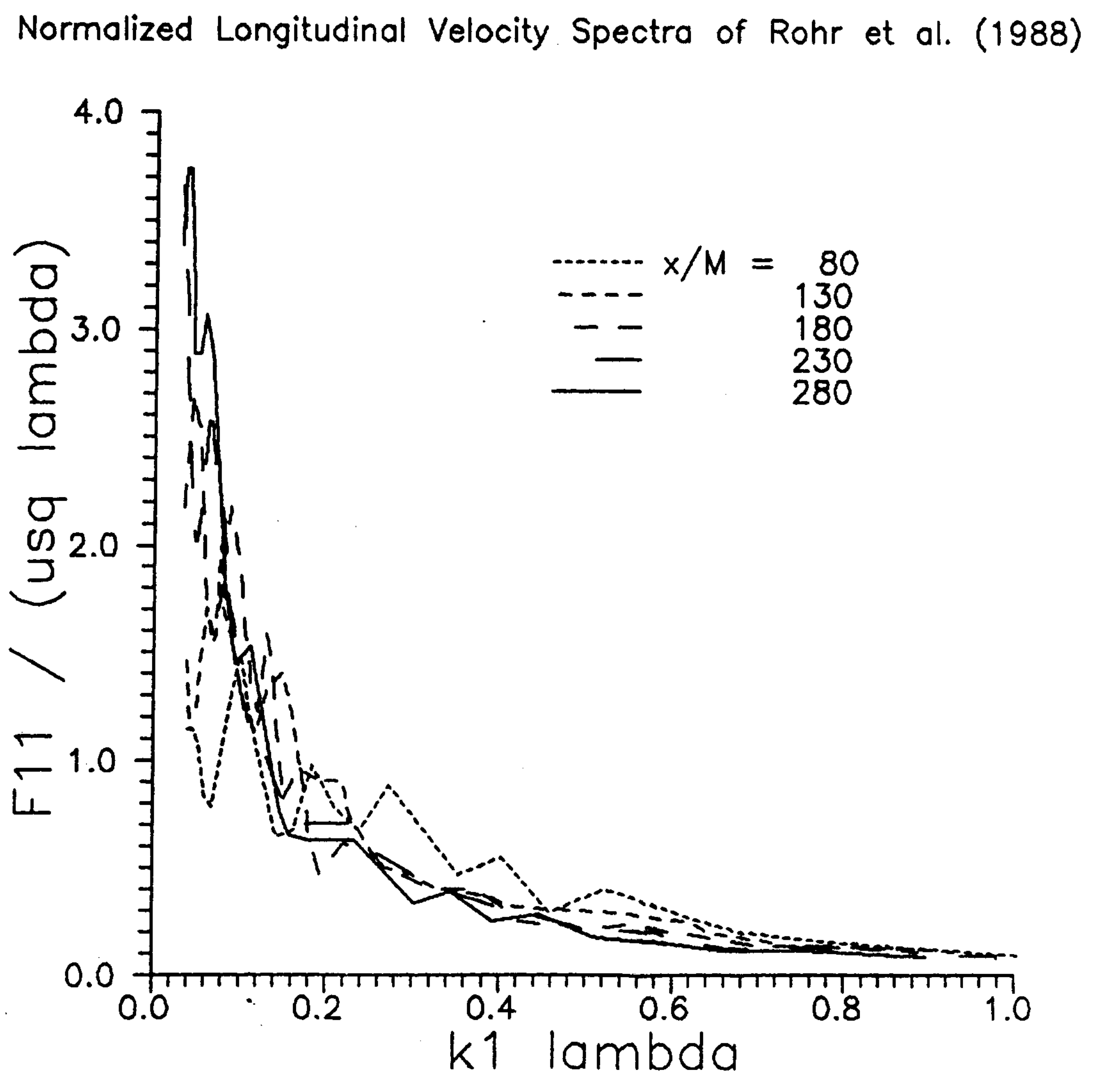


Figure 8. Normalized longitudinal velocity spectra of Rohr et al. (1988).

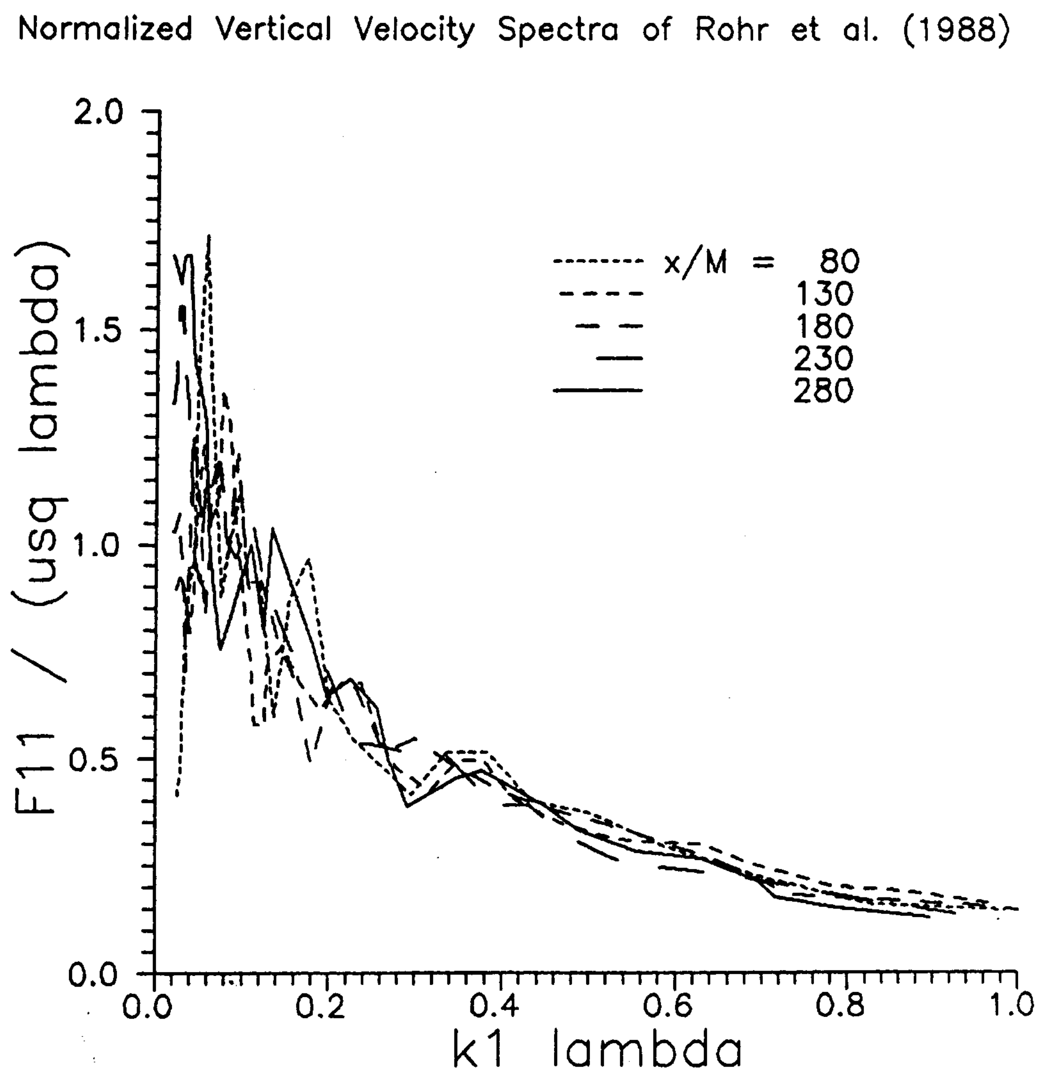


Figure 9. Normalized vertical velocity spectra of Rohr et al. (1988).

gives a more realistic picture of the situation at low wavenumbers than does a log plot. From these overlaid plots, it is apparent that even the low wavenumber data (to at least the largest wavenumbers for which the flow can be considered homogeneous) is consistent with the proposed scaling.

9. The Integral Scale

It follows that if the energy spectra are similar in a given flow, then the integral scale determined from the zero-wavenumber intercept must also obey the same scaling law. It must be admitted that there is not particularly strong support for this corollary of the similarity theory. Integral scales obtained from all the flows examined tend to increase with increasing distance downstream, but at different, and in most cases, diminishing rates. Dimensionless integral scales obtained from three sources are plotted against dimensionless distance (time) τ in Figure 10. In none of these flows is full development nearly approached for values of τ less than 6. Figure 10 shows that the rate of growth of L_1 is appreciable in the Tavoularis-Corrsin flow. On the other hand, Harris et al (1977), in an earlier realization of what was nominally the same flow, report near uniformity of L_1 at the last three downstream stations. The integral scales estimated by Gibson and Kanellopoulos (1987) mirror the quality of the spectral collapse in Figure 4, nearly equal values being obtained at the last two stations.

The largest eddies whose scale is L_1 take the longest time

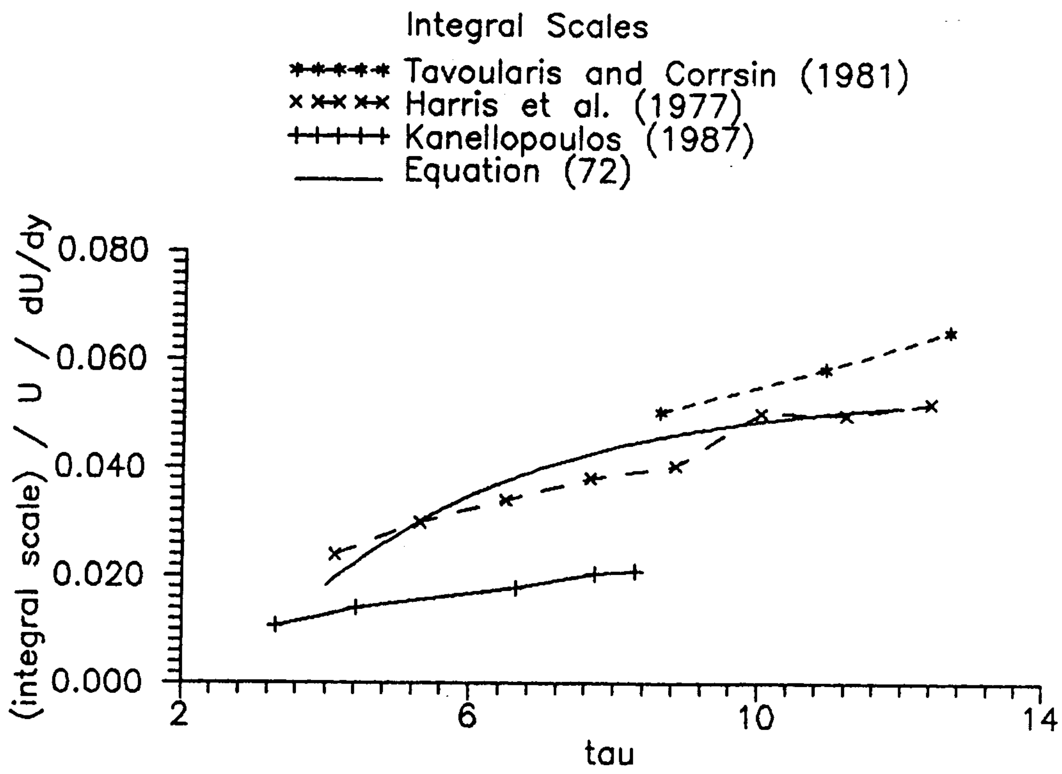


Figure 10. Variation of integral scales with dimensionless distance.

to adjust to changes in the mean flow, in this case to the imposition at the shear generator of a uniform rate of shear. Near the generator the initial "grid" turbulence decays, and the integral scale increases as the small scale motion decays fastest. The rate of evolution of the new large scale motions appropriate to homogeneous shear flow may be roughly estimated with the aid of a simple rate equation:

$$\alpha^{-1}\frac{dL_1}{dt_1} = L_{10} - L_1 \qquad (68)$$

where

$$t_1 = \frac{x_1}{U_1} \qquad (69)$$

is time measured in the convected frame, α is a rate constant, and L_{10} is the integral scale when the mean shear is imposed. The "adjustment time" of the largest eddies is roughly the ratio of turbulence energy to the rate of production, so that a suitable estimate of the rate constant is given by,

$$\alpha = \frac{-\overline{u_1u_2}}{q^2}\frac{dU_1}{dx_2} \qquad (70)$$

The solution to equation (68) is given by

$$L_1 = L_{10}\left[1 - e^{-\alpha t_1}\right] \qquad (71)$$

In Figure (10), we have plotted the variation

$$\frac{L_1}{U_1}\frac{dU_1}{dx_2} = 0.055\left\{1 - e^{-0.3(\tau-2.65)}\right\} \qquad (72)$$

which is based on the above arguments, and appears to correspond fairly well with the results of Harris et al. From equation (72) it is found that at the end of the tunnel used in the very similar experiments of Tavoularis and Corrsin (1981) and Harris et al. (1977) where τ=12.5, the integral scale reached roughly 98% of its asymptotic value. However, in the region from $8.5<\tau<12.5$ over which the spectral data were taken, the estimated change in the integral scale was from about 90% onwards, a change sufficient to explain the growth of L_1 in these experiments. Finally we note that at the end of the Gibson - Kanellopoulos flow with moderate shear, the integral scale is estimated to be at about 92% of its asymptotic value.

Perhaps the most persuasive evidence for the asymptotic constancy of the integral scale is provided by the measurements of Karnik and Tavoularis (1983) which extended to the largest values of τ (τ=29) of any of the experiments except for those of Rohr et al. (1988). Karnik and Tavoularis integrated the autocorrelation function up to its first zero with the results for their three basic experiments shown in their log-log plot reproduced in Figure (11a). The mean line through the data, which is plotted on linear scales in Figure (11b), shows a definite tendency to level off to some asymptotic value at large τ. The linear plot also shows clearly an initial high growth rate associated with the decay of "grid" turbulence created by the shear generator. This rate is diminished as the turbulence is dominated in the downstream flow by the turbulence newly created from the mean shear, and a point of inflexion appears.

All of the above considerations of the integral scales

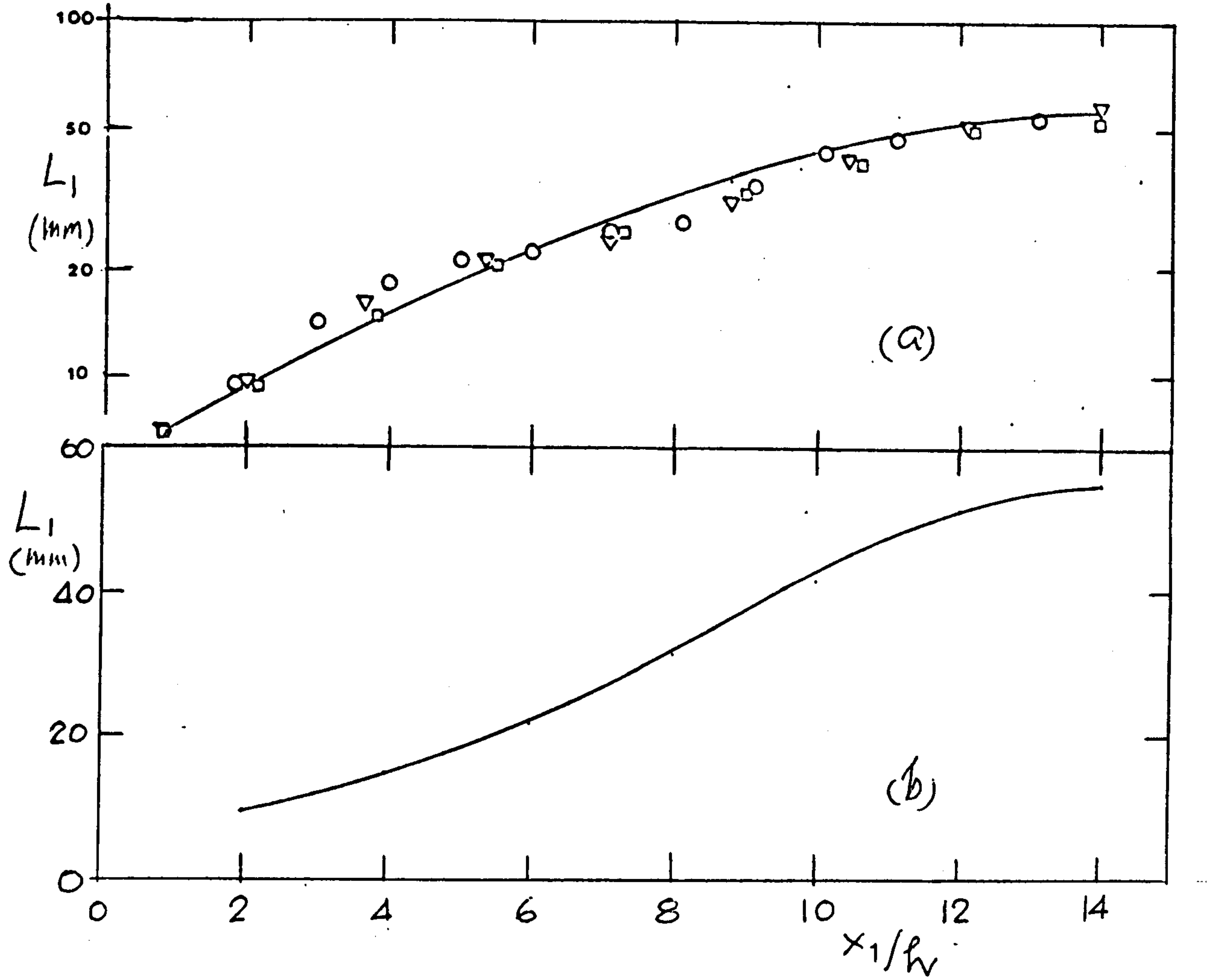


Figure 11. (a) Downstream development of streamwise velocity integral scales (from Karnik and Tavoularis 1983).
(b) Linear plot of same data.

ignore the question of whether or not the measured integral scales are indeed representative of a homogeneous shear flow developing in an infinite environment, or rather of one whose development is constrained by the tunnel in which it is generated. It is somewhat surprising that this question does not appear to have been previously addressed, particularly in view of the general concern in the various experiments for the behaviour of the integral scale. For example, Rohr et al. (1988) note their surprise at being able to collapse all of the statistical quantities from the different experiments by using the dimensionless time τ, except the measured integral scales. Similarly, Harris et al. (1977) note that their integral scales were nearly identical to the earlier ones of Champagne et al. (1970) even though the shear rates differed by a factor of 4. A value of 5-6 cm is typical of all the largest integral scales for the various experiments. While this is still relatively small compared to the tunnel height (30 cm), the integral scale determination from either spectrum or correlation intercept is strongly influenced by larger scales of motion which determine the tail of the autocorrelation and play an important role in determining the spectrum at low frequencies. As is clear from the spectral considerations above, there is substantial reason to doubt whether scales larger than the measured integral scales, yet so necessary to its determination, could have been reasonable approximations of the same scales in an unbounded flow. The observations of both Rohr et al. and Harris et al. would seem to indicate that they were not.

10. Initial Conditions

The importance of the initial conditions in determining the asymptotic character of the turbulence has already been discussed in detail by Rohr et al. (1988) who note: "More surprising is the continued downstream influence of the initial disturbance on the developing turbulence.... As long as the turbulence continues to interact with a constant mean shear, the data imply that the magnitude of the turbulence fluctuations should scale with the initial disturbance imposed at the inlet." Also of concern to them was the apparent inconsistency between different experiments of P/ϵ as well as $(-\overline{uv}/q^2)[1-\epsilon/P]$, both of which appeared to reach asymptotic values in the experiments considered, but changed from facility to facility in a manner which was not understood. Finally, the dependence on initial conditions of the velocity spectra from different experiments has already been noted in Figure (5),

All of the above observations are consistent with the theory presented here. In the light of the analysis, they are no longer "surprising", but can be seen to be the natural consequences of the tendency of the flow to settle into a fully self-preserving state which is determined by the initial conditions. Unfortunately, the theory (at least at this stage of development) can shed little light on what that dependence might be, and that only by speculating on the behavior of the turbulence at infinite Reynolds number.

It has long been believed that the velocity spectra at high wavenumbers (the dissipative range) should collapse in Kolmogorov variables ϵ and ν, and that the extent of the collapsed region should move toward lower wavenumbers with increasing Reynolds numbers. It is easy to show that this cannot be true in general if the theory proposed herein is valid, since if the spectra collapse in Taylor variables, they cannot collapse in Kolmogorov variables unless R_λ is constant. From equation (67), R_λ can be constant only if $P/\epsilon = 1$, which is clearly not the case for the experiments considered here.

Suppose we insist, however, that Kolmorgorov scaling must apply to the dissipative scales in the limit of infinite Reynolds number. Then we can simultaneously satisfy the conditions for full self-preservation in this limit if we also insist that $P = \epsilon$, regardless of the manner in which the flow is generated. Thus, for fixed geometry of the turbulence generator, we should expect P/ϵ to decrease toward unity as some appropriately defined source Reynolds number increases. Whether or not this speculation is correct remains for further experimentation to determine.

11. Summary and Conclusions

An analysis of the dynamical equations governing homogeneous shear flow has shown the possibility of solutions which are self-preserving at all scales of motion The appropriate velocity scale is the one obtained from the turbulence kinetic energy, and the length scale is the Taylor microscale. Two different cases of self-preserving flow were identified, the first corresponding to a constant mean shear, and the second to a mean shear which is inversely proportional to time. Only the first case was considered in detail, and the principal theoretical results were that the Taylor microscale must be a constant, while the kinetic energy changes exponentially with time. It also follows from the postulated similarity that the velocity spectra should scale over all wavenumbers with the turbulence kinetic energy and the Taylor microscale, and have a spectral shape determined by the initial conditions.

The experimental data support the theory. All of the measurements in flows with adequate development lengths show the Taylor microscale to be constant and the energy increase to be at least consistent with exponential growth. Collapse of the energy specta is better than might have been expected in advance of the theory. Only in the largest scales do the spectra measured at different locations differ significantly. While it is possible that the spectral differences at low wavenumber are genuine, it is more likely that they arise from two limitations in the experiments: inadequate development length and the influence of flow boundaries. The development distance required for the larger scales may be much longer than the experimenters realized. Unfortunately, when full development is allowed for and the zero-frequency intercepts of the one-dimensional spectra do tend to constant values, the larger scales become comparable to the tunnel dimensions and the theoretical assumption of unbounded turbulent shear flow may not be satisfied in the experiments. The theory proposed herein is consistent with the persistent influence of initial conditions, as well as with the variations reported from facility to facility.

The theory of self-preservation presented here suggests a number of possibilities for further experimentation. As for turbulence in the absence of shear, it can be shown that the spectral transfer and velocity derivative skewness depend inversely on R_λ. Since R_λ increases with distance, these quantities should decrease, the opposite of the behavior in the absence of shear (George 1987). Such measurements appear not to have been made, but would provide an important additional test of the theory. Also of particular interest would be experiments to examine the dependence of P/ϵ on the initial Reynolds number for fixed geometry. Finally, tests in larger tunnels would be of considerable value in evaluating the arguments made here regarding the integral scale.

Why the flow should choose to behave in a self-preserving manner remains a matter for conjecture. George (1988) has noted that turbulent flows whose governing equations admit to such fully self-preserving solutions always appear to asypmtotically relax to a fully self-preserving state. In such situations, theories of *local* similarity appear not to apply, except possibly as an asymptotic state in the limit of infinite Reynolds number. The homogeneous turbulent shear flow considered here appears to be no exception.

Acknowledgements

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