SPATIAL ATTENUATION OF VORTICITY MEASUMENTS BY MULTI-WIRE PROBES

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ABSTRACT

The attenuation introduced in measured vorticity quantities by the finite separation distances between the cluster of wires in nine and twelve wire vorticity probes is examined by using a three point model of the probe. The analysis demonstrates that the probes introduce a significant amount of spatial attenuation if the separation distances in the probe are significantly larger than three times the Kolmogorov length scale. In addition, the one dimensional spectra of the vorticity measured by the probe in isotropic turbulence do not satisfy the one-dimensional isotropic vorticity spectral relations for the high wave numbers due to the spatial attenuation. It is demonstrated that the effect of spatial attenuation on vorticity measurements is dependent on the shape of the three dimensional velocity spectrum which is being measured.

INTRODUCTION

Instantaneous measurement of the mean square vorticity or dissipation of the turbulent kinetic energy requires the simultaneous measurement of a number of spatial derivatives. However, the finite dimensions of the measuring devices which determine the spatial derivatives act as a spatial filter if they are not sufficiently small to resolve the smallest scales of the turbulence. The attenuation of the derivative measured using a parallel wire probe, which is the simplest geometry for measuring spatial derivatives, has been studied analytically or experimentally by a number of researchers including Wyngaard (1969), Klewicki and Falco (1991), Antonia et. al. (1993), and Ewing et. al. (1994). More complex geometries such as the nine and twelve wire probes (e.g. Honkan and Andrepolous, 1993 or Wallace et. al., 1992) used to measure the three components of vorticity simultaneously, however, have not been similarly analyzed. In the current analysis a three point model is studied using the techniques outlined by Wyngaard (1968, 1969) to determine an estimate of the attenuation introduced by these complex wire geometries.

Typical multi-wire probes utilize three or four wire probes centered around three points separated by finite distances. The analysis presented below examines the spatial attenuation caused by the finite separation of these points by modelling the probe as three measuring points. The analysis neglects the attenuation introduced by the finite dimensions of the individual wire clusters. The model will produce an accurate estimate of the attenuation if the dimensions of the wires in the cluster are significantly smaller than the separation distances; it is an estimate of the lower bound of the amount of attenuation when this is not true. Using this model it is possible to determine equations relating the three dimensional vorticity spectrum which would be measured by a finite size probe in an isotropic flow to the actual three dimensional vorticity spectrum in the flow. It is then possible to determine a filtering function which describes how the finite dimensions of the probe filter the three dimensional spectra of the derivative components in the vorticity.

Since it is not possible to measure a three-dimensional spectrum in an experiment, it is necessary to introduce models for the three dimensional spectrum in order to calculate numerical estimates of the amount of the attenuation which occurs in measured quantities. The current analysis uses three different isotropic models of the three dimensional spectrum to determine the sensitivity of the attenuation to the shape of the spectrum.

Initially the models are used calculate the attenuation of mean sqaure vorticity measurements. In addition to studying the magnitude of the attenuation of the individual vorticity components, the

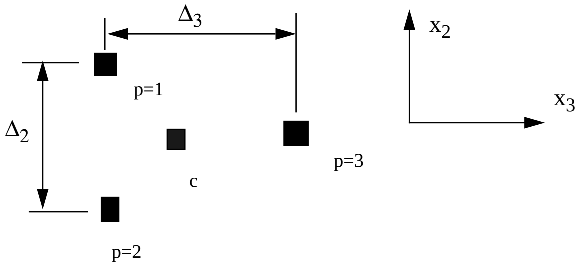

FIGURE 1 THREE POINT MODEL OF MULTI-WIRE PROBE

current study examines whether all of the components are attenuated by approximately the same amount. This is always an important consideration when measured data is compared to each other, because a failure to satisfy this constraint could lead an experimenter to false conclusions about the anisotropy (or isotropy) of the flow.

In the last section the attenuation of the one-dimensional spectra is examined. Recently, Ong et. al (1993) used measurements from multi-wire probe to test whether the smallest scales of the turbulence are locally isotropic using the isotropic one-dimensional vorticity spectra relationships. The equations for the attenuated three dimensional vorticity spectra are used here to determine the one-dimensional vorticity spectra which would be measured by a finite dimensioned probe in the flow. These one-dimensional spectra are substituted into the isotropic one-dimensional spectral relationship to determine if the attenuated spectra satisfy the same relationship as the unattenuated spectra.

THREE POINT MODEL

The calibration of the nine or twelve wire probes normally requires the use of sophisticated techniques (e.g. Wallace, 1986) which can not be modelled using the methods outlined by Wyngaard (1969). Instead a three point model, illustrated in figure 1, is used to model the nine and twelve wire vorticity probes. It is assumed the three components of velocity are known exactly at each of the three measuring points. Following the general method outlined by Wallace (1986), the spatial derivatives in the x_2 and x_3 directions can be determined by writing the velocities at each of the measuring points as a Taylor expansion of the velocity at the center of the probe and truncating the expansion after a single term, i.e.

$$u_i^{(p)} = u_i^c + h_j^{(p)} \frac{\partial u^c{}_i}{\partial x_j} \qquad (1)$$

where there is summation over j=1, 2. The velocity vector is $\boldsymbol{u}=(u_1, u_2, u_3)$ and $\boldsymbol{x}=(x_1, x_2, x_3)$ is the position vector. The quantities in this paper are non-dimensionalized by the Kolmogorov length scale $\eta=(\nu^3/\varepsilon)^{1/4}$ and the Kolmogorov velocity scale $u_k=(\nu\varepsilon)^{1/4}$, where ν is the kinematic viscosity and ε is the dissipation per unit mass. The superscript p=(1,2,3) is used to denote the individual measuring points and c indicates the center of the probe (see figure 1). The value of the coefficients in the Taylor expansions, $h_j^{(p)}$, are related to the separation distances, Δ_2 and Δ_3, in the x_2 and x_3 directions. The essence of the current analysis is to test if the truncation of the Taylor expansion at a single term introduces a significant amount of error.

The streamwise derivatives of the velocity components are calculated using Taylor's frozen field hypothesis. It is assumed that the derivatives calculated using this technique are exact. This may not be true in a real experiment (Lumley, 1965), and would add further error and uncertainty to the error due to the spatial attenuation. Taylor's hypothesis is applied to the average of the velocities at the three measuring points, so the resulting derivative is an estimate of the derivative at the center of the probe making it consistent with the derivatives calculated using equation (1).

Equation (1) can be rearranged to generate equations for the estimate of the each of the derivatives in the non-streamwise directions and an equation for the velocity at the center of the probe.

$$\frac{\partial u_i}{\partial x_2} = \frac{u_i^{(1)} - u_i^{(2)}}{\Delta_2} \qquad (2)$$

$$\frac{\partial u_i}{\partial x_3} = \frac{2u_i^{(3)} - (u_i^{(2)} + u_i^{(1)})}{2\Delta_3} \qquad (3)$$

$$u_i^c = \frac{u_i^{(1)} + u_i^{(2)} + u_i^{(3)}}{3} \qquad (4)$$

Δ_2 and Δ_3 are the separation distance in the x_2 and x_3 directions (figure 1) and are non-dimensionalized by the Kolmogorov length scale. The derivative in the x_2 direction is a difference equation involving only the velocities at measuring points 1 and 2. Therefore, the measured derivative is an estimate of the derivative centered between these two points. This is a problem for this method if the dimensions of the probe are not small relative to the variation of the statistical quantity being measured.

It is possible to use equations 2-4 to generate equations relating the measured three-dimensional derivative spectra to the three dimensional velocity spectra in the flow. The equation for the streamwise derivative spectra is

$$\Phi^m_{(i),1;(i),1} = \frac{1}{9}\left[3 + 2\cos(k_2\Delta_2) + 4\cos\left(\frac{k_2\Delta_2}{2}\right) \times \cos(k_3\Delta_3)\right] k_1^2 \Phi_{(i)(i)} \qquad (5)$$

where the brackets on the subscripts are used to indicate there is no summation over the i. $\boldsymbol{k}=(k_1, k_2, k_3)$ is the wave number vector non-dimensionalized by the Kolmogorov length scale. The symbol $\Phi_{(\iota),1;(\iota),1}$ is used to indicate a three-dimensional derivative spectrum and the superscript m is used to indicate that it is a measured spectrum. The two groups of subscripts separated by the semicolon indicate the two derivative quantities that the three dimensional spectrum corresponds to. In each of the groups the first subscript indicates the velocity component, while the subscript following the comma indicates the derivative coordinate. The symbol $\Phi_{(\iota)(\iota)}$ denotes the three dimensional velocity spectrum for i^{th} velocity component in the flow.

The equation for the measured three dimensional x_2 derivative spectra is

$$\Phi^m_{(i),2;(i),2} = \frac{2}{\Delta_2^2}[1-\cos(k_2\Delta_2)]\,\Phi_{(i)\,(i)} \qquad (6)$$

and the equation for the x_3 derivative spectra is

$$\Phi^m_{(i),3;(i),3} = \frac{1}{4\Delta_3^2}\left[6+2\cos(k_2\Delta_2)-8\cos\left(\frac{k_2\Delta_2}{2}\right)\times\cos(k_3\Delta_3)\right]\Phi_{(i)\,(i)} \qquad (7)$$

where there is no summation on the i in either equation.

To calculate the vorticity it is necessary to determine the three-dimensional spectrum of the product of two different derivative components. The term for the x_1 vorticity is

$$\Phi^m_{2,3;3,2}+\Phi^m_{3,2;2,3} = \frac{4}{\Delta_2\Delta_3}\sin\left(\frac{k_2\Delta_2}{2}\right)\sin(k_2\Delta_2)\,\Phi_{23} \qquad (8)$$

where the term on the left includes the two cross terms in the vorticity component. This term is straight-forward to derive, but the cross terms in the other two components must be handled carefully to ensure the correct value is derived since the exact derivative in the x_1 direction includes a factor of ik (where i is $\sqrt{-1}$) in Fourier space and the non-streamwise derivatives do not include a term with an i. The equation for the terms in the x_2 vorticity spectrum is

$$\Phi^m_{1,3;3,1}+\Phi^m_{3,1;1,3} = \frac{2k_1}{\Delta_3}\sin(k_3\Delta_3)\cos\left(\frac{k_2\Delta_2}{2}\right)\Phi_{13} \qquad (9)$$

and the equation for the terms in the x_3 vorticity spectrum is

$$\Phi^m_{2,1;1,2}+\Phi^m_{1,2;2,1} = \frac{4k_1}{3\Delta_2}\left[\sin(k_2\Delta_2)+\cos(k_3\Delta_3)\times\sin\left(\frac{k_2\Delta_2}{2}\right)\right]\Phi_{12} \qquad (10)$$

The measured three dimensional spectra for each vorticity components can be determined using the equations 5-10. The correct value of each of the derivative components can be determined using the equation

$$\Phi_{i,l;j,p} = k_l k_p \Phi_{ij} \qquad (11)$$

One check that equations 5-10 are correct is that they are equal to the corresponding member(s) of the fourth order tensor defined by equation 11 in the limit of the zero separation distance.

It is possible to determine the filtering functions of the geometry and gain insight into the nature of the filtering by taking the ratio of the measured three dimensional derivative spectrum and the exact three dimensional derivative spectrum. For example

$$\frac{\Phi^m_{1,3;1,3}}{\Phi_{1,3;1,3}} = \frac{6+2\cos(k_2\Delta_2)-8\cos(k_2\Delta_2/2)\cos(k_3\Delta_3)}{4\Delta_3^2k_3^2} \qquad (12)$$

This expression can be simplified by Taylor expanding the trigonometric functions. Using the current geometry this yields

$$\frac{\Phi^m_{1,3;(1,3)}}{\Phi_{1,3;1,3}} = 1-\left(\frac{1}{8}k_2^2\Delta_2^2+\frac{1}{12}k_3^2\Delta_3^2\right)+\ldots.. \qquad (13)$$

It is important to note that the filtering which occurs in this particular derivative is only a function of k_2, k_3, Δ_2, and Δ_3 and is not a function of k_1. The expression also indicates that the attenuation is significant for large values of k_2 or k_3. For this derivative it is the filtering in the k_3 which will normally be the most important since the exact derivative is the velocity spectrum multiplied by k_3^2. Therefore much of the spectral density, in the three dimensional spectrum, is concentrated at large values of k_3. A similar analysis can be carried out for the other derivatives in the measured vorticity components. For the current model the filtering functions are all functions of k_2, k_3, Δ_2, and Δ_3 only and independent of k_1.

THREE DIMENSIONAL VELOCITY SPECTRUM

The calculations in this paper require a three dimensional velocity spectrum to simulate the turbulence field which would be measured by the probe. The only models available currently are isotropic, so these are utilized to estimate the error introduced in the measured quantities. Three different models are used to test the sensitivity of the results to the three-dimensional turbulence model.

The three dimensional velocity spectrum, $\tilde{\Phi}_{ij}$, can be defined in terms of the three dimensional energy spectrum function $\tilde{E}(\tilde{k})$, (Batchelor, 1953),

$$\tilde{E}(\tilde{k}) = \frac{1}{2}\oint\int_{|\tilde{\boldsymbol{k}}|=\tilde{k}} \tilde{\Phi}_{ii}(\tilde{\boldsymbol{k}})\,d\sigma(\tilde{\boldsymbol{k}}) \qquad (14)$$

where the integration is carried out over spherical shells of radius $\tilde{k}$ in wave number space. $\tilde{k}$ is the magnitude of the wave number vector $\tilde{\boldsymbol{k}}=(\tilde{k}_1, \tilde{k}_2, \tilde{k}_3)$. For isotropic turbulence the two are related by (Batchelor, 1953)

$$\tilde{\Phi}_{ij}(\tilde{\boldsymbol{k}}) = \frac{\tilde{E}(\tilde{k})}{4\pi\tilde{k}^4}\left(\tilde{k}^2\delta_{ij}-\tilde{k}_i\tilde{k}_j\right) \qquad (15)$$

where δ_{ij} is the Kronecker delta. By definition, $\tilde{E}(\tilde{k})$ must integrate to the turbulent kinetic energy and the dissipation per unit mass, ε,

$$\varepsilon = 2\nu\int_0^\infty \tilde{k}^2\tilde{E}(\tilde{k})\,d\tilde{k} \qquad (16)$$

Wyngaard (1968,1969) used Pao's (1965) model for the energy spectrum, i.e.

$$E(k) = \alpha k^{-5/3} exp\{-\frac{3}{2}\alpha k^{4/3}\} \qquad (17)$$

to study the attenuation of probes where $E = (\nu^{5/4}\varepsilon^{1/4})^{-1}\tilde{E}$ and $k = \tilde{k}\eta$ $(\eta=(\nu^3/\varepsilon)^{1/4})$, represents the spectrum function and wave number non-dimensionalized by Kolmogorov variables, respectively. The value of alpha is chosen as 1.6 in this study, which is in the acceptable range for this value.

Pao's spectrum function is not a realistic model of the high wave number region ($k > 0.1$) because it distributes too much energy to this region. In addition, the one-dimensional spectrum for Pao's spectrum function does not roll off as fast as experimental data in the far dissipation region (see Driscoll, 1982). This led to the introduction of models which roll off faster in this region such as

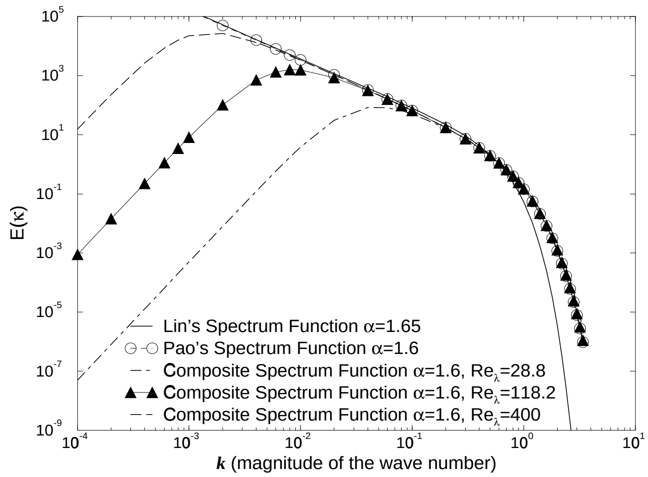

FIGURE 2 COMPARISON OF THREE DIMENSIONAL ENERGY SPECTRUM FOR THE MODELS

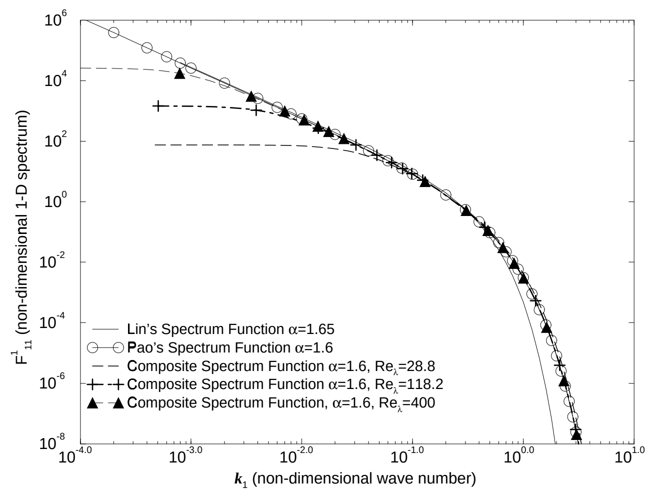

FIGURE 3 COMPARISON OF THE ONE-DIMENSIONAL STREAMWISE VELOCITY SPECTRUM

Lin's (1972) model of the energy spectrum

$$E(k) = \alpha(k^{-5/3} + k^{-1})\, exp\{-\alpha\left(\frac{3}{2}k^{4/3} + k^2\right)\} \qquad (18)$$

The value of alpha is chosen as 1.65 (see e.g. Driscoll, 1982). Ewing et. al. (1994) demonstrated that the amount of attenuation, for parallel wires, was significantly less sensitive to changes in the value of α than it was to the change of model.

The previous two spectra models are infinite Reynolds number models of the equilibrium and dissipation regions. To include the possibility of finite Reynolds number effects (i.e. roll off at low wave numbers), a composite spectrum is included in the analysis (Hellend et al., 1977) which is the product of von Karman's (1948) empirical model for the energy containing eddies and Pao's model. The model for the spectrum is

$$E(k) = \alpha\left(\frac{L}{\eta}\right)^{5/3} \frac{[k(L/\eta)]^4}{\{1 + [k(L/\eta)]^2\}^{-17/6}} exp\{-\frac{3}{2}\beta k^{4/3}\} \qquad (19)$$

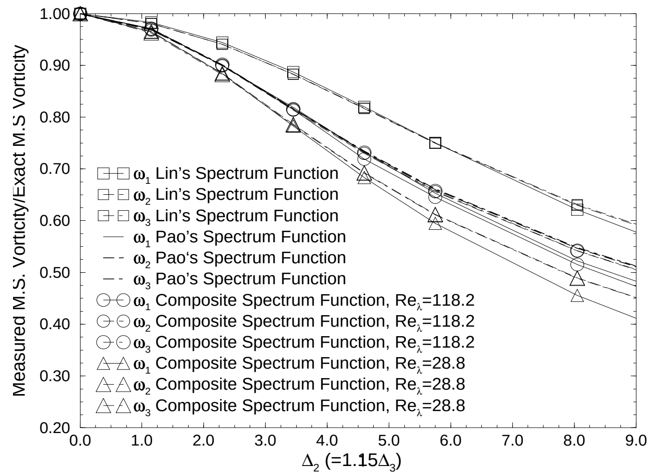

FIGURE 4 ATTENUATION OF THE MEAN SQUARE VORTICITY FOR Δ_2 = 1.15 Δ_3

L is a charteristic length scale of the energy containing eddies, α a model constant chosen as 1.6 in the current analysis and β is determined by satisfying the dissipation integral. For L/η, greater than 200 the value of β is approximately equal to α. The turbulence can be charterized using the Reynolds number based on the Taylor microscale. This Reynolds number can be calculated using the expression

$$Re_\lambda = \frac{u\lambda}{\nu} = \left(\frac{20}{3}\right)^{1/2}\int_0^\infty E(k)\,dk \qquad (20)$$

where $u^2 = (\overline{u_i u_i})/3$.

The three dimensional energy spectrum function for the three spectral models is shown in figure 2. Note the roll off of the composite spectrum at low wave numbers, and the more rapid roll off of Lin's spectrum at the high wave numbers. Figure 3 illustrates the one-dimensional spectrum of the streawise component, normalized by Kolmogorov variables, of the turbulence. This is the one-dimensional spectrum which is normally measured in an experiment (if Taylor's frozen field hypothesis is utilized to convert the time spectrum to a wave number spectrum). Notice that all of the spectrum functions which utilize Pao's model for the high wave number region roll off slower at the high wave numbers. In addition the one-dimensional spectrum of the infinite Reynolds number models continues to increases in the low wave number region, while the composite spectrum levels off in a manner more characteristic of measured turbulence.

ATTENUATION OF THE MEAN SQUARE VORTICITY

The attenuation of the mean sqaure vorticity can be calculated by integrating the equations for the measured three dimensional derivative spectra. Figure 4 illustrates the attenuation introduced by the probe when the probe geometry is chosen to model the geometry of Wallace's (1992) twelve wire probe and Honkan and Andreopoulos's (1993) nine wire probe, with Δ_2=1.15 Δ_3. Each of the measured vorticity components have been normalized by the exact value of the vorticity component for the spectrum to demon-

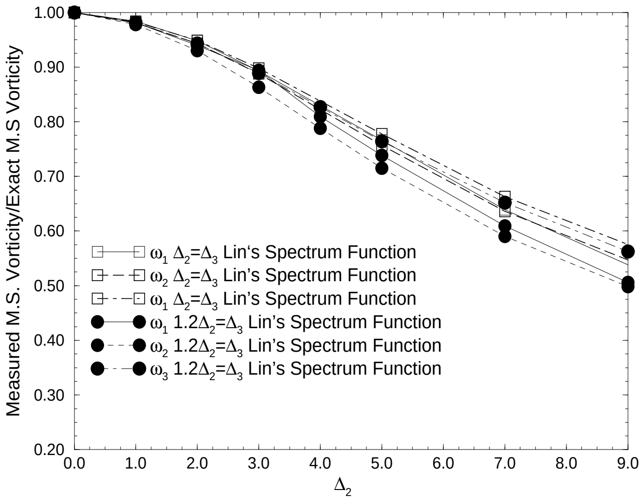

FIGURE 5 ATTENUATION OF THE MEAN SQUARE VORTICITY FOR OTHER GEOMETRIES

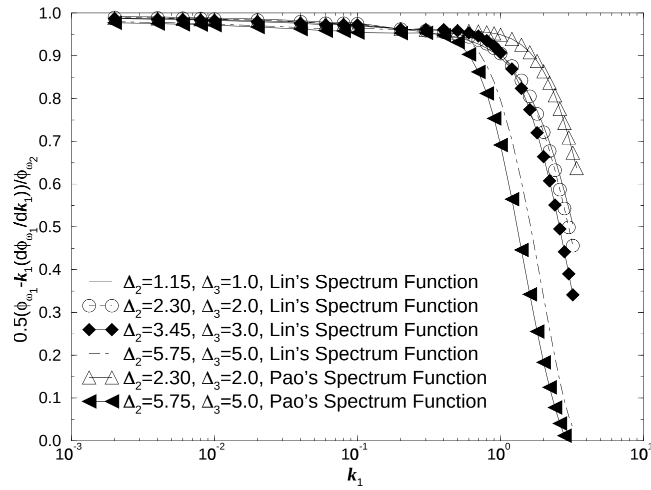

FIGURE 6 RATIO OF TERMS IN THE ONE-DIMENSIONAL ISOTROPIC VORTICITY RELATIONS

strate the relative attenuation of the signal. The figure illustrates that the amount of attenuation is highly dependent on the choice of the model for the high wave number portion of the spectrum function. The error in the signal is 10 percent when the separation distance in the x_2 direction is approximately 3 times the Kolmogorov length scale for Lin's spectrum function but the same attenuation occurs when the separation distance is only 2.25 times the Kolmogorov length scale for Pao's spectrum function. The attenuation introduced by the geometry for Lin's spectrum function is much smaller that the other two spectrum functions because Lin's rolls off faster in the high wave number region. This causes less of the relative spectral density to be in regions where k_2 and k_3 are large (i.e. where attenuation is large). In addition, the normalized attenuation of the measured vorticity components calculated using the composite spectrum (Re_λ>120) does not differ significantly from the normalized attenuation calculated using Pao's spectrum. For lower Reynolds numbers, such as Re_λ=28.8 , it is clear that the model of the low wave numbers can have a significant effect on the estimate of the amount of attenuation which occurs in the measured vorticity.

It is important to note that the difference in the amount of attenuation between the models increases as the separation distance increases. This indicates that data correction is not an effective technique of handling spatial attenuation, since the uncertainty of the correction factor increases significantly as the correction factor increases.

This geometry, with Δ_2=1.15 Δ_3, (for the Reynolds numbers tested) attenuates all three of the mean sqaure vorticity components by approximately the same amount for a given probe dimension and spectrum function. Therefore, the relative magnitude of the components is conserved during the measurement despite the attenuation due to spatial filtering. The calculations were repeated for two other geometries (Δ_2=Δ_3 and 1.2 Δ_2=Δ_3) to determine whether the three components of vorticity are equally attenuated for a wide range of geometries. The attenuation of the normalized mean sqaure vorticity components, calculated using Lin's spectrum function, is shown in figure 5. In the first case when Δ_2=Δ_3, the three components of vorticity are attenuated by approximately the same amount except when the separation distances are large. In the second case (1.2 Δ_2=Δ_3) the attenuation of the three components differs by 3 percent when Δ_2 is 3 times the Kolmogorov length scale, despite the fact that two of the components are only attenuated by 10 percent. It is clear that the three components of the mean sqaure vorticity measured simultaneously using the current three point model are not attenuated by approximately the same amount for all geometries. It is important to realize that this criterion has only been tested for isotropic turbulence in the current analysis and it is not possible to generalize this result to anisotropic turbulence without an appropriate model for anisotropic turbulence.

ONE-DIMENSIONAL SPECTRA

Recently, Ong et. al. (1993) measured the one dimensional spectra of the vorticity to determine if the spectra satisfy the isotropic one-dimensional vorticity relationships (e.g Ong et. al. 1993)

$$\phi_{\omega_2}(k_1) = \phi_{\omega_3}(k_1) = \frac{1}{2}\left[\phi_{\omega_1}(k_1) - k_1\frac{d\phi_{\omega_1}(k_1)}{dk_1}\right] \qquad (21)$$

The measured one-dimensional spectra can be related to the three dimensional vorticity spectra by the equation

$$\phi_{\omega_i} = \iint \varepsilon_{(i)jp}\varepsilon_{(i)lm}\Phi_{j,p;l,m}(\boldsymbol{k})\,dk_2dk_3 \qquad (22)$$

where ε_{ijk} is the alternating unit tensor. The results of the experiment demonstrated good agreement over an intermediate range of wave numbers, but the relationship was not satisfied at the highest wave numbers or at the low wave numbers.

The spectra measured for a probe of finite dimensions were calculated by substituting the appropriate measured three dimensional spectra into equation 21. It is important to note that the one-dimensional spectra in equation 21, at a particular value of k_1, are determined by integrating the three dimensional spectra over a k_2

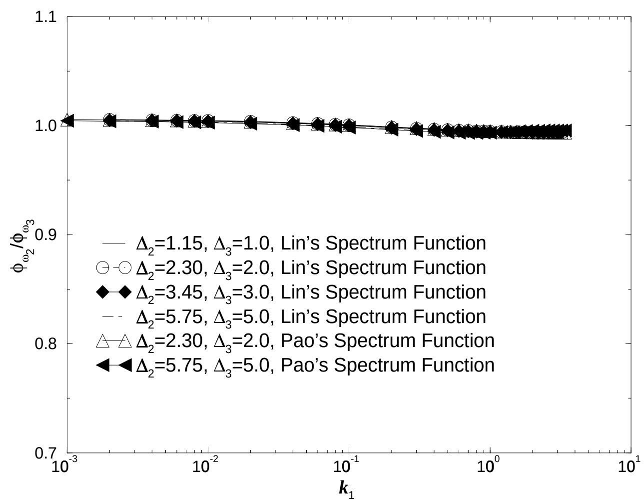

FIGURE 7 RATIO OF TERMS IN THE ONE-DIMENAIONAL ISOTROPIC VORTICITY RELATIONS

k_3 plane for a fixed k_1. Therefore an one-dimensional spectrum is the sum of quantities which have been attenuated by different amounts (since the local attenuation is a function of k_2 and k_3) so that it is not possible to generate a simple filtering of the one-dimensional spectra in terms of the finite dimensions of the probe, Δ_2 and Δ_3, since the attenuation must also depend on the three dimensional spectrum of the turbulence.

The measured one-dimensional vorticity spectra for a single geometry with Δ_2=1.15 Δ_3 were used to test whether the finite attenuation of the probe causes the measured data to fail to satisfy the one-dimensional vorticity relations in equation 20. The derivative term in equation 20 was determined analytically by differentiating equation 21, so it does not incorporate any of the errors which would occur using a finite difference method on the one-dimensional spectra ϕ_{w_1}.

Figure 6 illustrates the ratio of the first and final terms in equation 20 for various separation distances. The calculations were carried out for both Lin's and Pao's spectrum functions. In all cases the measured data fail to satisfy the relationship at the highest wave numbers although, it does satisfy the isotopic relationship at the lower wave numbers. As the finite dimensions of the probe increase, the region over which the relationship is not satisfied increases. This is similar to the result determined by Wyngaard (1968) for one-dimensional velocity spectra measured using cross wires. Wyngaard demonstrated that the finite dimensions of the probe would cause the measured one-dimensional spectra to have a different ratio than occurs in the flow, although the primary cause investigated by Wyngaard was cross talk between the components which does not occur in the present model.

A comparison of the attenuation curves for the two different spectral models demonstrates that the amount of attenuation which occurs is dependent on the spectrum function used to model the turbulence. Figure 7 demonstrates that the ratio of the ϕ_{w_2} / ϕ_{w_3} is nearly unity over the entire range of wave numbers, for all choices of separation distances and both spectrum models.

It is clear that the spatial attenuation can cause the measured one-dimensional vorticity spectra to fail to satisfy the isotropic one-dimensional vorticity relations at the high wave numbers, even if the underlying turbulence is isotropic. Therefore, comparisons such as those shown in figure 6 do not exclude the possibility that the turbulence being measured is isotropic if the dimensions of the probe used to make the measurement are large relative to the smallest scales of the turbulence. The region where the attenuation is large should not be used to confirm the local isotropy theory because the figure illustrates that the effect of spatial attenuation is dependent on the three dimensional spectrum of the turbulence which is being measured, so it is not possible to predict what the curve for the ratio will be with knowing the actual three-dimensional spectra. It must be noted that all of the analyses in the current paper were carried out using isotropic turbulence models. In an experimental measurement a significant amount of attenuation could cause an isotropic flow to appear anisotropic (the case examined in this paper), or the opposite may occur where anisotropic turbulence appears to satisfy the isotropic one-dimensional relations due to spatial attenuation.

SUMMARY AND CONCLUSIONS

The analysis demonstrated that the amount of attenuation in the mean sqaure vorticity component measured using the three point model is significant when the separation distance is greater than 3 times the Kolmogorov length. In addition the amount of attenuation which occurs is dependent on the shape of the spectrum function at high wave numbers for all Reynolds numbers as well as the model of the energy containing scales for lower Reynolds numbers. The attenuation calculated in the current paper does not include the attenuation due to the finite length measuring devices. This should be valid if the finite dimensions in the cluster of wires are significantly smaller than the wire separation distances. Looking at the different spectral models, it is clear that since the turbulence spectrum is unknown a priori, the uncertainty in the mean sqaure vorticity measurements increases as the separation distance increases. This indicates that data correction may not be an effective method of handling the problem.

The analysis also demonstrates that attenuation introduced by the finite dimensions of the probe causes the measured one-dimensional vorticity spectra to fail to satisfy the one-dimensional isotropic vorticity spectra relations, even when the actual turbulence being measured is isotropic. The ratio of ϕ_{w_2} / ϕ_{w_3} is approximately unity for all of the cases studied in this paper, so these one-dimensional spectra are affected in the same way by the attenuation. The spatial attenuation causes the term involving the streamwise vorticity to fail to be equal to either of these two components over a range of the largest wave numbers. The range of wave numbers increases as the dimension of the probe increases.

It is clear from the curves shown that it is not possible to generate a general correction for the spatial attenuation effects. Therefore, the region where attenuation is significant should be ignored when confirming the existence of local isotropy because it may be difficult to distinguish between the effects of spatial attenuation and the aberrations which occur due to the anisotropy of the flow.

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