Examination of LSE-based velocity field estimations using instantaneous, full field measurements in an annular mixing layer

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Abstract

The data base measured by Citriniti (1995) is used to examine the accuracy of using Linear Stochastic Estimation to estimate the velocity field in the annular mixing layer. This data base is used to compute the velocity field that would be measured if the field was measured on two radial rings in the shear layer and the velocities on four other rings were estimated using Linear Stochastic Estimation. This result is compared to the actual velocity field measured at 138 points on the 6 radial positions by Citriniti (1995). The estimated and actual velocity fields are projected onto the orthogonal basis deduced by Citriniti (1995) and only the information from the most energetic modes are retained. It is found that the estimated field is capable of accurately reproducing the gross features of both the ring structures and the inter-ring regions found in the shear layer.

Introduction

It is widely recognized that large energetic structures play an important role in the dynamics of the near field in many free-shear flows, such as the axisymmetric shear layer occurring near the exit of a round jet. Although there are many different techniques that can be used to identify the coherent structures (*cf* Bonnet and Glauser 1994), only a few of these can be used to study the dynamics of the structures. One technique that can be used for this purpose is the Proper Orthogonal Decomposition (POD) technique introduced by Lumley (1967).

One of the primary difficulties of studying the *instantaneous* dynamics of the large structures with this technique (and others) is that it requires the simultaneous measurement of the velocity over the region of interest. As a result, there have been few attempts to experimentally study the instantaneous dynamics of the structures on an entire plane in a free-shear flow. In one attempt to investigate these features Citriniti (1995) simultaneously measured the velocity field at 138 points in a high-Reynolds-number annular shear layer using hot-wire anemometers. Using the POD technique he found evidence that the shear layer was made up of both ring vortices and streamwise rib vortices that occur between the vortex rings, similar to the streamwise vortices that occur in the planar mixing layers braid region (*cf* Rogers and Moser 1994). The experimental evidence also suggests that these streamwise modes may entrain more ambient air into the shear layer than the ring vortices (the commonly assumed entrainment mechanism).

Although this experiment yielded very useful physical information about the axisymmetric shear layer, the effort required to perform the experiment effectively precludes its use on a wide variety of flows. One alternative approach suggested by Bonnet *et al.* (1994) is to measure the velocity at a small number of points and use Linear Stochastic Estimation to approximate velocity over the rest of the field. This estimated velocity field could then be used as an input into POD technique and an estimate for the dynamics of the large structures could be deduced. Bonnet *et al.* (1994) demonstrated that this technique was capable of accurately reproducing the dynamics of the large structures when applied to one non-homogeneous direction in a shear layer. They also suggested that the method could be extended to more spatial dimensions in a flow, however as yet, there has not been an attempt to confirm this.

The objective of this investigation is to examine if LSE can be used in conjunction with the POD technique to produce an accurate estimate of the dynamics of the large structures on a full $r-\theta$ plane in the axisymmetric shear layer. The predictions for the dynamics of the structures from the estimated field are compared with the results reported by Citriniti (1995) to determine how well the technique reproduces the dynamics of both the ring and inter-ring regions.

Background

Annular Shear Layer Data Base

In order to study the large structures in the axisymmetric shear layer Citriniti (1995)[1] constructed an array of 138 single-wire transducers that was placed at a distance of 3 diameters downstream of a round jet. Glauser (1987) had shown previously that the large structures in flow could be resolved using the POD if the velocity was measured at 6 radial positions across the layer at $r/d = 0.15, 0.28, 0.41, 0.54, 0.67$, and 0.80. He also found that the azimuthal dependence of the structures could be recovered if the velocity was measured at 6 azimuthal positions at the inner most radial position and 12, 24, 32, 32, and 32 positions at the other 5 radial positions, respectively. This information was used to locate the probes within the 138 wire array. The mean velocity normalized by the exit velocity ranged from 0.9 to 0.15 over these 6 radial points indicating that array of hot wires measured the streamwise velocity component in a region that spanned most of the shear layer.

The exit velocity profile of the jet was top hat within 0.1% with a turbulence intensity of 0.35%. The outlet velocity of the jet was set such that the exit Reynolds number based on the nozzle diameter, d, was 80,000. For these conditions, Glauser (1987) demonstrated that the large structures could be well resolved in the first 800 Hz of the spectra so the signals from all of the wires in the array were filtered at 800 Hz and simultaneously sampled at 2,028 Hz. The data was gathered in 300 blocks of 1024 points, which allowed the accurate computation of spectral information.

Proper Orthogonal Decomposition

Lumley (1967) argued that the functions used to represent the large structures in a turbulent flow should be defined in an objective manner using information from the flow only. For example, Lumley suggested these functions could be defined as those that make the largest contribution to the turbulent energy in the region of interest. Using this definition, it can be shown that functions for the homogeneous and stationary directions are simply Fourier modes so these directions can be transformed out the problem in the standard manner.

For the axisymmetric shear layer, the functions to represent the information in the radial direction that satisfy this definition, $\Phi^n(r,m,f)$, are solutions to the integral eigenvalue problem given by (Citriniti 1995)

$$\int_0^\infty \psi(r,r',m,f)\Phi^n(r',m,f)r'dr' = \lambda^n(m,f)\Phi^n(r,m,f), \qquad (1)$$

where $\psi(r,r',m,f)$ is the Fourier transform of the two-point velocity correlation in both the azimuthal direction and time, m is the azimuthal mode number and f is the frequency. It is straightforward to show that these functions are orthogonal (*cf* Lumley 1967). Thus, the integral eigenvalue can be solved using only statistical measures of the flow to yield an orthogonal basis to optimally describe the flow in the radial direction.

The information about the dynamics of the field is, however, contained in the coefficients of the orthogonal basis which are given by

$$a^n(m,f) = \int_0^\infty \hat{u}(r,m,f)\Phi^{n*}(r,m,f)rdr, \qquad (2)$$

where $\hat{u}(r,m,f)$ is the Fourier transform of the *instantaneous* velocity field in the azimuthal direction and time. Thus, it is necessary to simultaneously sample the velocity field over the entire domain of interest in order to determine these coefficients.

Once the coefficients for the orthogonal functions are determined, the dynamics of the large-scale structures in the field can then be studied by reconstructing the Fourier coefficients; *i.e.*,

$$\hat{u}_{rec}(r,m,f) = \sum_{n=1}^{N} a^n(m,f)\Phi^n(r,m,f), \qquad (3)$$

where N is a user specified parameter. If all the available modes are used (*i.e.*, N is set to its highest value) then all the information from the original signal can be recovered. If on the other hand N is set to a small number then only the information from the most energetic modes is retained, thus reducing the small-scale information in the signal. The amount of small-scale information in the reconstructed velocity field can be further reduced by including the information from only a small number of azimuthal modes. The coefficients for the other modes are set to zero in order to remove them from the signal. This reduced signal can then be inverse Fourier transformed in time and the azimuthal direction to yield a low-order model for the instantaneous velocity field in the axisymmetric shear layer.

Citriniti (1995) demonstrated that the dynamics of the large-scale structures could be modeled if only the first POD mode is used (*i.e.*, N=1) and only the coefficients for azimuthal mode numbers 0 and $3-6$ are retained in the reconstructed field. Examples of the reconstructed instantaneous fluctuating velocity fields computed using these modes are show in figures 1 and 2. In both of these figures the colour scale corresponds to the value of the instantaneous fluctuating velocity. The light colours correspond to velocities that are larger than the local mean while the dark colours correspond to velocities less than the local mean.

It is clear that there is a band of faster-than-average moving fluid inside a band of slower-than-average moving fluid in figure 1, which is consistent with the pattern one would expect with the passage of a vortex ring like structure. On the other hand there are neighbouring regions of fast and slow moving fluid in figure 2, suggesting that regions of fast flow moving out of the core neighbour regions of slow moving fluid being entrained into the core. This is consistent with the pattern one would expect in a braid region of counter-rotating streamwise vortices. Thus, the velocity field

[1] See also the paper by Citriniti in this volume.

$(x', x'', \ldots)$; *i.e.*,

$$u_e(x_n) = A(x_n)u(x') + B(x_n)u(x'') + \ldots \quad (4)$$

The coefficients, $A(x_n), B(x_n), \ldots$ are then determined by minimizing the mean square error in the estimated velocity; *i.e.*, $\overline{(u_e(x_n) - u(x_n)^2)}$.

For the current application, it is more useful to estimate the Fourier coefficients $\hat{u}(r, m, f)$ at a radius in the axisymmetric shear layer using the Fourier coefficients measured at one or two radial positions. If the Fourier coefficients are measured at a single radial position it follows that the Fourier coefficients at the other positions can be approximated as

$$\hat{u}_e(r_n, m, f) = C(r_n, m, f)\hat{u}(r', m, f), \quad (5)$$

where $\hat{u}(r', m, f)$ are the Fourier coefficients at the measured points. In this case, the mean square error in the estimated Fourier coefficient, $\overline{(\hat{u}_e - \hat{u})(\hat{u}_e - \hat{u})^*}$, is minimized if

$$C(r_n, m, f) = \frac{F(r_n, r', m, f)}{F(r', r', m, f)}, \quad (6)$$

where $F(r_n, r', m, f) = \overline{\hat{u}_e(r_n, m, f)\hat{u}^*(r', m, f)}$. Thus, the coefficients used to estimate the velocity are determined using statistical measures of the flow that can be easily measured with a small number of hot-wires.

If the Fourier coefficients are measured at two radial positions it follows that the coefficients at the other radial positions can be approximated as

$$\begin{aligned} \hat{u}_e(r_n, m, f) \quad = \quad & A(r_n, m, f)\hat{u}(r', m, f) + \\ & B(r_n, m, f)\hat{u}(r'', m, f), \end{aligned} \quad (7)$$

where $\hat{u}(r', m, f)$ and $\hat{u}(r'' m, f)$ are the measured Fourier coefficients. In this case, the error in the estimated coefficients are minimized if

$$A(r_n, m, f) = \frac{F(r_n, r')F(r'', r'') - F(r_n, r'')F(r'', r')}{F(r'', r'')F(r', r') - |F(r'', r')|^2} \quad (8)$$

and

$$B(r_n, m, f) = \frac{F(r_n, r'')F(r', r') - F(r_n, r')F(r', r'')}{F(r'', r'')F(r', r') - |F(r'', r')|^2}. \quad (9)$$

The dependence of F on m and f has been left out of these last equations in order to shorten their length.

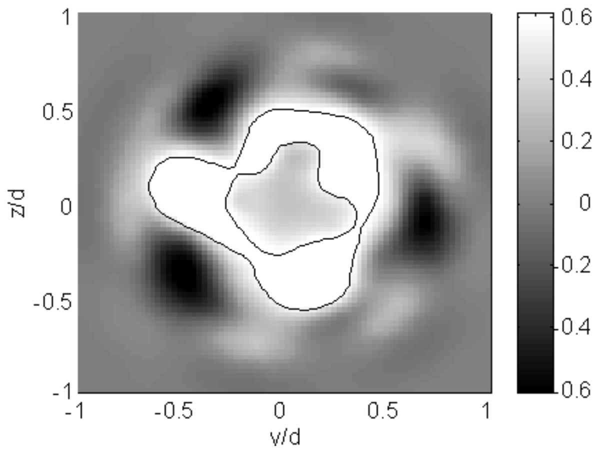

Figure 1: Ring-like structure from the low-order reconstruction of the axisymmetric shear layer.

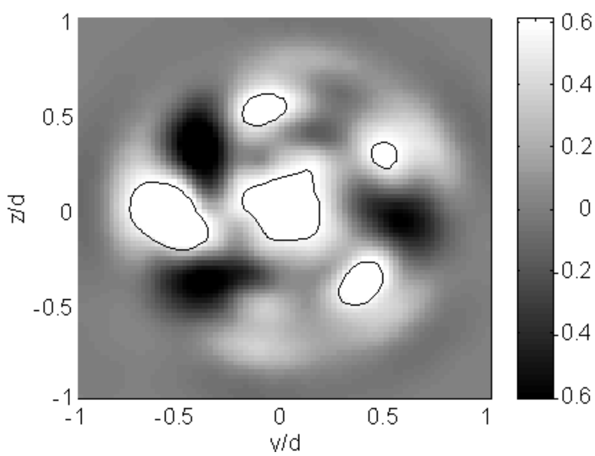

Figure 2: The inter-ring region that appears to contain a series of counter-rotating streamwise vortices.

reconstructed with only a few modes on the $r-\theta$ plane showed evidence of both ring vortices and streamwise vortices in the inter-ring regions.

Linear Stochastic Estimation

Although these results shed insight into the dynamics of the structures in the axisymmetric shear layer, it is not trivial to simultaneously measure the velocity field at 138 points in the field. Thus, as Bonnet *et al.* (1994) suggested, it would be very useful if a similar picture of this flow (and others) could be deduced by applying the POD to an instantaneous field that was measured at a smaller number of points and estimated at the other points necessary to resolve the POD coefficients. Bonnet *et al.* suggested using Linear Stochastic Estimation to extrapolate information from the measured points and fill out the field sufficiently to use the POD.

In this approach, the instantaneous velocity at a position x_n is approximated as a linear combination of the velocities at the points where it is measured

Results

In the axisymmetric shear layer, the LSE technique could be use to interpolate or extrapolate information in two different spatial directions, r or θ. Since the signals are correlated over a greater distance in the radial direction than the azimuthal direction (relative to the characteristic length scales in those directions), it follows that the LSE should work more effectively in radial direction than in the azimuthal direction. Thus, the first step in this investigation was to examine how effectively the LSE could interpolate or extrapolate information in the radial direction. It was assumed that the Fourier coefficients $\hat{u}(r,m,f)$ are known at one or two radial positions. This information is then used to estimate the Fourier coefficients at the other radial positions. Physically, this corresponds to doing the experiment carried out by Citriniti (1995) using only one or two rings of hot-wires instead of the 6 used in that experiment.

A number of different configurations were examined in order to determine which radial positions were the optimal to use. Initially the suitability of different positions was examined by considering the value of the means square error in the estimated coefficient normalized by the mean square value of the actual coefficients; *i.e.*,

$$\frac{\overline{|(\hat{u}_e(r_n,m,f)-\hat{u}(r_n,m,f))|^2}}{F(r_n,r_n,m,f)}.$$

It was found that none of the individual radial positions could be used to extrapolate the Fourier coefficients for both the 0 mode and modes $3-6$. The mean square error in the estimated Fourier coefficient for the 0 mode was generally small over the dominant energy frequencies when a point on the inner or high-speed side of the layer was used as the measured point. This was not unexpected since the ring vortex structure, which contributes to the 0 mode, is the most dominant feature of the inner radial positions. Similarly, the normalized error in the Fourier coefficients for modes $3-6$ was significantly lower when a point on the outer or slow speed side of the layer was used as the measured point. Again, this is expected since the inter-ring region is more dominant on the outside of the layer than the inside.

Thus, it was suspected that the most accurate estimated coefficients could be derived if two measuring points were used, with one on each side of the shear layer. A variety of combinations of these points were studied. It was found that that there was little variation in the accuracy of the estimated Fourier coefficients (at least from the perspective of the normalized mean square error) when the two measured points were situated on opposite sides of the mixing layer. The best results, however, seemed to occur when the two radial positions are chosen near center of the shear layer, $r/d = 0.41$ and $r/d = 0.67$, which is analogous to the result deduced by Bonnet *et al.* (1994). The normalized mean square error in the estimated Fourier coefficients for azimuthal modes 0 at the other four radii in the shear layer are shown in figure 3. It is important to note the spectra for the 0 mode has a dominant peak at approximately 100 hz so this is the region where the Fourier coefficients need to be well predicted. It is clear that this occurs for all the radial positions except for the position at the center of the mixing layer. It is not clear why the prediction at this point is so poor however, a similar result was noted by Bonnet *et al.* when they used the LSE in a single non-homogeneous direction.

The normalized mean square error in the Fourier coefficients for azimuthal mode 5 are shown in figure 4. The error is only shown for the outer three positions because the Fourier coefficient for mode 5 is not measured in the inner most radius. The spectra for this mode peaks at the lowest frequency measured so it is

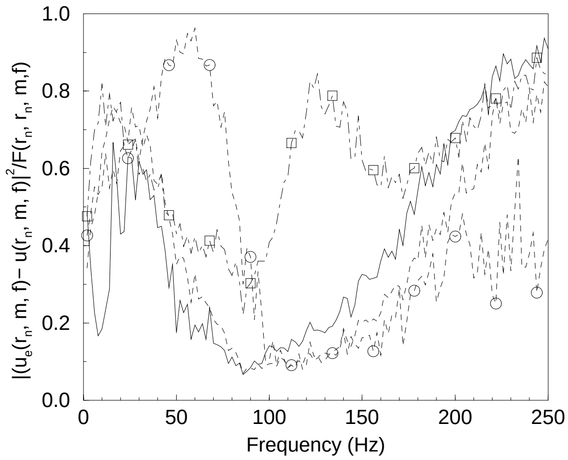

Figure 3: Normalized mean square error in the coefficients for azimuthal mode number 0. —— $r_1/d = 0.15$, ---- $r_2/d = 0.28$, —-— $r_4/d = 0.54$, and ---- ○ $r_5/d = 0.8$.

important that the Fourier coefficients for this mode number are accurately predicted in this region. Although the normalized mean square error is higher for these Fourier coefficients than for the 0 mode, the level of the error is still small in the low frequency region at the outer radial positions where the higher modes play a large role in the dynamics of the flow.

Although these results show that the normalized mean square errors in the estimated coefficients are small for the important ranges of frequencies, they do not provide any information about how well the instantaneous coefficients are predicted. In order to examine this question the estimated and actual coefficients were projected onto the orthogonal basis deduced by Citriniti (1995) to determine the coefficients for the POD modes. Following the same procedure outlined by Citriniti (1995), the coefficient of the first POD mode computed from the estimated field was used to generate a partial reconstruction of the field. The information from azimuthal modes $0, 3, 4, 5$, and 6 were retained and the resulting coefficients were inverse Fourier transformed in time and the azimuthal direction in order to produce a low-order estimate of the velocity field.

A comparison of several realizations from the estimated and original fields are shown in figures 5 - 10. The realizations from the estimated field are shown in figures 5 - 7 while corresponding realizations from the original field are shown in figures 8 - 10. The colour is again used to denote the level of the instantaneous fluctuating velocity with light colours corresponding to velocities larger than the local mean and dark colours corresponding to velocities less than the local mean.

Comparing figures 5 and 8 it is clear that the estimated realization has almost all of the gross features of the original field. As one would expect the estimated field appears to be a filter version of the original field, removing some of the finer features of the flow and smoothing the topology. This is evident if the highest speed contour from the two realizations are

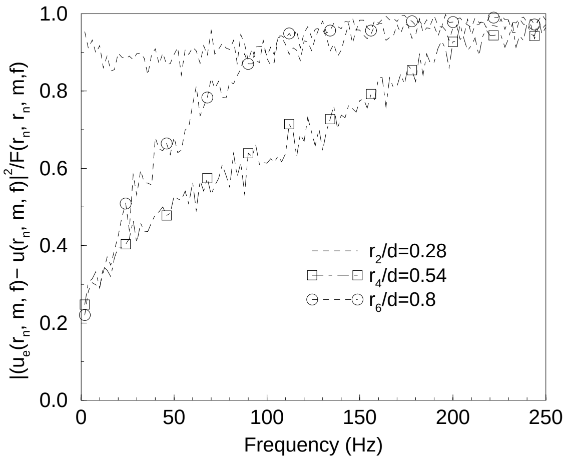

Figure 4: Normalized mean square error in the coefficients for azimuthal mode number 5. ---- $r_2/d = 0.28$, —— $r_4/d = 0.54$, and ---- $\circ$ $r_5/d = 0.8$.

compared. The shape of the contour is smoother for the realization from the estimated field. Two different realizations in the inter-ring region are compared in figures 6 and 9 and figures 7 and 10. It can be seen that the overall levels of the fluctuating velocity in the estimated field are smaller than the levels in the actual field. However, the topology of the structures in the region are well preserved showing the distinctive feature characteristic of the streamwise vortices. Thus, the estimated field seems to be quite capable of reproducing the major features of the shear layer.

Summary

The accuracy of using estimated velocity fields to study the dynamics of the large structures in the axisymmetric shear layer is examined using the data base measured by Citriniti (1995). It was found that the dynamics of the ring vortices and the inter-ring regions could be well reproduced if the Fourier coefficients are measured on 2 radii and estimated on 4 others. This reduced the number of probes necessary to carry out the experiment from 138 to 56, thus effecting a 60 percent savings in the number of probes. Further refinement of the estimation technique will be considered in order to try to increase the savings in the number of probes. In particular interpolation in the azimuthal direction will be considered.

Acknowledgements

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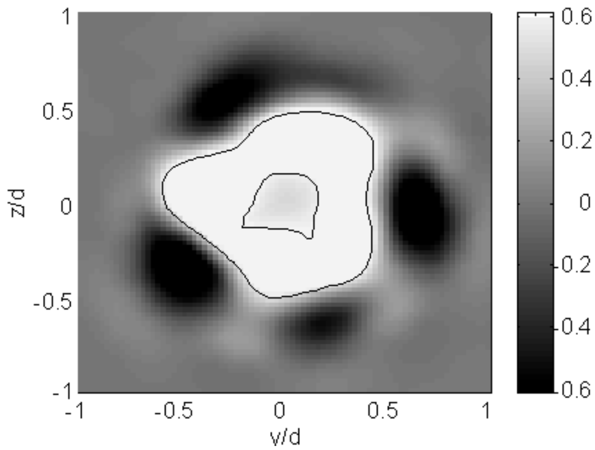

Figure 5: Ring-like structure from the low-order reconstruction produced using the estimated field.

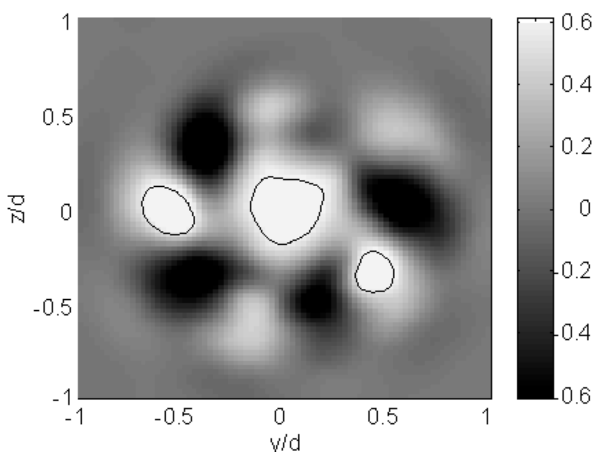

Figure 6: The inter-ring region from the low-order reconstructing produced using the estimated field. Appears to be a series of counter-rotating vortices.

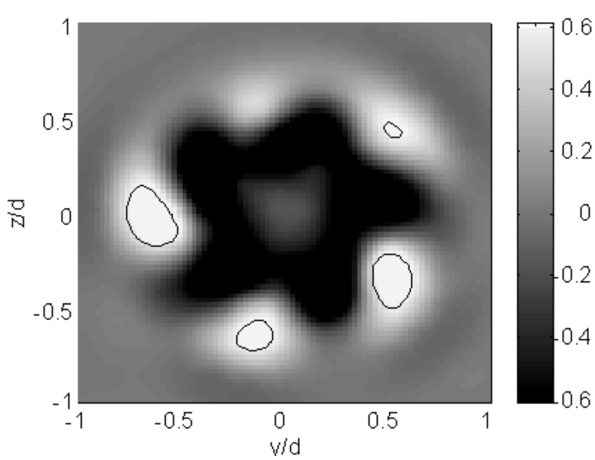

Figure 7: A second realization in the inter-ring region from the low-order reconstruction produced using the estimated field.

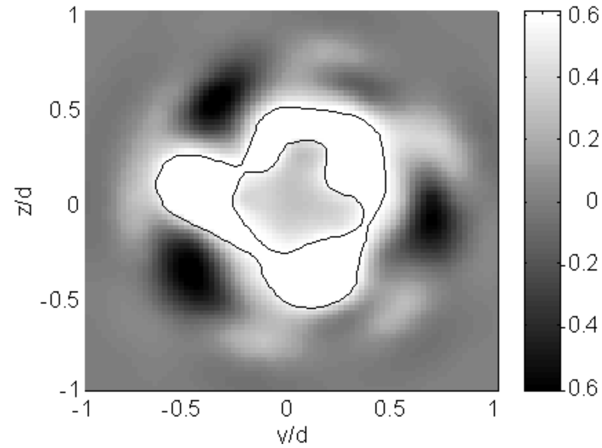

Figure 8: Ring-like structure from the low-order reconstruction produced using the original field.

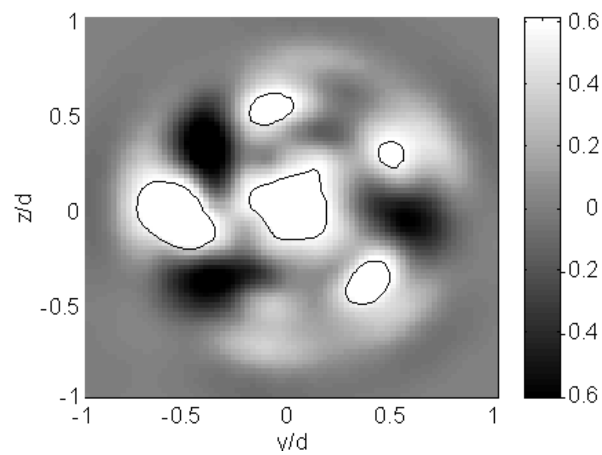

Figure 9: The inter-ring region from the low-order reconstruction produced using the original field. Appears to be a series of counter-rotating vortices.

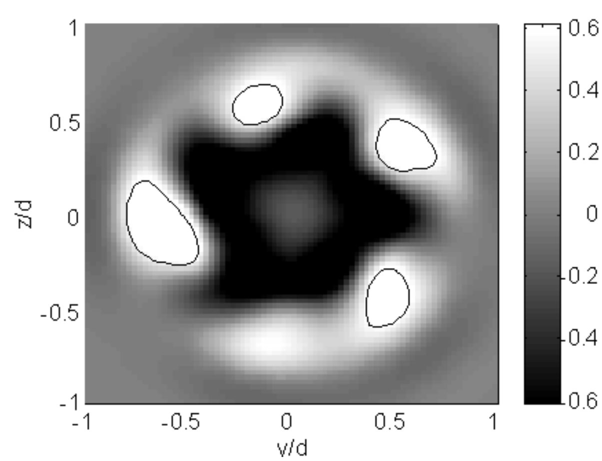

Figure 10: A second realization in the inter-ring region from the low-order reconstruction produced using the original field.