POD coefficient interaction and its relation to structure evolution in a turbulent axisymmetric shear layer

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Abstract

The coefficients of the orthogonal basis obtained from the application of the Proper Orthogonal Decomposition (POD) to the streamwise velocity field in the axisymmetric mixing layer are presented. The coefficients are utilized to explain the interaction between azimuthal modes in the shear layer and their relation to the large scale structure at a position 3 diameters downstream from the exit of a round jet. The large scale structure in the axisymmetric mixing layer has been found to be well represented by the first radial POD mode and a further breakdown of the structure into azimuthal modes reveals more insight into the structure dynamics. It is shown that higher azimuthal Fourier mode structures (m =3,4,5 and 6) interact in the layer and a proposed phase relationship between these structures at different parts of the layer is discussed.

Introduction

The POD technique has been used in the eduction of coherent structures in the axisymmetric mixing layer by Glauser (1987). These experiments showed conclusively that only three POD modes were necessary to describe the *radial* dependence of all but the smallest structures present in the flow. Moreover, only a single POD mode captured all the essential characteristics of the large scale structure. A surprising result of the Glauser experiment, however, was that the *azimuthal* Fourier modes necessary to describe the flow varied greatly with radial position. (Note that the periodicity of the flow in the azimuthal direction, θ, dictates that the azimuthal POD modes are Fourier modes.) Specifically, near the potential core, the 0-azimuthal mode contained almost all the energy, while outside the energy was distributed in modes centered around mode-5.

The short-coming of the Glauser experiment was that although it was able to determine which modes were important *to* the flow, it could not determine *how* they were assembled *in* the flow. For example, it was not possible to tell whether the mode-0 (which dominated the statistics of the inner shear layer and core region) occurred simultaneously with the mode-5 contributions on the outside, with a time lag, or simply randomly phased with respect to it. This could only have been determined if the eigenfunctions could have been projected back onto the entire profile to determine the coefficients for a single ensemble. And this would have been possible only if all the measurements at *all* locations had been made *simultaneously.* The coefficients so determined could then have been used to reconstruct the instantaneous spatial and temporal flow at the cross-section, one mode at a time.

The experiment of Citriniti (1996) was designed specifically to provide the missing information to find out how the radial POD and azimuthal Fourier modes fit together in time. This was a non-trivial undertaking, even by comparison to the Glauser experiment since what was required was simultaneous measurement in an entire cross-section of a very high Reynolds number flow. And, most importantly, the measurements had to have sufficient spatial resolution to avoid aliasing the higher spatial azimuthal and POD modes into the lower ones which were of primary interest.

The results of this effort to obtain the dynamical relationships between azimuthal Fourier modes in the axisymmetric shear layer are presented below. Specifically, the phase relationship between azimuthal modes 0,3,4,5 and 6 in the inner and outer portions of the layer are determined.

Methods

Proper Orthogonal Decomposition

The present investigation uses the POD-reconstructed velocity database obtained in the experiment of Citriniti (1996). In that experiment, the POD technique was applied to an ensemble of realizations of the streamwise velocity field, three diameters downstream from the exit of a round jet. The velocity field was reconstructed using the calculated eigenfunctions and random coefficients.

The structure, in the POD application, is assumed to be represented by the first of an ordered set of orthogonal eigenfunctions, $\vartheta_i(\vec{x},t)$, that are defined by the maximization of their normalized mean square projection on the velocity vector, $u_i(\vec{x},t)$ (Lumley, 1970). The maximization is performed via the calculus of variations and the result is an integral eigenvalue equation of the Fredholm type (Lumley, 1970),

$$\int R_{i,j}(\vec{x},\vec{x}',t,t')\vartheta_j(\vec{x}',t')d\vec{x}'dt' = \lambda\vartheta_i(\vec{x}) \quad (1)$$

where the symmetric kernel of this equation is the two point correlation tensor

$$R_{i,j}(\vec{x},\vec{x}',t,t') = < u_i(\vec{x},t)u_j(\vec{x}',t') > \quad (2)$$

and ϑ_j are the eigenfunctions and λ is the eigenvalue.

Solution of this equation produces the eigenfunctions and Galerkin projection of the instantaneous velocity on the set determines the coefficients of the eigenfunctions. The velocity field can then be reconstructed and the form of this equation for the axisymmetric mixing layer, $(x_1,x_2,x_3) = (x,r,\theta)$, is (Citriniti and George, 1997),

$$\hat{u}_i^{nmf}(r,m,f) = \sum_{n=1}^{N} \hat{a}_n(m,f)\phi_i^{(n)}(r,m,f) \quad (3)$$

where $n = 1,2,3\ldots$ represents the discrete nature of the solution set and $\phi_i^{(n)}(r,m,f)$ and $\hat{a}_n(m,f)$ are the POD eigenfunctions and coefficients, respectively, decomposed in frequency, f, and azimuthal mode number, m. An equation for the coefficients is obtained by using the orthogonality of the eigenfunctions, *i.e.*

$$\hat{a}_n(m,f) = \int \hat{u}_i^{nmf}(r,m,f)\phi_i^{(n*)}(r,m,f)dr. \quad (4)$$

Performing partial sums in equation 3, *i.e.* $N = 1,2,3,\ldots$, provides a way to visualize different energy weighted views of the flow. It has been shown, *v.* Citriniti and George (1997), that setting $N=1$ effectively filters out the small scale structure and leaves an unobscured view of the large scale, or coherent, structure in the axisymmetric mixing layer. Also, since the eigenfunctions are made orthonormal, the magnitude of the streamwise velocity is determined by the coefficients. Therefore, by studying the instantaneous coefficient dynamics on a mode-by-mode basis, a clear view of the structure interaction is obtained.

Experiment

The flow field at 3 diameters downstream of the nozzle is representative of the fully developed mixing layer. At this position, it has been shown that 138 measuring positions is sufficient to properly resolve the flow field for the application of the POD to the streamwise velocity field (Citriniti and George, 1996). The hot-wire probes are distributed across the mixing layer at 6 radial position with the azimuthal distribution of probes on the 6 radii, starting at the center and proceeding outward, 6, 12, 24, 32, 32, 32 thus totalling 138 positions. The transducers were simultaneous operating, single-wire, hot-wire anemometer probes powered by in-house anemometers, *v.* Citriniti *et al.* (1994). The sampling rate of each of the hot-wires was set at 2,048 Hz to satisfy the Nyquist criterion which must be greater than twice the 800 Hz corner frequency on the low-pass anti-alias filters. There were 300 blocks of 1024 samples producing a bandwidth of 2 Hz and a block length of 0.5 s.

The Reynolds number based on nozzle diameter, d, for the jet is 80,000. The free-stream turbulence intensity at the jet exit is 0.35% and the boundary layer at the jet exit was turbulent with an approximate thickness of 1.2 mm. The mean velocity profile was flat to within 0.1%.

The statistics of the streamwise velocity field, as measured by the sampling grid, demonstrate that an axisymmetric shear layer has been formed, *i.e.* the mean and rms contours are circular *v.* Citriniti (1996). The mean velocity normalized by the exit velocity is about 0.9 at the inner radius of the sampling array and falls to 0.15 at the outside of the mixing layer. The spectral character of the velocity field demonstrates the fully developed, turbulent character of the mixing layer, *v.* Citriniti (1996).

Coefficient Dynamics

The magnitude and phase of the random coefficients are shown in sequential order in figure 1 to figure 3. The first column shows the temporal evolution of the magnitude of the first radial POD coefficient while the second shows its phase angle. The third column is a plot of the reconstructed velocity field by the POD (*v.* equation 3) with $N=1$ and using the first 7 azimuthal modes, $m = 0,1,2,3,4,5,6$. The third column shows the effect the large scale structures on the streamwise velocity field in the mixing layer. Each row in the 3 figures represents one step in the temporal evolution of the structures. There is no discontinuity between figures so they could, in essence, be placed top to bottom to obtain 12 straight time steps in the flow evolution. Each row is separated by about 1 ms in real time.

In figure 1 the amplitude of the sixth azimuthal mode is first seen to grow and then begin to decay (column 1). Directly following the peak in this mode, there appears to be a mode-transfer process that begins with the increase in the amplitude of mode 5 and proceeding to modes 4, 3 and 2. This transfer is interesting because it suggests that the smaller length scale modes may act as triggers, or perhaps indicators, for the emergence of the lower, more energetic mode-number structures in the mixing layer. This mode interplay has also been observed in the POD-based dynamical system simulations of Glauser *et al.* (1992).

Citriniti and George (1997) have shown that the higher azimuthal mode structure is associated with streamwise vortices which advect high-momentum fluid from the potential core to the outside of the layer and low-momentum, ambient fluid toward the potential core. The fact that the mode-6 structures precede the lower mode structures indicates that these streamwise vortices are important players in the energy cascade process of turbulence. It is interesting to note that the mode-6 structure, which is correlated over a shorter spatial scale than the mode-4 structure, should be the trigger mechanism since this would imply that the energy cascade proceeds from smaller to larger scales. However, it should be noted that these temporal visualizations do not follow individual energy paths but rather the entire field so the mode-6 trigger may just be the consequence of the temporal sequencing of the large scale structure in the layer at $x/d = 3$. These results are consisitent both with the low-mode dynamical systems models of Glauser *et al.* (1992) as well as the proposed mechanism for turbulent structure interaction in the mixing layer proposed by Glauser and George (1987). This model proposes that the dynamics of the large scale structure in the layer parallels the leapfrog mechanism of two interacting vortex rings.

The phase of the random coefficients also shows interesting trends. A pattern of large phase lag followed by a continuous evolution toward zero phase and then to large phase lead is seen at all azimuthal modes indicating a highly repetitive pattern of structures exists in the layer.

Other useful measures of the large scale structure in the turbulence are evident in the movies generated by the sequential plotting of the figures presented earlier. In these movies, the true evolution of the large scale structures is evident. Especially the bursting of a highly coherent ring-like structure near the potential core of the layer. This event is seen in the third row of figure 2 and is followed (and proceeded because of the repetitive nature of the flow) by the straining of streamwise structures which are believed to be formed by the breakup of the Kelvin-Helmholtz rings formed at the edge of the shear layer.

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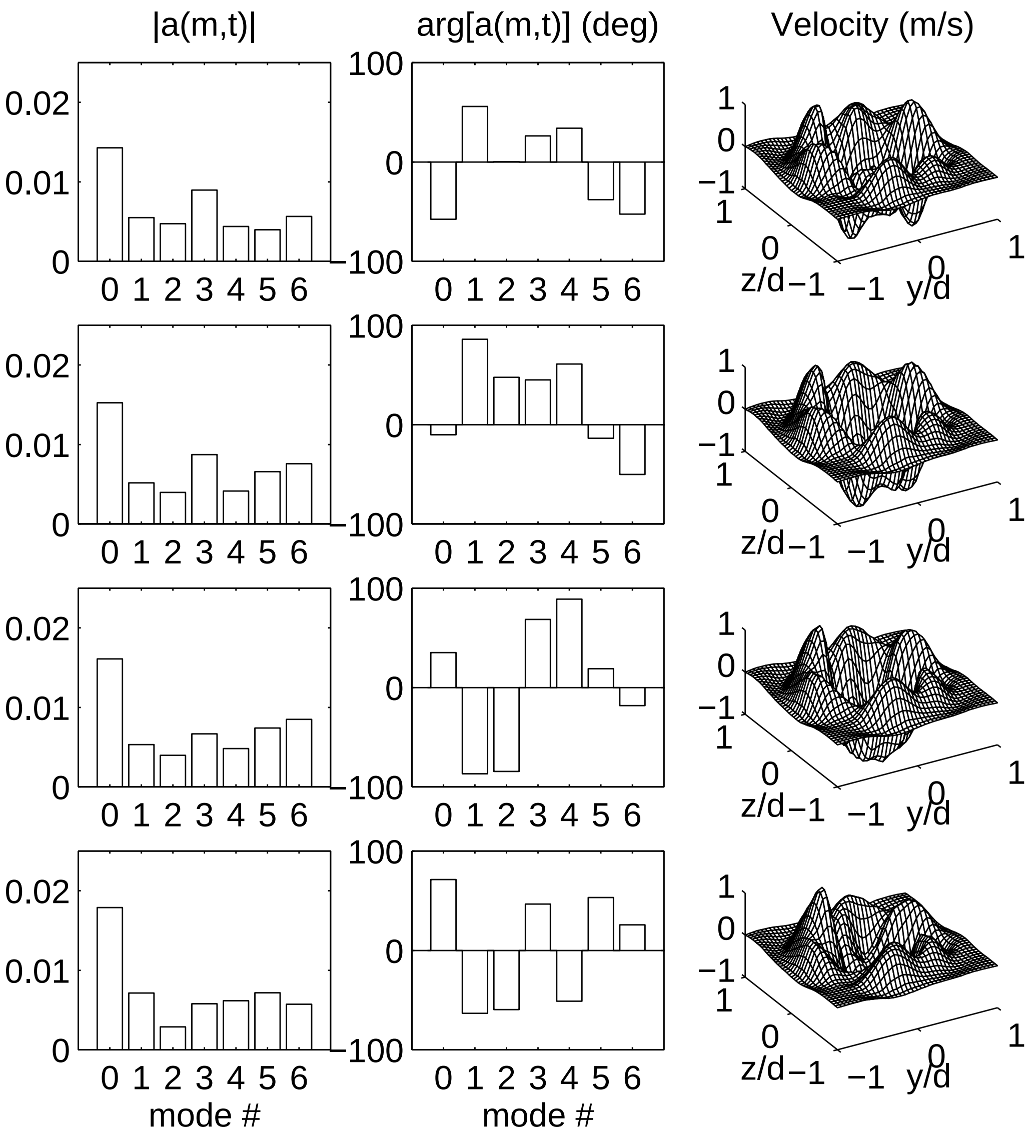

Figure 1: See figure 3 for extended caption

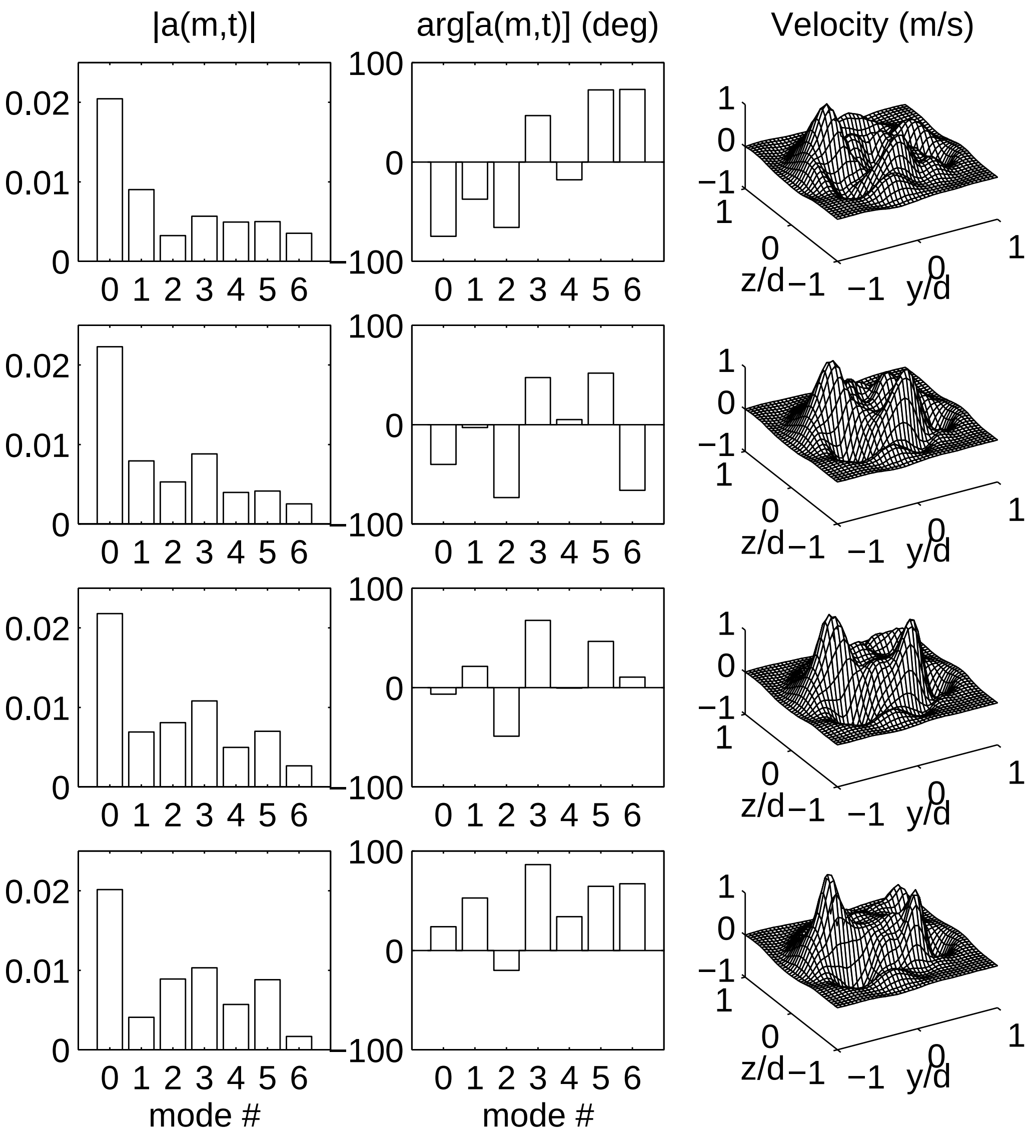

Figure 2: See figure 3 for extended caption

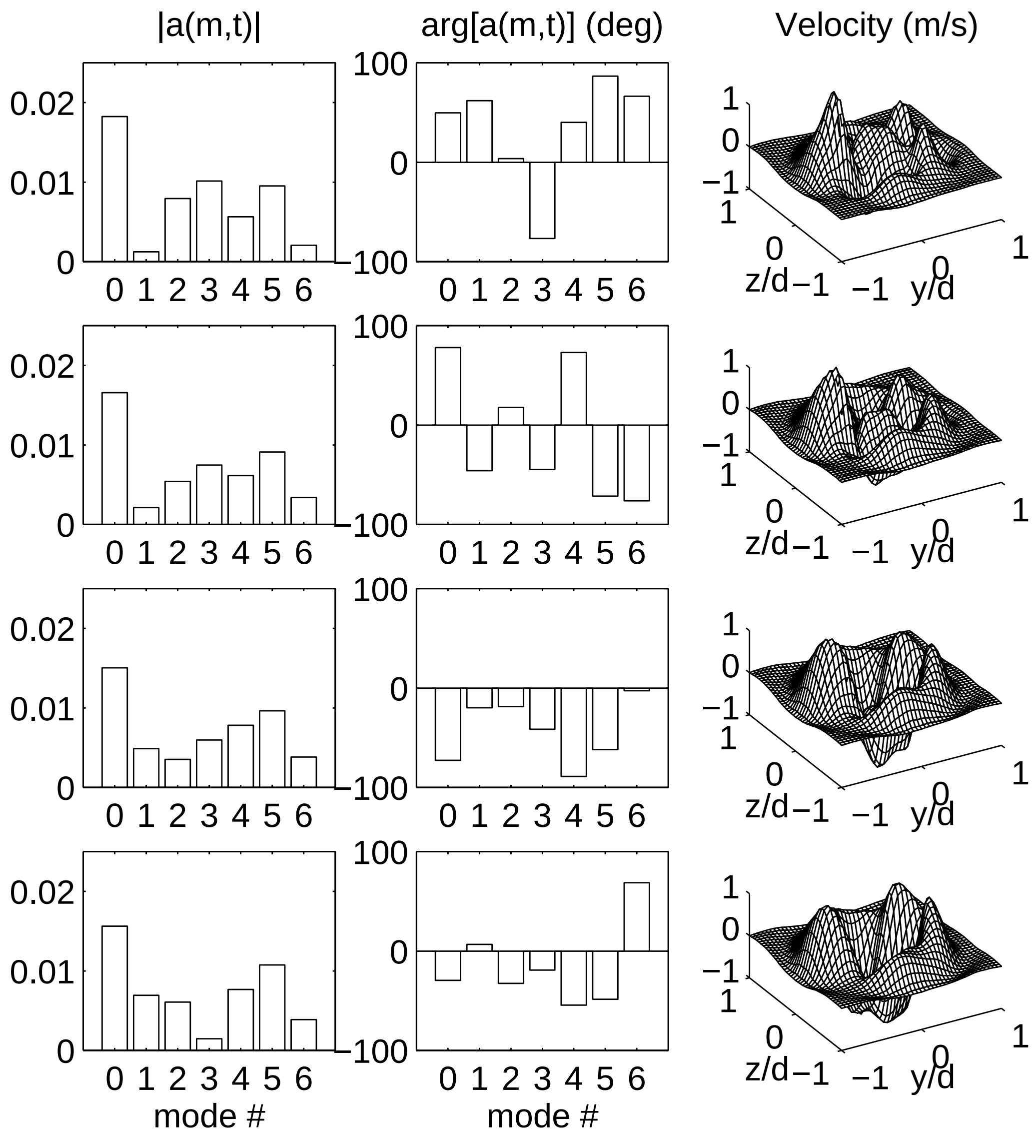

Figure 3: Sequential plots of the coefficient magnitude and phase and the projected velocity for the first POD mode and azimuthal modes 0-6. Column 1: magnitude of the first 6 azimuthal modes. Column 2: phase angle of the first 6 azimuthal modes. Column 3: projection of the streamwise velocity field using the first POD mode and azimuthal modes 0-6 (*v.* equation 3). The rows display the temporal evolution of the various quantities and are separated in real time by approximately 1 ms.