Experiments in Fluids 29 (2000) 418–428 © Springer-Verlag 2000

The effect of cross-flow velocity on mean-square derivatives measured using hot wires

D. Ewing, W. K. George

Abstract It is well known that significant errors occur in the velocity derivative moments measured in turbulent flows when the measuring transducer is too large or Taylor's hypothesis is used in high-turbulence-intensity flows. An additional error occurs when velocity derivative moments are measured with hot wires in high-turbulence-intensity flows, because the wires cannot resolve the individual velocity components in these flows. Estimates of the error this causes in the derivative moments measured with single-, cross-, and parallel-wire probes are developed herein. The errors are significant in the derivative moments measured with cross-wire probes, but are smaller in derivative moments measured with single- and parallel-wire probes. For example, the relative errors in $\overline{(\partial u_2/\partial x_1)^2}$ measured in the far field of the round jet are 30–50% smaller than predicted in previous analyses.

1 Introduction

Velocity derivative moments are used to determine many important characteristics of turbulent flows, such as the rate of dissipation of turbulent kinetic energy or the mean-square vorticity. It is challenging to measure these moments accurately, in part, because of the errors introduced by the finite size of the transducers used in the measurements. For example, Wyngaard (1969) showed that measurements of $\overline{(\partial u_1/\partial x_2)^2}$ with a parallel-wire probe are 10–25% smaller than the actual moments when the distance between the wires is three to six times the Kolmogorov length scale. Here, u_1 is the fluctuating velocity in the mean-flow direction, and x_2 is the coordinate in the cross-stream direction.

The moments of the velocity derivatives in the mean-flow direction, x_1, are normally measured using Taylor's frozen-field hypothesis (Taylor 1938). In this approach, it is assumed that the small-scale turbulent motions that contribute to the derivative moments are frozen as they are convected past the measuring transducer. It is also assumed that they are convected at a constant velocity, U, so the variance of the derivative moments in the mean-flow direction can be approximated as (Monin and Yaglom 1975)

$$\overline{\left(\frac{\partial u_\alpha}{\partial x_1}\right)^2_m} = \frac{1}{U^2}\overline{\left(\frac{\mathrm{d}u_\alpha^m}{\mathrm{d}t}\right)^2} \qquad (1)$$

where $\mathrm{d}u_\alpha^m/\mathrm{d}t$ is the time derivative of the fluctuating velocity measured by the transducer.

There have been numerous analytical and experimental investigations that have examined when the assumptions in Taylors' hypothesis are approximately satisfied in turbulent flows and have developed estimates of the errors that occur in measurements with Taylors' hypothesis (e.g., Lumley 1965; Wyngaard and Clifford 1977; George et al. 1989; Mi and Antonia 1994). For example, it can be easily shown that the small-scale motions are approximately frozen as they are convected past the transducer in many turbulent flows (e.g., Lumley 1965). However, these motions are advected by the large-scale motions so the advection velocity is not even approximately constant in high-turbulence-intensity flows. Lumley (1965) developed a mathematical model to estimate the size of the error this causes and showed that the value of $\overline{(\partial u_1/\partial x_1)^2}$ measured using Taylor's hypothesis could be approximated as

$$\overline{\left(\frac{\partial u_1}{\partial x_1}\right)^2_m} = \overline{\left(\frac{\partial u_1}{\partial x_1}\right)^2}\left[1+\frac{\overline{u_1^2}+2\overline{u_2^2}+2\overline{u_3^2}}{U^2}\right] \qquad (2)$$

for "locally" isotropic turbulence. Here, u_1, u_2, and u_3 are the components of the fluctuating velocity in the x_1, x_2, and x_3 directions respectively, and U is mean velocity in the x_1 direction. The error is significant in moments measured in moderate- or high-turbulence-intensity flows. For example, it exceeds 20% in the entire far field of the axisymmetric jet (George et al. 1989). Wyngaard and Clifford (1977) extended Lumley's model to estimate the value of $\overline{(\partial u_2/\partial x_1)^2}$ and the scalar derivative moment, $\overline{(\partial \theta/\partial x_1)^2}$, which would be measured using Taylor's

Received: 10 March 1995/Accepted: 07 December 1999

D. Ewing
Department of Mechanical Engineering, McMaster University
Hamilton, Ontario, L8S 4L7, Canada

W. K. George
Department of Mechanical and Aerospace Engineering, State University of New York at Buffalo, Amherst, NY, 14260, USA

Correspondence to: D. Ewing

The authors would like to acknowledge the contribution that P. D. Beuther and H. J. Hussein made to earlier research on Taylor's frozen-field hypothesis at the SUNY at Buffalo. The assistance and suggestions of J. Acton, J. Citriniti, and A. Pollard are also acknowledged. Finally, the first author would also like to acknowledge the support of the Natural Science and Engineering Research Council of Canada.

hypothesis. George et al. (1989) later showed that the estimates of the measured derivative moments developed by Lumley (1965) and Wyngaard and Clifford (1977) could be deduced using less restrictive assumptions than previously thought.

George et al. (1989) also measured $\overline{(\partial u_1/\partial x_1)^2}$ in the far field of a round jet using both a stationary single wire and a single wire on a flying probe. They assumed the errors in the flying-wire measurements were negligible relative to those in the stationary-wire measurements and showed the relative errors in the stationary-wire measurements were in good agreement with the errors predicted using Eq. (2). Later measurements showed, however, that the small-scale motions in the round jet were not "locally" isotropic, as assumed in the derivation of Eq. (2), but instead were "locally" axisymmetric (George and Hussein 1991). In a similar experiment, Mi and Antonia (1994) examined the accuracy of the predicted errors for scalar derivative moments using measurements from the far field of an axisymmetric jet. They found little difference between the predicted errors for "locally" isotropic and "locally" axisymmetric turbulence and showed that the predictions for both cases were in good agreement with the measurements from the jet. The difference between the errors in the velocity derivative moments for "locally" isotropic and "locally" axisymmetric turbulence in the round jet will be examined here.

In all the aforementioned analyses, it was assumed that the measuring transducers could exactly resolve the individual velocity components in turbulent flows. This, however, is never true for single- and cross-wire probes in moderate- and high-turbulence-intensity flows. Instead, the velocities measured with these probes include contributions from the cross-flow velocity component. It is well known that this causes significant errors in the velocity moments measured with these probes in moderate- and high-turbulence-intensity flows (e.g., Hinze 1975, Hussein et al. 1994). Heretofore, however, the error this causes in measurements of the velocity derivative moments has not been considered.

A model is developed here to estimate the error from the cross-flow velocity in measurements with single-, cross-, and parallel-wire probes. This model also includes the error introduced by the unsteady convection velocity in measurements using Taylor's hypothesis (e.g., Lumley 1965; George et al. 1989) and the spatial resolution error in measurements with parallel-wire probes (e.g., Wyngaard 1969). Measurements from the far field of the axisymmetric jet reported by George and Hussein (1991) and Hussein et al. (1994) are then used to evaluate the relative size of the errors that occur in measurements of the velocity derivative moments for this flow.

1.1
Background on the model

The analysis here follows the approach outlined by George et al. (1989), which, in turn, includes many aspects of Lumley's model. For example, following Lumley (1965), it divides the flow into two types of motions: small-scale statistically homogeneous motions that contribute to the velocity derivative measured by the wire, but make a negligible contribution to the instantaneous velocity measured by the wire; and large-scale unsteady motions that contribute essentially all the instantaneous velocity, but make a negligible contribution to the derivative of the velocity measured by the wire. As in Lumley's model, it is also assumed that the velocities from these two sets of motions are statistically independent. Although these assumptions are not formally correct in turbulent flows, they are approximately valid in high-Reynolds-number flows, where the large- and small-scale motions differ significantly in size, and the small-scale motions are "locally" homogeneous.

However, unlike Lumley's approach, it is not necessary to specify a probability density function for the unsteady convection velocity in George et al.'s approach. In this approach, the system is modeled as a probe moving with a fluctuating velocity through a frozen small-scale field. George et al. (1989) argued that this model was dynamically similar to Lumley's model from the perspective of a hot wire on the probe if the velocity of the probe was equal in magnitude but opposite in direction to the velocity from the large-scale motions in a turbulent flow. Using this approach, George et al. deduced the estimates of the measured derivative moments developed earlier by Lumley (1965) and Wyngaard and Clifford (1977) without specifying the probability density function for the large-scale velocity field. This approach is followed here, since it yields the desired results with the least restrictive assumptions.

2
Measurement using Taylor's hypothesis

2.1
Single-wire measurements

A single hot wire positioned normal to the direction of the probe's average velocity, x_1 here, primarily measures the instantaneous velocity normal to the wire, particularly when the ratio of its length to its diameter is large (Champagne et al. 1967). Thus, the velocity measured by a "long" single wire aligned in the x_3-direction on a probe moving through a small-scale field can be approximated as

$$u_1^m(t) = \{[\tilde{v}_1(t) + \tilde{u}_1(\tilde{x}_\alpha^p)]^2 + [\tilde{v}_2(t) + \tilde{u}_2(\tilde{x}_\alpha^p)]^2\}^{1/2} \quad (3)$$

where $\tilde{v}_1$, $\tilde{v}_2$, and $\tilde{v}_3$ are the components of the probe's velocity in the negative x_1, x_2, and x_3 directions, $\tilde{u}_1$, $\tilde{u}_2$, and $\tilde{u}_3$ are the components of the velocity from the small-scale frozen field in the positive x_1, x_2, and x_3 directions, and $\tilde{x}_\alpha^p(t)$ is the position of the probe in the frozen field. This expression differs from corresponding expressions in the previous analyses (e.g., George et al. 1989) where it was assumed that only the component $\tilde{u}_1(\tilde{x}_\alpha^p)$ is measured from the small-scale field. The positive direction of the probe's velocity components are chosen in the negative coordinate directions here, so that a positive probe velocity corresponds to a positive velocity from the large-scale motions when the results of this model are used to estimate the error in a turbulent flow.

The time derivative of the velocity measured by the wire is given by

$$\frac{du_1^m}{dt} = \frac{\tilde{v}_n}{[\tilde{v}_1^2 + \tilde{v}_2^2]^{1/2}} \frac{d\tilde{u}_n}{dt} + O\left(\frac{\tilde{u}_\alpha}{\tilde{v}_\alpha}\right) + O\left(\frac{d\tilde{v}_\alpha/dt}{d\tilde{u}_\alpha/dt}\right) \tag{4}$$

where summation is implied for repeated subscripts except for Greek letters. Here, though, n is only summed from 1 to 2. The last two terms in this expression are of higher order, since it has been assumed that the small-scale motions make a negligible contribution to the measured velocity, and the time derivative of the probe's velocity is small compared with the rate of change of the velocity measured from the small-scale field.

Following George et al., the position of the probe in the small-scale frozen field can be determined by integrating the velocity of the probe along its path, i.e.,

$$\tilde{x}_\alpha^p(t) = \tilde{x}_\alpha^p(t_0) - \int_{t_0}^{t} \tilde{v}_\alpha(t)\, dt \ . \tag{5}$$

The latter term is negative since the positive components of the probe's velocity are in the negative coordinate directions. Thus, the velocity from the frozen homogeneous small-scale field at the position of the probe can be written as

$$\tilde{u}_n(\tilde{x}_\alpha^p(t)) = \iiint_{-\infty}^{\infty} \hat{u}_n(k_\beta) \times e^{ik_m[\tilde{x}_m^p(t_0) - \int_{t_0}^{t} \tilde{v}_m(t)\, dt]}\, dk_1\, dk_2\, dk_3 \tag{6}$$

where the inverse Fourier transform is defined in the sense of generalized functions (Lighthill 1958). Here, k_1, k_2, and k_3 are the components of the wavenumber vector in the x_1, x_2, and x_3 directions. The time derivative of the velocity measured from the small-scale field can then be written as

$$\frac{d\tilde{u}_n}{dt} = \iiint_{-\infty}^{\infty} -ik_j\tilde{v}_j\hat{u}_n(k_\alpha) \times e^{ik_m[\tilde{x}_m^p(t_0) - \int_{t_0}^{t} \tilde{v}_m(t)\, dt]}\, dk_1\, dk_2\, dk_3 \ . \tag{7}$$

It follows that the time derivative of the velocity measured by the wire in Eq. (4) can be rewritten as

$$\frac{du_1^m}{dt} = \frac{\tilde{v}_n}{[\tilde{v}_1^2 + \tilde{v}_2^2]^{1/2}} \iiint_{-\infty}^{\infty} -ik_j\tilde{v}_j\hat{u}_n(k_\alpha) \times e^{ik_m\tilde{x}_m^p(t)}\, dk_1\, dk_2\, dk_3 \tag{8}$$

and its variance is given by

$$\overline{\left(\frac{du_1^m}{dt}\right)^2} = \overline{\frac{\tilde{v}_l\tilde{v}_n}{v_1^2 + v_2^2}\tilde{v}_j\tilde{v}_i \iiint_{-\infty}^{\infty}\iiint_{-\infty}^{\infty} k_i'k_j\hat{u}_l(k_\alpha)\hat{u}_n^*(k_\alpha')e^{i(k_m - k_m')\tilde{x}_m^p(t)}\, dk_1\, dk_2\, dk_3\, dk_1'\, dk_2'\, dk_3'} \ . \tag{9}$$

Here, the subscripts l and n are summed from 1 to 2, while i and j are summed from 1 to 3. This expression reduces to

$$\overline{\left(\frac{du_1^m}{dt}\right)^2} = \overline{\frac{\tilde{v}_l\tilde{v}_n}{v_1^2 + v_2^2}\tilde{v}_j\tilde{v}_i} \iiint_{-\infty}^{\infty} k_ik_j\Phi_{ln}(k_\alpha)\, dk_1\, dk_2\, dk_3 \tag{10}$$

$$= \overline{\frac{\tilde{v}_l\tilde{v}_n}{\tilde{v}_1^2 + \tilde{v}_2^2}\tilde{v}_j\tilde{v}_i}\ \overline{\frac{\partial\tilde{u}_l}{\partial x_i}\frac{\partial\tilde{u}_n}{\partial x_j}} \tag{11}$$

since it has been assumed the probe's velocity and the velocity from the small-scale field are statistically independent. Here, $\Phi_{ln}(k_\alpha)$ is the three-dimensional spectrum of the small-scale velocity field given by (cf., Monin and Yaglom 1975)

$$\Phi_{ln}(k_\alpha)\delta(k_\alpha - k_\alpha') = E\{\hat{u}_l(k_\alpha)\hat{u}_n^*(k_\alpha')\} \ . \tag{12}$$

Applying Taylor's hypothesis to the variance of the measured time derivative yields

$$\overline{\left(\frac{\partial u_1}{\partial x_1}\right)_m^2} = \frac{1}{U^2}\overline{\left(\frac{du_1^m}{dt}\right)^2} = \overline{\frac{\tilde{v}_l\tilde{v}_n}{\tilde{v}_1^2 + \tilde{v}_2^2}\frac{\tilde{v}_j}{U}\frac{\tilde{v}_i}{U}}\ \overline{\frac{\partial\tilde{u}_l}{\partial\tilde{x}_i}\frac{\partial\tilde{u}_n}{\partial x_j}} \tag{13}$$

where, again, l and n are summed from 1 to 2, while i and j are summed from 1 to 3. This expression can be reduced to the expression developed by George et al. (1989) if the term $\tilde{v}_l\tilde{v}_n/[\tilde{v}_1^2 + \tilde{v}_2^2]$ is replaced with $\delta_{l1}\delta_{n1}$, the term for a hot wire that only measures the velocity component in the mean-flow direction. Physically, the term $\tilde{v}_l\tilde{v}_n/[\tilde{v}_1^2 + \tilde{v}_2^2]$ accounts for the contribution of the cross-flow component from the small-scale velocity field, $\tilde{u}_2$, when the large-scale motions convect the small-scale motions past the wire at an angle relative to the mean-flow direction.

Equation (13) can be further simplified by approximating $1/(\tilde{v}_1^2 + \tilde{v}_2^2)$ using a binomial expansion given by

$$\frac{1}{\tilde{v}_1^2 + \tilde{v}_2^2} = \frac{1}{U^2}\left[1 - 2\frac{v_1}{U} + 3\frac{v_1^2}{U^2} - \frac{v_2^2}{U^2} - 4\frac{v_1^2}{U^3} + 4\frac{v_2^2 v_1}{U^3}\cdots\right] \tag{14}$$

where U and v_i are the mean and fluctuating probe velocity given by $E\{\tilde{v}_i\} = U\delta_{i1}$ and $v_i = \tilde{v}_i - U\delta_{i1}$. (A similar binomial expansion is used to estimate the error that the contribution from the cross-flow velocity causes in the velocity moments measured with hot wires; e.g., Hinze 1975). This expansion should yield reasonable results if most realizations satisfy the convergence criterion given by

$$\left|\frac{2v_1}{U} + \frac{v_1^2}{U^2} + \frac{v_2^2}{U^2}\right| < 1 \tag{15}$$

Thus, the value of $\overline{(\partial u_1/\partial x_1)^2}$ measured with a single wire using Taylor's hypothesis can be approximated by

$$\overline{\left(\frac{\partial u_1}{\partial x_1}\right)^2_m} = \left[1 - \underbrace{\frac{\overline{v_2^2}}{U^2}} + \frac{\overline{v_1^2}}{U^2}\right]\overline{\left(\frac{\partial \tilde{u}_1}{\partial x_1}\right)^2} + \left[\frac{\overline{v_2^2}}{U^2}\right]\overline{\left(\frac{\partial \tilde{u}_1}{\partial x_2}\right)^2} + \left[\frac{\overline{v_3^2}}{U^2}\right]\overline{\left(\frac{\partial \tilde{u}_1}{\partial x_3}\right)^2}$$
$$+ 2\left[\frac{\overline{v_1 v_2}}{U^2} - \underbrace{\frac{\overline{v_2^3}}{U^3}}\right]\overline{\frac{\partial \tilde{u}_1}{\partial x_1}\frac{\partial \tilde{u}_1}{\partial x_2}} + 2\left[\frac{\overline{v_1 v_3}}{U^2} - \underbrace{\frac{\overline{v_3 v_2^2}}{U^3}}\right]\overline{\frac{\partial \tilde{u}_1}{\partial x_1}\frac{\partial \tilde{u}_1}{\partial x_3}} + 2\left[\frac{\overline{v_2 v_3}}{U^2}\right]\overline{\frac{\partial \tilde{u}_1}{\partial x_2}\frac{\partial \tilde{u}_1}{\partial x_3}}$$
$$+ \underbrace{\left[\frac{\overline{v_2^2}}{U^2}\right]\overline{\left(\frac{\partial \tilde{u}_2}{\partial x_1}\right)^2}} + \underbrace{2\left[\frac{\overline{v_2^3}}{U^3}\right]\overline{\frac{\partial \tilde{u}_2}{\partial x_1}\frac{\partial \tilde{u}_2}{\partial x_2}}} + \underbrace{2\left[\frac{\overline{v_2^2 v_3}}{U^3}\right]\overline{\frac{\partial \tilde{u}_2}{\partial x_1}\frac{\partial \tilde{u}_2}{\partial x_3}}}$$
$$+ \underbrace{2\left[\frac{\overline{v_1 v_2}}{U^2} - \frac{\overline{v_2^3}}{U^3}\right]\overline{\frac{\partial \tilde{u}_1}{\partial x_1}\frac{\partial \tilde{u}_2}{\partial x_1}}} + \underbrace{2\left[\frac{\overline{v_2^3}}{U^3}\right]\overline{\frac{\partial \tilde{u}_1}{\partial x_2}\frac{\partial \tilde{u}_2}{\partial x_2}}} + \underbrace{2\frac{\overline{v_2 v_3^2}}{U^3}\overline{\frac{\partial \tilde{u}_1}{\partial x_3}\frac{\partial \tilde{u}_2}{\partial x_3}}}$$
$$+ \underbrace{4\frac{\overline{v_2^2}}{U^2}\overline{\frac{\partial \tilde{u}_1}{\partial x_1}\frac{\partial \tilde{u}_2}{\partial x_2}}} + \underbrace{4\frac{\overline{v_2 v_3}}{U^2}\overline{\frac{\partial \tilde{u}_1}{\partial x_1}\frac{\partial \tilde{u}_2}{\partial x_3}}} + \underbrace{4\frac{\overline{v_2^2 v_3}}{U^2}\overline{\frac{\partial \tilde{u}_1}{\partial x_3}\frac{\partial \tilde{u}_2}{\partial x_2}}} + O\left(\frac{v^4}{U^4}\right) \quad (16)$$

where $v = (\overline{v_i v_i}/3)^{1/2}$.[1] The underbraces in this expression highlight the terms not included in the previous estimates of the measured moment (e.g., George et al. 1989), and thus represent the error caused by the cross-flow velocity.

Equation (16) can be used to estimate the errors in derivative moments measured in a turbulent flow by evaluating the moments of the probe velocity using the velocity moments from the flow and evaluating the velocity derivative moments from the small-scale field using the derivative moments from the flow. Several of the derivative moments in this expression are difficult to measure and, hence, are often not measured. In these cases, it will be necessary to approximate these terms by making an assumption about the small-scale motions such as that the motions are "locally" isotropic or "locally" axisymmetric.

2.2 Cross-wire probes

The response of a cross-wire probe, with wires at an angle of 45° to the mean-flow direction, can be approximated as

$$\begin{aligned}(u_1^m + u_2^m)^2 + k^2(u_1^m - u_2^m)^2 &= [(\tilde{v}_1 + \tilde{u}_1) + (\tilde{v}_2 + \tilde{u}_2)]^2 + 2(\tilde{v}_3 + \tilde{u}_3)^2 \\ &\quad + k^2[(\tilde{v}_1 + \tilde{u}_1) - (\tilde{v}_2 + \tilde{u}_2)]^2 \end{aligned} \quad (17)$$

and

$$\begin{aligned}(u_1^m - u_2^m)^2 + k^2(u_1^m + u_2^m)^2 &= [(\tilde{v}_1 + \tilde{u}_1) - (\tilde{v}_2 + \tilde{u}_2)]^2 + 2(\tilde{v}_3 + \tilde{u}_3)^2 \\ &\quad + k^2[(\tilde{v}_1 + \tilde{u}_1) + (\tilde{v}_2 + \tilde{u}_2)]^2 \end{aligned} \quad (18)$$

where again $\tilde{v}_\alpha$ and $\tilde{u}_\alpha$ are the components of the probe's velocity and the velocity from the small-scale frozen field, and k^2 describes the sensitivity of the wire to flow along its length. It is assumed that the cross-wire probe is oriented in the x_1–x_2 plane, and k^2 is independent of flow angle (cf. Tutu and Chevray 1975; Champagne et al. 1967). It follows that the velocity components measured by the cross-wire probe are given by

$$\begin{aligned} u_1^m = \tfrac{1}{2}\{&[(\tilde{v}_1 + \tilde{u}_1 + \tilde{v}_2 + \tilde{u}_2)^2 + h^2(\tilde{v}_3 + \tilde{u}_3)^2]^{1/2} \\ &+ [(\tilde{v}_1 + \tilde{u}_1 - \tilde{v}_2 - \tilde{u}_2)^2 + h^2(\tilde{v}_3 + \tilde{u}_3)^2]^{1/2}\} \end{aligned} \quad (19)$$

and

$$\begin{aligned} u_2^m = \tfrac{1}{2}\{&[(\tilde{v}_1 + \tilde{u}_1 + \tilde{v}_2 + \tilde{u}_2)^2 + h^2(\tilde{v}_3 + \tilde{u}_3)^2]^{1/2} \\ &- [(\tilde{v}_1 + \tilde{u}_1 - \tilde{v}_2 - \tilde{u}_2)^2 + h^2(\tilde{v}_3 + \tilde{u}_3)^2]^{1/2}\} \end{aligned} \quad (20)$$

where

$$h^2 = \frac{2}{1 + k^2} \quad . \quad (21)$$

The time derivatives of these measured velocities are given by

$$\begin{aligned} \frac{du_1^m}{dt} = \frac{1}{2}\Bigg\{ &\frac{1}{[(\tilde{v}_1 + \tilde{v}_2)^2 + h^2\tilde{v}_3^2]^{1/2}} \\ &\times \left[(\tilde{v}_1 + \tilde{v}_2)\left(\frac{d\tilde{u}_1}{dt} + \frac{d\tilde{u}_2}{dt}\right) + h^2\tilde{v}_3\frac{d\tilde{u}_3}{dt}\right] \\ &+ \frac{1}{[(\tilde{v}_1 - \tilde{v}_2)^2 + h^2\tilde{v}_3^2]^{1/2}} \\ &\times \left[(\tilde{v}_1 - \tilde{v}_2)\left(\frac{d\tilde{u}_1}{dt} - \frac{d\tilde{u}_2}{dt}\right) + h^2\tilde{v}_3\frac{d\tilde{u}_3}{dt}\right]\Bigg\} \\ &+ O\left(\frac{\tilde{u}_\alpha}{\tilde{v}_\alpha}\right) + O\left(\frac{\partial\tilde{v}_\alpha/\partial t}{\partial\tilde{u}_\alpha/dt}\right) \end{aligned} \quad (22)$$

and

$$\begin{aligned} \frac{du_2^m}{dt} = \frac{1}{2}\Bigg\{ &\frac{1}{[(\tilde{v}_1 + \tilde{v}_2)^2 + h^2\tilde{v}_3^2]^{1/2}} \\ &\times \left[(\tilde{v}_1 + \tilde{v}_2)\left(\frac{d\tilde{u}_1}{dt} + \frac{d\tilde{u}_2}{dt}\right) + h^2\tilde{v}_3\frac{d\tilde{u}_3}{dt}\right] \\ &- \frac{1}{[(\tilde{v}_1 - \tilde{v}_2)^2 + h^2\tilde{v}_3^2]^{1/2}} \end{aligned}$$

[1] Equation (16) has been simplified by noting that in statistically homogeneous turbulence (see Monin and Yaglom 1975)

$$\overline{\frac{\partial \tilde{u}_\alpha}{\partial x_\gamma}\frac{\partial \tilde{u}_\beta}{\partial x_\epsilon}} = \overline{\frac{\partial \tilde{u}_\beta}{\partial x_\gamma}\frac{\partial \tilde{u}_\alpha}{\partial x_\epsilon}}$$

$$\times\left[(\tilde{v}_1-\tilde{v}_2)\left(\frac{\mathrm{d}\tilde{u}_1}{\mathrm{d}t}-\frac{\mathrm{d}\tilde{u}_2}{\mathrm{d}t}\right)+h^2\tilde{v}_3\frac{\mathrm{d}\tilde{u}_3}{\mathrm{d}t}\right]\Bigg\} + O\left(\frac{\tilde{u}_\alpha}{\tilde{v}_\alpha}\right)+O\left(\frac{\partial\tilde{v}_\alpha/\partial t}{\partial\tilde{u}_\alpha/\mathrm{d}t}\right) . \tag{23}$$

Following the approach outlined for the single-wire measurements, it can be shown that the variance of the velocity derivatives measured with a cross-wire probe using Taylor's hypothesis can be approximated by

$$\begin{aligned}\overline{\left(\frac{\partial u_1}{\partial x_1}\right)^2_m} &= \left(1+\frac{\overline{v_1^2}}{U^2}-\underbrace{\frac{2}{(1+k^2)}\frac{\overline{v_3^2}}{U^2}}\right)\overline{\left(\frac{\partial\tilde{u}_1}{\partial x_1}\right)^2}+\frac{\overline{v_2^2}}{U^2}\overline{\left(\frac{\partial\tilde{u}_1}{\partial x_2}\right)^2}+\frac{\overline{v_3^2}}{U^2}\overline{\left(\frac{\partial\tilde{u}_1}{\partial x_3}\right)^2}\\
&\quad+2\frac{\overline{v_1v_2}}{U^2}\overline{\frac{\partial\tilde{u}_1}{\partial x_1}\frac{\partial\tilde{u}_1}{\partial x_2}}+2\frac{\overline{v_1v_3}}{U^2}\overline{\frac{\partial\tilde{u}_1}{\partial x_1}\frac{\partial\tilde{u}_1}{\partial x_3}}+2\frac{\overline{v_2v_3}}{U^2}\overline{\frac{\partial\tilde{u}_1}{\partial x_2}\frac{\partial\tilde{u}_1}{\partial x_3}}+\underbrace{\frac{4}{(1+k^2)^2}\frac{\overline{v_3^2}}{U^2}\overline{\left(\frac{\partial\tilde{u}_3}{\partial x_1}\right)^2}}\\
&\quad+\underbrace{\frac{4}{(1+k^2)}\frac{\overline{v_1v_3}}{U^2}\overline{\frac{\partial\tilde{u}_1}{\partial x_1}\frac{\partial\tilde{u}_3}{\partial x_1}}}+\underbrace{\frac{8}{(1+k^2)}\frac{\overline{v_2v_3}}{U^2}\overline{\frac{\partial\tilde{u}_1}{\partial x_1}\frac{\partial\tilde{u}_3}{\partial x_2}}}+\underbrace{\frac{8}{(1+k^2)}\frac{\overline{v_3^2}}{U^2}\overline{\frac{\partial\tilde{u}_1}{\partial x_3}\frac{\partial\tilde{u}_3}{\partial x_1}}}+O\left(\frac{v^3}{U^3}\right)\end{aligned} \tag{24}$$

and

$$\begin{aligned}\overline{\left(\frac{\partial u_2}{\partial x_1}\right)^2_m} &= \left(1+\frac{\overline{v_1^2}}{U^2}-\underbrace{h^2\frac{\overline{v_2^2}}{U^2}}\right)\overline{\left(\frac{\partial\tilde{u}_2}{\partial x_1}\right)^2}+\frac{\overline{v_2^2}}{U^2}\overline{\left(\frac{\partial\tilde{u}_2}{\partial x_2}\right)^2}\\
&\quad+\frac{\overline{v_3^2}}{U^2}\overline{\left(\frac{\partial\tilde{u}_2}{\partial x_3}\right)^2}+2\frac{\overline{v_1v_3}}{U^2}\overline{\frac{\partial\tilde{u}_2}{\partial x_1}\frac{\partial\tilde{u}_2}{\partial x_3}}+2\frac{\overline{v_2v_3}}{U^2}\overline{\frac{\partial\tilde{u}_2}{\partial x_2}\frac{\partial\tilde{u}_2}{\partial x_3}}\\
&\quad+2\frac{\overline{v_1v_2}}{U^2}\overline{\frac{\partial\tilde{u}_2}{\partial x_1}\frac{\partial\tilde{u}_2}{\partial x_2}}-\underbrace{\frac{4}{(1+k^2)}\frac{\overline{v_3v_2}}{U^2}\overline{\frac{\partial\tilde{u}_2}{\partial x_1}\frac{\partial\tilde{u}_3}{\partial x_1}}}+O\left(\frac{v^3}{U^3}\right)\end{aligned} \tag{25}$$

The underbraces in these equations again highlight terms that are not included in previous estimates of the moments measured using Taylor's hypothesis (e.g., George et al. 1989), and thus represent the error caused by the contribution from the cross-flow velocity. It is evident that the estimate of $\overline{(\partial u_1/\partial x_1)^2}$ measured with the single- and cross-wire probes differ when the effect of the cross-flow velocity is considered. Thus, the error in the measurements of $\overline{(\partial u_1/\partial x_1)^2}$ depends on the transducer used in the measurements, as one would expect. This differs from the previous analyses where predicted errors were independent of the transducer used in the measurements.

3 Parallel-wire measurements

The unsteady convection of the small-scale motions by the large-scale motions also introduces an error in measurements of $\overline{(\partial u_1/\partial x_2)^2}$ with a parallel-wire probe because the wires in the probe measure a contribution from the cross-flow velocity. The error in these measurements can be approximated by considering a parallel-wire probe moving through a frozen small-scale field. The lateral derivative measured by this moving probe, with two wires aligned in the x_3 direction at $\tilde{x}^p_\alpha \pm \Delta/2\delta_{\alpha 2}$, can be written as

$$\left(\frac{\partial u_1}{\partial x_2}\right)_m = \frac{u_1^m(\tilde{x}^p_\alpha+\Delta/2\delta_{\alpha 2})-u_1^m(\tilde{x}^p_\alpha-\Delta/2\delta_{\alpha 2})}{\Delta} \tag{26}$$

where u_1^m is the velocity measured by each of the wires. These velocities can be approximated using a binomial expansion, i.e.,

$$\begin{aligned}u_1^m &= (\tilde{v}_1^2+\tilde{v}_2^2)^{1/2}\\
&\quad\times\left[1+2\frac{\tilde{v}_1\tilde{u}_1}{\tilde{v}_1^2+\tilde{v}_2^2}+\frac{\tilde{u}_1^2}{\tilde{v}_1^2+\tilde{v}_2^2}+2\frac{\tilde{v}_2\tilde{u}_2}{\tilde{v}_1^2+\tilde{v}_2^2}+\frac{\tilde{u}_2^2}{\tilde{v}_1^2+\tilde{v}_2^2}\right]^{1/2}\\
&= (\tilde{v}_1^2+\tilde{v}_2^2)^{1/2}\Bigg[1+\frac{\tilde{v}_1}{(\tilde{v}_1^2+\tilde{v}_2^2)^{1/2}}\frac{\tilde{u}_1}{(\tilde{v}_1^2+\tilde{v}_2^2)^{1/2}}\\
&\quad+\frac{\tilde{v}_2}{(\tilde{v}_1^2+\tilde{v}_2^2)^{1/2}}\frac{\tilde{u}_2}{(\tilde{v}_1^2+\tilde{v}_2^2)^{1/2}}+O\left(\frac{\tilde{u}_\alpha^2}{\tilde{v}_\alpha^2}\right)\Bigg]\end{aligned} \tag{27}$$

Thus, the lateral derivative measured by the parallel-wire probe moving through the small-scale frozen field can be written as

$$\begin{aligned}\left(\frac{\partial u_1}{\partial x_2}\right)_m &= \frac{\tilde{v}_1}{[\tilde{v}_1^2+\tilde{v}_2^2]^{1/2}}\frac{[\tilde{u}_1(\tilde{x}_\alpha+\Delta/2\delta_{\alpha 2})-\tilde{u}_1(\tilde{x}_\alpha-\Delta/2\delta_{\alpha 2})]}{\Delta}\\
&\quad+\frac{\tilde{v}_2}{[\tilde{v}_1^2+\tilde{v}_2^2]^{1/2}}\frac{[\tilde{u}_2(\tilde{x}_k+\Delta/2\delta_{k2})-\tilde{u}_2(\tilde{x}_k-\Delta/2\delta_{k2})]}{\Delta}\\
&\quad+O\left(\frac{\tilde{u}_\alpha}{\tilde{v}_\alpha}\right)+\left(\frac{\Delta\tilde{v}_\alpha}{\Delta}\right)\end{aligned} \tag{28}$$

Following the approach outlined in the previous sections, the variance of this measured derivative can be written as

$$\overline{\left(\frac{\partial u_1}{\partial x_2}\right)^2_m} = \frac{\overline{\tilde{v}_1^2}}{\overline{\tilde{v}_1^2 + \tilde{v}_2^2}} \iiint\limits_{-\infty}^{\infty} 2\frac{[1-\cos(k_2\Delta)]}{\Delta^2}\Phi_{11}\,\mathrm{d}k_1\,\mathrm{d}k_2\,\mathrm{d}k_3 + \frac{\overline{\tilde{v}_2^2}}{\overline{\tilde{v}_1^2 + \tilde{v}_2^2}} \iiint\limits_{-\infty}^{\infty} 2\frac{[1-\cos(k_2\Delta)]}{\Delta^2}\Phi_{22}\,\mathrm{d}k_1\,\mathrm{d}k_2\,\mathrm{d}k_3 + 2\frac{\overline{\tilde{v}_1\tilde{v}_2}}{\overline{\tilde{v}_1^2 + \tilde{v}_2^2}} \iiint\limits_{-\infty}^{\infty} 2\frac{[1-\cos(k_2\Delta)]}{\Delta^2}\Phi_{12}\,\mathrm{d}k_1\,\mathrm{d}k_2\,\mathrm{d}k_3 \tag{29}$$

It is clear that this expression reduces to the expression developed by Wyngaard (1969) as the turbulence intensity approaches zero. Thus, this expression includes both the error caused by the separation distance between the wires and the error introduced by the contribution from the cross-flow velocity.

It is useful to let the separation distance approach zero in order to isolate the cross-flow error. In this case, Eq. (29) reduces to

$$\begin{aligned}\overline{\left(\frac{\partial u_1}{\partial x_2}\right)^2_m} &= \frac{\overline{\tilde{v}_1^2}}{\overline{\tilde{v}_1^2 + \tilde{v}_2^2}}\overline{\left(\frac{\partial u_1}{\partial x_2}\right)^2} + \frac{\overline{\tilde{v}_2^2}}{\overline{\tilde{v}_1^2 + \tilde{v}_2^2}}\overline{\left(\frac{\partial u_2}{\partial x_2}\right)^2} + 2\frac{\overline{\tilde{v}_1\tilde{v}_2}}{\overline{\tilde{v}_1^2 + \tilde{v}_2^2}}\overline{\frac{\partial u_1}{\partial u_2}\frac{\partial u_2}{\partial x_2}} \\ &= \left[1 - \frac{\overline{v_2^2}}{U^2} + 2\frac{\overline{v_2^2 v_1}}{U^3}\right]\overline{\left(\frac{\partial u_1}{\partial x_2}\right)^2} + \left[\frac{\overline{v_2^2}}{U^2} - 2\frac{\overline{v_2^2 v_1}}{U^3}\right] \\ &\quad\times \overline{\left(\frac{\partial u_2}{\partial x_2}\right)^2} - 2\left[\frac{\overline{v_1 v_2}}{U^2} - \frac{\overline{v_2^3}}{U^3} - \frac{\overline{v_1^2 v_2}}{U^3}\right] \\ &\quad\times \overline{\frac{\partial u_1}{\partial u_2}\frac{\partial u_2}{\partial x_2}} + O\left(\frac{v^4}{U^4}\right)\end{aligned} \tag{30}$$

The coefficient of the first term is smaller than unity, so that the contribution from the actual derivative moment in the flow is attenuated as the turbulence intensity of the flow increases. The second term, which is normally the larger of the remaining terms, is positive, so it will offset a portion of this attenuation. Normally, though, it will not completely offset the attenuation, so the measured derivative moment will usually underestimate the actual moment.

4
Estimate of errors in typical measurements

It is useful to evaluate the relative importance of the error introduced by the cross-flow velocity, using data from a typical flow. Detailed measurements in the self-similar region of a high-Reynolds-number axisymmetric jet were reported by Hussein (1988), Hussein et al. (1994), and George and Hussein (1991). These can be used to evaluate the errors predicted with the expressions developed herein, as well as from the corresponding expressions of previous analyses, which ignored the contribution of the cross-flow velocity (e.g., Lumley 1965; Wyngaard and Clifford 1977; George et al. 1989). The difference between these two calculated errors is the error caused by the cross-flow velocity or the "cross-flow" error.

The velocity moments in the aforementioned jet were measured using both a flying hot wire and a laser Doppler anemometer, and the measurements from the two techniques were shown to be in good agreement (Hussein et al. 1994). They also measured the velocity derivative moments with a flying hot-wire probe to reduce errors associated with the use of Taylor's hypothesis and the cross-flow error. The experiment was designed so that the length of the wire and the separation distances in the probes were only slightly larger than one Kolmogorov length scale. The major source of error in the experiment was precisely defining the effective separation distance between the wires, particularly when multiple angle-wire probes were required to measure the moment (e.g., $\overline{(\partial u_2/\partial x_3)^2}$). Hussein (1988) analyzed this source of error and reported that the experimental uncertainty in the measurements was 15–20%, with smaller uncertainties in the single-wire measurements. Further details about the experimental facilities and the experimental procedure are given by Hussein (1988), George and Hussein (1991), and Hussein et al. (1994).

George and Hussein (1991) showed that the measured derivative moments satisfied the conditions for "locally" axisymmetric turbulence with an axis of symmetry in the mean-flow direction. They deduced the four invariants that determine all the of velocity derivative moments for this case by curve-fitting the measured derivative moments. The predicted errors examined here were evaluated using both the measured derivative moments and the derivative moments computed from the invariants reported by George and Hussein (1991), shown in Fig. 1. Both approaches yielded very similar results, and all the sig-

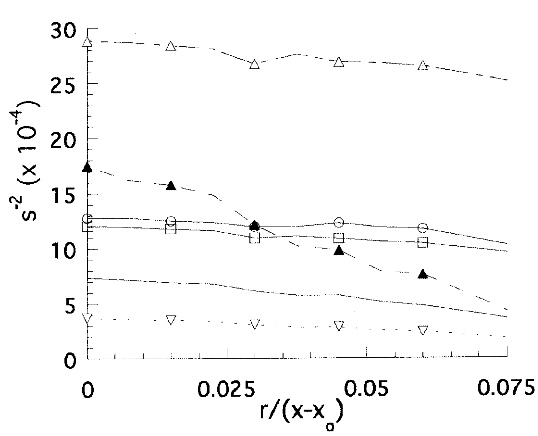

Fig. 1. Derivative moment profile in the region of the round jet considered here computed from the invariants reported by George and Hussein (1991): — $\overline{(\partial u_1/\partial x_1)^2}$; □ $\overline{(\partial u_2/\partial x_2)^2}$; ○ $\overline{(\partial u_1/\partial x_2)^2}$ and $\overline{(\partial u_1/\partial x_3)^2}$; ▲ $\overline{(\partial u_2/\partial x_1)^2}$ and $\overline{(\partial u_3/\partial x_1)^2}$; △ $\overline{(\partial u_2/\partial x_3)^2}$; ▽ $-\overline{(\partial u_1/\partial x_1)(\partial u_2/\partial x_2)}$ and $-\overline{(\partial u_1/\partial x_1)(\partial u_3/\partial x_3)}$

nificant results outlined here were observed for both cases. It was also found that the significant conclusions were unaffected by variations in the derivative moments within the range of the reported experimental uncertainty.

The velocity and velocity derivative moments from the round jet are used to estimate the errors that would occur in the mean-square velocity derivatives measured with stationary hot-wire probes in the jet. The errors are examined in the region $r/(x - x_0) < 0.075$, where $U/U_m > 0.7$ and the local turbulence intensity varies approximately from 25 to 35%. Outside this region, the binomial expansions used to simplify the estimates of the errors in this analysis are not accurate because the turbulence intensity is too large.

In many experiments, far fewer velocity derivative moments are measured than reported by George and Hussein (1991). Therefore, it is often necessary to assume that the flow is "locally" isotropic in order to approximate the error in the measurements using Taylor's hypothesis. The accuracy of the error computed using this approach is assessed here by comparing the errors for "locally" isotropic turbulence and "locally" axisymmetric turbulence predicted using the model developed in this analysis.

4.1
Single-wire measurements

The comparison of the predictions for the errors in $\overline{(\partial u_1/\partial x_1)^2}$ measured with a single wire is shown in Fig. 2. The predicted errors for the current model, which includes the effect of the cross-flow velocity, are only 20% smaller than the error predicted using the previous models, which ignored it. Thus, the cross-flow errors in these measurements are not as large as the errors caused by the unsteady convection of the small-scale motions. This difference could be important if the data were to be corrected using the errors predicted with the previous analyses. For example, the results here indicate that measurements corrected in this manner would under-predict the actual moment by almost 10% at $r/(x - x_0) = 0.075$ and by larger amounts at larger radii.

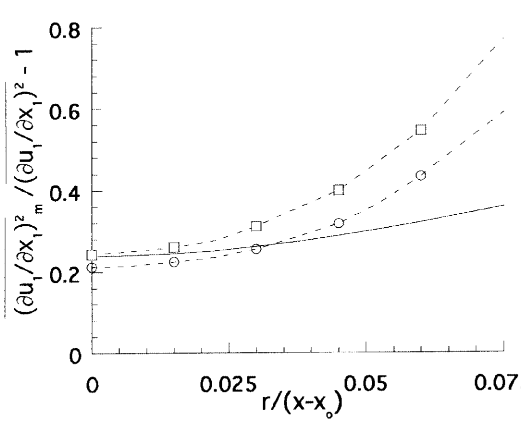

Fig. 2. Comparison of the errors in $\overline{(\partial u_1/\partial x_1)^2}$ measured with a single wire in the far field of the axisymmetric jet predicted (○) with the current model, which includes the contribution from the cross-flow velocity, and (□) with the previous models, which ignore the cross-flow velocity (e.g., George et al. 1989): — local turbulence intensity, v/U

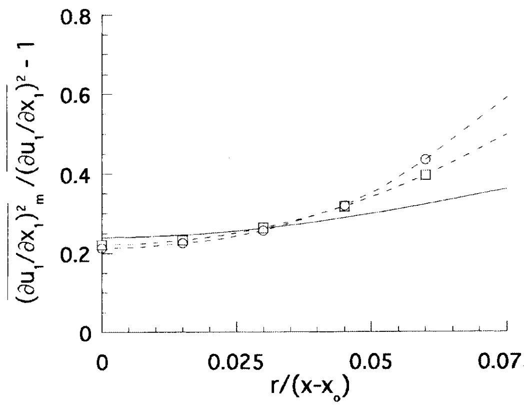

Fig. 3. Comparison of the errors in $\overline{(\partial u_1/\partial x_1)^2}$ measured using a single wire for (○) "locally" axisymmetric turbulence and for (□) "locally" isotropic turbulence. The predicted errors include the cross-flow error: — local turbulence intensity in the jet, v/U

A comparison of the errors predicted using the current model for "locally" isotropic turbulence and for "locally" axisymmetric turbulence is shown in Fig. 3. The predicted errors are nearly equal near the centerline, where the derivative moments included in the expression for the error approximately satisfy the isotropic relationships (cf., George and Hussein 1991). Away from the centerline, where the smal-scale motions are more anisotropic, the predicted errors for "locally" isotropic turbulence are up to 15% smaller than those for "locally" axisymmetric turbulence. The difference between the predicted errors is also increasing with radius, so the difference is likely to be even larger in the outer region of the jet.

4.2
Cross-wire measurements

A comparison of the predictions for the errors $\overline{(\partial u_1/\partial x_1)^2}$ measured with single- and cross-wire probes in the round jet is shown in Fig. 4. The predicted errors in the cross-wire measurements for the current model were computed using a value of $k = 0.15$ (cf., Tutu and Chevray 1975;

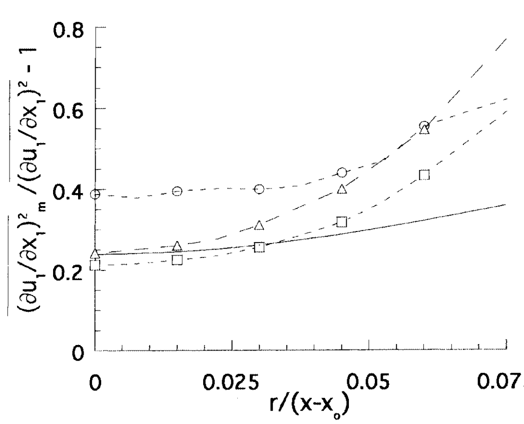

Fig. 4. Comparison of the errors in $\overline{(\partial u_1/\partial x_1)^2}$ measured with (○) a cross-wire probe and (□) a single-wire probe predicted with the current model, which includes the cross-flow error: △ errors in $\overline{(\partial u_1/\partial x_1)^2}$ measured with both probes predicted using the previous models: — turbulence intensity v/U

Champagne et al. 1967). They are typical of the results for the normal range of k. The errors predicted using the previous model are also shown in the figure. A single curve is shown for this case because the predicted errors are independent of the measuring transducer when the effect of the cross-flow velocity is ignored. It is clear, though, that there are significant differences between the errors for single- and cross-wire measurements when the contribution from the cross-flow velocity is considered. The errors in the cross-wire measurements are up to twice as large as the error in the single-wire measurements. It can also been shown that, for a "locally" isotropic flow, the error in the cross-wire measurements will be about twice the error in the single-wire measurements. The errors in the cross-wire measurements are much larger than in the single-wire measurements because the cross-wire probe is more sensitive to the cross-flow velocity and, as a result, the coefficients in the expression for the error are much larger. This is not unexpected, since the cross-flow errors in the velocity moment $\overline{u_1^2}$ measured with a cross-wire probe are considerably larger than the errors in the single-wire measurements. The results here show that it is more accurate to measure $\overline{(\partial u_1/\partial x_1)^2}$ with a single wire.

The relative errors in $\overline{(\partial u_2/\partial x_1)^2}$ measured with a cross-wire probe using Taylor's hypothesis are shown in Fig. 5. The errors predicted with the current model are up to 50% smaller than the errors predicted using the result from the previous models, which ignored the cross-flow error. Data corrected using the error predicted in the previous analyses would under-predict the actual moment by 10% in the entire region of the jet considered here.

The relative errors for "locally" isotropic turbulence predicted using the current model are also shown in Fig. 5. These errors are 50–70% smaller than the errors for "locally" axisymmetric turbulence. Much of this difference is caused by the large deviation of the moment $\overline{(\partial u_2/\partial x_3)^2}$ from the value expected for "locally" isotropic turbulence (cf., Fig. 1). The other derivative moments do not deviate from "local" isotropy nearly as much, nor does $\overline{(\partial u_2/\partial x_3)^2}$ appear in the expressions for the errors in the other moments. Hence, the large difference between the predicted error for "locally" isotropic and "locally" axisymmetric turbulence is only seen in $\overline{(\partial u_2/\partial x_1)^2}$. The term $\overline{(\partial u_2/\partial x_3)^2}$ is also part of the error caused by the unsteady convection velocity. So similar difference between the error for "locally" isotropic and "locally" axisymmetric turbulence would be observed in the predictions from the previous models.

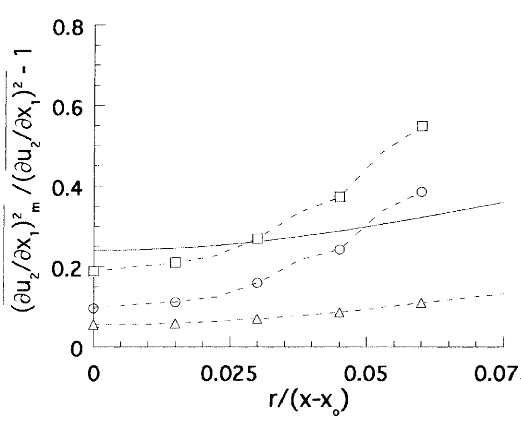

Fig. 5. Comparison of the errors in $\overline{(\partial u_2/\partial x_1)^2}$ measured with a cross-wire probe in the round jet predicted (○) with the current model, which includes cross-flow error, (□) with the previous models, which ignore the cross-flow: △ error in $\overline{(\partial u_2/\partial x_1)^2}$ for "locally" isotropic turbulence predicted using the current model; — turbulence intensity, v/U

4.3 Parallel-wire measurements

The predictions of the errors in $\overline{(\partial u_1/\partial x_2)^2}$ measured with a parallel wire probe in the round jet are shown in Fig. 6. Here, the separation distance between the wires has been set to zero in order to isolate the cross-flow error. The errors for "locally" axisymmetric measurements do not exceed 1% in the region of the jet considered here. The predicted errors for "locally" isotropic turbulence are larger but do not exceed 5%. Thus, the errors caused by the cross-flow velocity in measurements with the parallel-wire probe are smaller than the errors in measurements using Taylor's hypothesis. In many measurements, this error will also be smaller than the error caused by the finite separation distance between the wires in the probe. However, the cross-flow and spatial resolution errors both act to reduce the measured moment in most flows. Thus, it may be necessary to consider both sources of error in high-turbulence-intensity flows if care is being exercised to minimize the error in the measured derivative moments.

4.4 Comparison of the predicted errors with measurements

George et al. (1989) measured $\overline{(\partial u_1/\partial x_1)^2}$ in the far field of the round jet using both a stationary single wire and a single wire moving through the flow on a rotating arm. Flying the wire on the rotating arm increased the mean velocity measured by the wire, thereby reducing the turbulence intensity of the measured velocity. Therefore, $\overline{(\partial u_1/\partial x_1)^2}$ could be measured using Taylor's hypothesis at the same location with single hot-wire probes experiencing two different turbulence intensities, and the difference in the measurements could be compared with the predictions from the models. In their comparison, George et al. noted

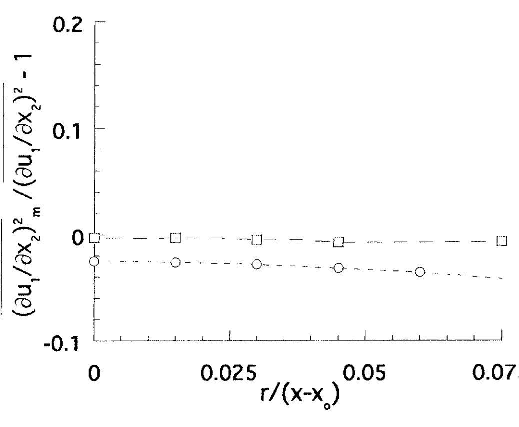

Fig. 6. Comparison of the errors in $\overline{(\partial u_1/\partial x_2)^2}$ measured using a parallel wire (with zero wire separation) for (□) "locally" axisymmetric turbulence and for (○) "locally" isotropic turbulence

that the errors in the flying-wire measurements were considerably smaller than those in the stationary-wire measurements, and assumed the errors in the flying-wire measurements could be neglected when they compared the two measurements. They also assumed that the turbulence was "locally" isotropic, and predicted that the normalized difference between the measurements was given by

$$\frac{\overline{[(\partial u_1/\partial x_1)_m^2]}_{\text{sta}} - \overline{[(\partial u_1/\partial x_1)_m^2]}_{\text{mov}}}{\overline{[(\partial u_1/\partial x_1)_m^2]}_{\text{mov}}} = \left[\frac{\overline{u_1^2} + 2\overline{u_2^2} + 2\overline{u_3^2}}{U^2}\right] \tag{31}$$

where the subscripts "mov" and "sta" denote the moments measured in the moving- and stationary-wire experiments, respectively. They found reasonable agreement between the predictions of the theory and the measurements.

However, as mentioned previously, later measurements in the round jet showed the small-scale motions were not "locally" isotropic, but rather were "locally" axisymmetric. The errors in the flying-wire measurements were also 3–5%, so it is more accurate to incroporate the errors for these measurements in the estimate of the normalized difference yielding

$$\frac{\overline{[(\partial u_1/\partial x_1)_m^2]}_{\text{sta}} - \overline{[(\partial u_1/\partial x_1)_m^2]}_{\text{mov}}}{\overline{[(\partial u_1/\partial x_1)_m^2]}_{\text{mov}}} = \left[\frac{U_{\text{mov}}^2}{U_{\text{sta}}^2} - 1\right] \frac{\overline{u_1^2} + (\beta+\gamma+\eta)\overline{u_2^2} + \gamma\overline{u_3^2}}{U_{\text{mov}}^2 + \overline{u_1^2} + (\beta+\gamma+\eta)\overline{u_2^2} + \gamma\overline{u_3^2}} \ . \tag{32}$$

When the effect of the cross-flow velocity is included in the model $\beta = 3$,

$$\gamma = \overline{\left(\frac{\partial u_1}{\partial x_2}\right)^2} \left[\overline{\left(\frac{\partial u_1}{\partial x_1}\right)^2}\right]^{-1} \tag{33}$$

and

$$\eta = \overline{\left(\frac{\partial u_2}{\partial x_1}\right)^2} \left[\overline{\left(\frac{\partial u_1}{\partial x_1}\right)^2}\right]^{-1} \tag{34}$$

and when the cross-flow is ignored $\beta = 0$, $\eta = 0$, and

$$\gamma = \overline{\left(\frac{\partial u_1}{\partial x_2}\right)^2} \left[\overline{\left(\frac{\partial u_1}{\partial x_1}\right)^2}\right]^{-1} \ . \tag{35}$$

George et al. (1989) performed their experiment in the same jet used by Hussein et al. (1994) and George and Hussein (1991), so these measurements can be used to evaluate the predicted values for the normalized differences.

The comparison of the predictions for the two cases and the measurements reported by George et al. (1989) is shown in Fig. 7. The error bars indicate the uncertainty in the predictions due to the experimental uncertainty in the derivative moments. Noise was the primary source of error in the single-wire measurements, other than error associated with Taylor's hypothesis that has been accounted for in Eq. (32). This was discussed in detail in Hussein

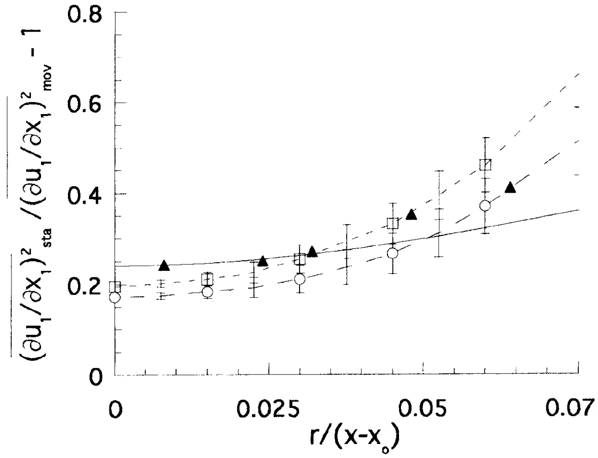

Fig. 7. Comparison of the relative difference between $\overline{(\partial u_1/\partial x_1)^2}$ measured with a stationary and flying hot-wire (○) predicted with the current model, which includes the cross-flow error, (□) predicted with the previous models, which ignore the cross-flow error, and (▲) calculated from measurements reported by George et al. (1989): — turbulence intensity, v/U

(1988), who concluded that the error caused by noise in these measurements was negligible. The noise in the flying-wire measurements was larger than in the stationary measurements, so the actual normalized differences would be larger than those presented in Fig. 7 if noise were a significant factor.

It is clear that the predictions from both models are smaller than the measured differences in the inner portion of the jet. The predictions and the measured differences are in better agreement when $r/(x - x_0) > 0.4$. Overall, however, the agreement between the predictions from both models and the measurements is not particularly good, since the discrepancies near the centerline exceed levels that could be explained by experimental uncertainty. This may represent a shortcoming of the theory or a problem with the data. It would be useful to perform this comparison in another flow, but the measurements reported by George et al. (1989) seem to be the only direct measurements of the error that occurs in velocity derivative moments measured using Taylor's hypothesis. Mi and Antonia (1994) recently performed a similar experiment for the scalar derivative moments and found better agreement between the measurements and the predictions from the theory.

4.5
Effect of finite wire length

The finite length of the wires in the probes has been ignored so far in the analysis. Following the approach outlined by Wyngaard (1968), it is straightforward to show that the mean-square velocity derivatives in the expression for the value of $\overline{(\partial u_1/\partial x_1)^2}$ measured with a single-wire (Eq. 16) should be written as

$$\overline{\frac{\partial \tilde{u}_l}{\partial x_i}\frac{\partial \tilde{u}_n}{\partial x_j}} = \iiint_{-\infty}^{\infty} \left[\frac{\sin(k_3 l_3/2)}{k_3 l_3/2}\right]^2 k_i k_j \Phi_{ln}\, \mathrm{d}k_1\, \mathrm{d}k_2\, \mathrm{d}k_3 \tag{36}$$

where l_3 is the length of the wire in the x_3 direction. The averaging along the wire will reduce the derivative moments in the x_1 and x_2 direction, such as $(\partial u_1/\partial x_2)^2$ or $(\partial u_1/\partial x_1)^2$, by only 3–8% for wires with lengths from 3 to 6η. It will reduce the derivative moments in x_3 direction by a larger amount. For example, it will reduce $(\partial u_1/\partial x_3)^2$ by approximately 10–25% when the length of the wire is 3–6η (cf., Wyngaard 1969; Ewing et al. 1995). The term that includes this moment contributes 30–40% of the predicted error in $(\partial u_1/\partial x_1)^2$. Thus, it may be necessary to consider the length of the wire when long wires are used in the measurements. Similar results could developed for the measurements with the cross-wire and parallel-wire probes.

5 Summary and concluding remarks

It is well known that significant errors occur in the velocity moments measured with single- or cross-wire probes in high-turbulence-intensity flows because the probes cannot exactly resolve the velocity components from the flows. A similar error occurs in the velocity derivative moments measured with these probes because they also do not exactly resolve the velocity components from the small-scale motions. Estimates of the error this causes in the value of $(\partial u_1/\partial x_1)^2$ and $(\partial u_2/\partial x_1)^2$ measured with single- and cross-wire probes using Taylor's hypothesis and in $(\partial u_1/\partial x_2)^2$ measured with a parallel-wire probe were developed here. These estimates also include the errors introduced by the unsteady convection of the small-scale motions in measurements using Taylor's hypothesis and the size of the measuring transducer in measurements with parallel-wire probes. Thus, the estimates of the errors developed here are more general than previous estimates of the errors in measurements with Taylor's hypothesis (e.g., Lumley 1965; George et al. 1989) or in measurements with the parallel-wire probe (e.g., Wyngaard 1969). The approach outlined here can also be extended to higher-order derivative moments measured with these probes.

Measurements from the far field of a high-Reynolds-number round jet reported by Hussein et al. (1994) and George and Hussein (1991) were also used to estimate the size of the different errors that would occur in the derivative moments measured with stationary wires in the jet. The predicted errors were computed for the range $0 < r/(x - x_0) < 0.75$ where the turbulence intensity varied from 25 to 35%. Outside this range, the binomial expansions used to simplify the expressions for the predicted errors developed here are not accurate.

The estimates of the cross-flow error in $\overline{(\partial u_1/\partial x_1)^2}$ measured with a single wire, using Taylor's hypothesis, were only about 20% of the error caused by the unsteady convection velocity. The cross-flow error also decreased the overall error, so that the predictions from the previous analyses yielded conservative estimates of the overall error. The cross-flow error could be important, however, if measurements in a high-turbulence-intensity region were corrected using errors predicted from the previous analyses. For example, measurements at $r/(x - x_0) = 0.075$ in the jet corrected in this manner would under-predict the actual moments by 10%.

The cross-flow errors were larger in measurements with cross-wire probes. For example, the overall errors in $(\partial u_1/\partial x_2)^2$ measured with a cross-wire probe were up to twice as large as the errors in the single-wire measurements when the effect of the cross-flow velocity is considered. Thus, it is simpler and more accurate to measure $(\partial u_1/\partial x_1)^2$ with a single-wire probe. The estimates of the cross-flow errors in measurements of $(\partial u_2/\partial x_1)^2$ were also up to 50% of the error caused by the unsteady convection velocity. The cross-flow error did reduce the overall errors, though, so predictions from the previous analyses are again conservative. However, measurements corrected using errors predicted with the previous analyses would under-predict the actual moment by 10% in the region of the jet considered.

It was also found that the errors predicted by assuming that the flow was "locally" isotropic could yield poor estimates of the actual errors in some circumstances. For example, the overall errors in measurements of $\overline{(\partial u_2/\partial x_1)^2}$ predicted assuming the flow was "locally" isotropic turbulence were smaller than 10%, while the predicted errors for "locally" axisymmetric flow were 10–40%. Much of this difference is caused by the large departure of $(\partial u_2/\partial x_3)^2$ in the round jet from the value for "locally" isotropic turbulence. This term is part of the error caused by the unsteady convection velocity, so similar differences would occur in the errors predicted using the previous analyses. The large deviation from isotropy may not be observed in all flows, but the results here show that, when it does occur, the errors predicted using the simpler assumption of "locally" isotropy may significantly underestimate the actual errors.

The cross-flow errors in the measurements of $\overline{(\partial u_1/\partial x_2)^2}$ with a parallel-wire probe were considerably smaller than the errors in the measurements with Taylor's hypothesis. These errors did not exceed 1% in the region of the jet considered here. The predicted errors for "locally" isotropic turbulence were also less than 5% in this range. Thus, the cross-flow errors will often be smaller than the errors caused by finite separation between the wires in the parallel-wire probe. However, both errors reduce the measured moments in most flows, so it may be necessary to consider both when estimating the overall error in the parallel-wire measurements.

The accuracy of the predictions from both this analysis and the previous analyses were tested by comparing them with direct measurements reported by George et al. (1989). In this experiment $\overline{(\partial u_1/\partial x_1)^2}$ was measured using Taylor's hypothesis with both a stationary wire and moving wire at the same point in the round jet. The relative differences between the measured moments were compared with the predictions from the analyses. Although there was qualitative agreement between the measurements and predictions from both models, there were some discrepancies that exceeded the experimental uncertainty. Further experimental investigations should be performed to determine whether the observed discrepancies are caused by errors in the measurements or shortcomings in the models.

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