ORGANIZED STRUCTURE DYNAMICS IN A TURBULENT ROUND JET

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Abstract

The dynamics of large scale azimuthally-coherent structures obtained in a high Reynolds-number axisymmetric mixing layer are presented. The structure dynamics are obtained by application of the Proper Orthogonal Decomposition (POD) to an ensemble of realizations of the streamwise velocity field obtained from 138 probes located 3 diameters downstream of the nozzle exit. The velocity field is measured at all 138 positions simultaneously, making it possible to obtain the instantaneous coefficients of the POD modes as well as to extract the large scale structure.

Pictures showing the dynamics of the structure interaction in the mixing layer demonstrate the importance of the mode-0, 3, 4, 5 and 6 azimuthal modes in both the entrainment and advection of fluid in the layer. Azimuthally coherent structures, which are remnants of circular vortex rings produced in the shear layer, appear near the potential core at regular intervals corresponding to the Strouhal frequency. The region between rings is dominated by a high azimuthal mode structures which appear near the outside of the layer. These structures do not appear to be singular but rather are made up of pairs of counter-rotating streamwise vortices that are very similar to the ribs seen in simulations of the axisymmetric layer and in plane mixing layers. The vortices advect high-momentum fluid toward the outside of the layer and low-momentum fluid toward the potential core.

1 Introduction

Understanding the dynamics of coherent structures in the axisymmetric mixing layer is of primary concern in many engineering applications such as understanding and controlling noise suppression, mixing enhancement, near-field entrainment and shear layer growth. Vital to the success of these endeavors is a clear understanding of the naturally occurring structures in the layer and, especially, the structure dynamics. The difficulty associated with eduction of large scale structures is the failing of most eduction techniques to provide an objective means to identify the structure while allowing a study of the structure dynamics. One technique which has been very successful in accomplishing both objectives is the Proper Orthogonal Decomposition (POD).

The POD has been applied to simultaneous measurements of the streamwise velocity field in the axisymmetric mixing layer at 138 positions with hot-wire anemometer probes. The POD technique is used to extract the large scale structure by using the first POD eigenmode to objectively define the large scale structure. The instantaneous velocity field is reconstructed using random coefficients obtained through Galerkin projection of the instantaneous velocity field on the POD eigenfunctions. The velocity field can then be partially reconstructed using only the first POD mode and, in this way, a view of the velocity field as produced by the large scale structure in the layer can be objectively analyzed. The temporal sequencing of events and a discussion of the naturally occurring structure in the layer are also presented.

2 Methods

2.1 Proper orthogonal decomposition

The POD seeks the most energetic fluctuations in a random vector field; its ability to extract large scale structure in a turbulent velocity field is well established, (Herzog 1986, Glauser *et al.* 1987, Moin and Moser 1989, Delville *et al.* 1990, Citriniti and George 1997) The structure is represented by an ordered set of orthogonal eigenfunctions, $\vartheta_i(\vec{x},t)$, that are defined by the maximization of their normalized mean square projection on the velocity vector, $u_i(\vec{x},t)$,(Lumley, 1967).

The maximization is performed via the calculus of variations and the result is an integral eigenvalue equation of the Fredholm type,

$$\int R_{i,j}(\vec{x},\vec{x}',t,t')\vartheta_j(\vec{x}',t')d\vec{x}'dt' = \lambda\vartheta_i(\vec{x},t) \tag{1}$$

where the symmetric kernel of this equation is the two point correlation tensor

$$R_{i,j}(\vec{x},\vec{x}',t,t') =< u_i(\vec{x},t)u_j(\vec{x}',t') > \tag{2}$$

and λ is the eigenvalue, (Lumley, 1967).

Solution of this equation produces the eigenfunctions and Galerkin projection of the instantaneous velocity on the basis determines the coefficients of the eigenfunctions. The velocity field can then be reconstructed via the POD; the form of this equation for the axisymmetric mixing layer is,

$$\hat{u}_i^{nmf}(r,m,f) = \sum_{n=1}^{N} \hat{a}_n(m,f)\phi_i^{(n)}(r,m,f) \tag{3}$$

where $n = 1,2,3\ldots$ represents the discrete nature of the solution set, $(x_1,x_2,x_3) = (x,r,\theta)$. The POD eigenfunctions, $\phi_i^{(n)}(r,m,f)$, and coefficients, $\hat{a}_n(m,f)$, are decomposed in frequency, f, and azimuthal mode number, m, (Citriniti, 1996). Performing partial sums in equation 3 (*i.e.* $N = 1,2,3,\ldots$) provides a way to visualize

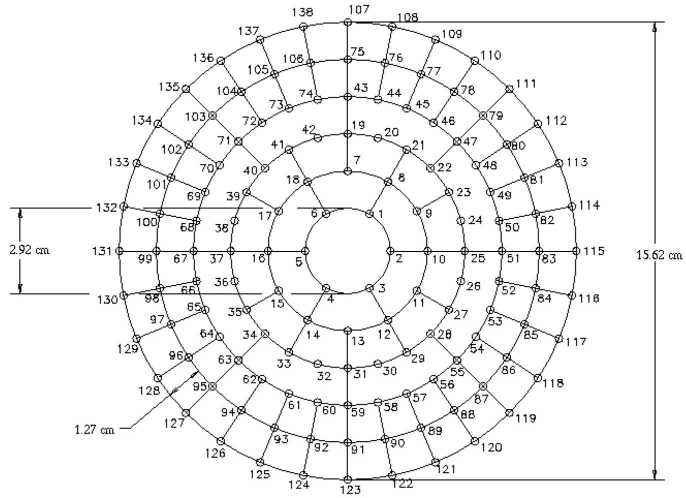


Figure 1: Spatial sampling grid for application of the POD to the axisymmetric mixing layer at $x/d = 3$ incorporating 138 simultaneous-sample, single-wire hot-wire probes.

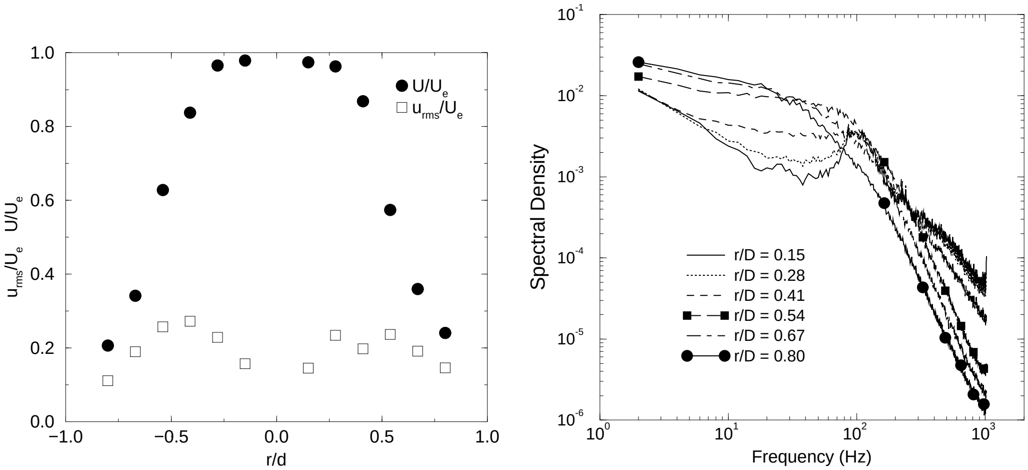
different energy-weighted views of the flow. It will be shown that setting $N = 1$ effectively filters out the small scale structure and leaves an unobscured view of the large scale, or coherent, structure in the axisymmetric mixing layer.

2.2 Experiment

The flow field at 3 diameters downstream of the nozzle is representative of the fully developed mixing layer. At this position, the POD technique has been applied to realizations of the flow field made with 138 simultaneous operating single-wire hot-wire anemometer probes, *v.* figure 1. For the exit velocity of 12.5 m/s, the Reynolds number based on nozzle diameter, d, is 80,000. The free-stream turbulence intensity at the jet exit is 0.35% and the boundary layer at the jet exit was turbulent with an approximate thickness of 1.2 mm. The mean velocity profile was flat to within 0.1%.

The sampling rate of each of the simultaneously sampled 138 hot-wires was 2048 Hz to satisfy the Nyquist criterion which must be greater than twice the 800 Hz corner frequency on the low-pass anti-alias filters. There were 300 blocks of 1024 samples producing a bandwidth of 2 Hz and a block length of 0.5 s.

The statistics of the streamwise velocity field, as measured by the sampling grid, demonstrate that a fully-developed axisymmetric shear layer has been formed, *v.* figure 2(a). The spectral character of the velocity field is presented in figure 2(b). The energy distribution at the potential core, $r/d = 0.15$, is found to peak at about 100 Hz which is the natural passage frequency of the ring instability at



(a) Normalized mean and rms velocity statistics in the shear layer.

(b) Spectra at the 6 radial positions across the shear layer.

Figure 2: Statistical nature of the velocity field in the shear layer at $x/d = 3$.

the potential core, (Zaman and Hussain, 1984). This corresponds to a Strouhal frequency ($=fd/U_e$) $= 0.78$.

3 Results

3.1 Distribution of kinetic energy

The Hilbert-Schmidt theory which governs the solutions to equation 1 dictates that in this case the summation of all eigenvalues over n, m and f produces the total streamwise kinetic energy in the flow (Lumley, 1970). The distribution of the kinetic energy between radial POD and azimuthal Fourier modes can then be calculated by summing over various terms. A parameter, ξ, which is such a measure is defined by,

$$\xi = \frac{\sum_f \lambda^{(n)}(m,f)}{\sum_n \sum_m \sum_f \lambda^{(n)}(m,f)}. \tag{4}$$

The azimuthal mode energy distribution for the first 5 radial POD modes is plotted in figure 3 for the streamwise velocity field measured at $x/d = 3$, (Citriniti and George, 1997). The predominance of the first POD mode (as determined by the first eigenvalue) is indicated by the fact that 67% of the kinetic energy in the flow is contained in this eigenvalue. More importantly, the distribution of energy in azimuthal mode numbers within the first POD mode demonstrates the importance of the 0, 3, 4, 5 and 6 modes in the shear layer at $x/d = 3$.

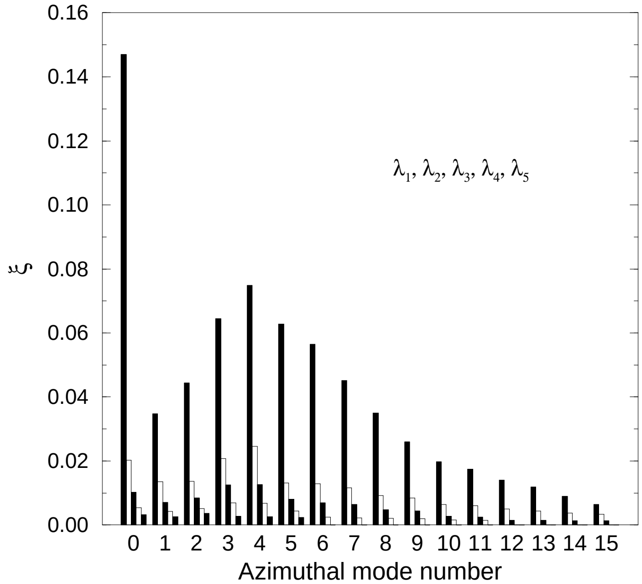


Figure 3: Azimuthal-mode kinetic energy distribution in the first 5 POD eigenvalues, from Citriniti and George (1997).

3.2 Extraction of large scale structure

The POD reconstructed velocity field can be compared to the original velocity to see if it is extracting structure from the seemingly random flow. Figure 4 shows surface mesh plots of the magnitude of the streamwise velocity at a single time step, $t_n = 413$, obtained directly form the hot-wire probes. Also shown are the corresponding velocity fields resulting from the POD reconstruction using only the first radial POD mode (*i.e.* $N = 1$ in equation 3). Part (a) of this figure shows the original velocity which contains many peaks. It is difficult to determine which peaks are due to spatially coherent structures and which are due to small scale perturbations. In part (b) of this figure, the velocity field resulting from the POD reconstruction is shown in which only the first radial POD mode and all azimuthal modes are included (*i.e.* $N = 1$ and $m = 0-15$ in equation 3). The POD has filtered out many of the small scale perturbations while retaining the features representative of the spatially coherent structure. Since the higher azimuthal modes ($m \geq 7$) will not contribute significantly to the spatially coherent structures, they can be removed as is done in figure 4(d).

Further refinement of the large scale structure is accomplished by removing other unimportant azimuthal modes from the POD reconstruction. The results of § 3.1 indicated that much of the energy in the flow was contained in azimuthal modes 0, 3, 4, 5 and 6 (*v.* figure 3). If only these modes are included in the representation, a further filtering of the small scale structure can be accomplished, *v.* (d) in figure 4. It should be noted that this decision is not made *ad hoc* since the POD has provided the

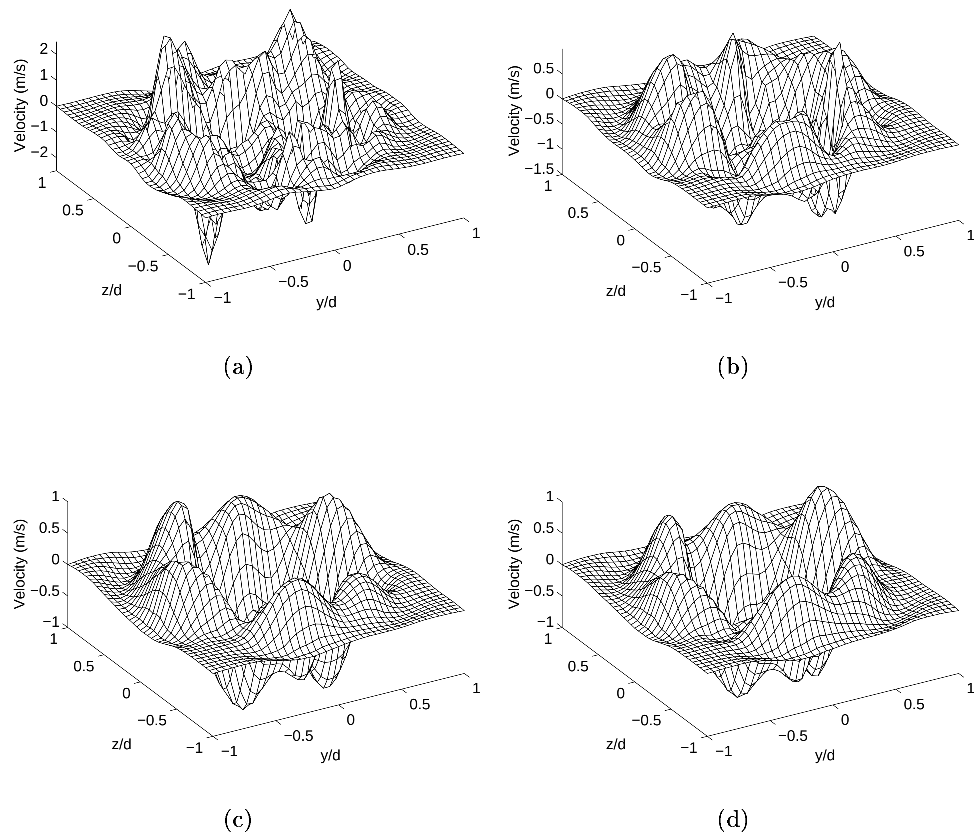


Figure 4: Surface mesh of streamwise velocity field at x/d = 3 with $t_n = 413$ from (a) the original hot-wire measurements and (b) the POD reconstruction (*v.* equation 3) using only the first POD mode and including; all azimuthal modes, *i.e.* $m = 0 - 15$ (c) $m = 0 - 6$ and (d) $m = 0, 3, 4, 5, 6$

information that led to the exclusion of these unimportant modes, namely through figure 3. Fig. 4(d) shows that the first POD mode including the azimuthal modes 0, 3, 4, 5 and 6 sufficiently recovers the large scale structure. Thus the POD has reduced an infinitely dimensional problem to one involving only 5 parameters.

3.3 Structure dynamics

Five sequential frames of the reconstructed streamwise velocity (with the mean subtracted off) using the first POD mode, *i.e.* $N = 1$ in equation 3, are shown in figure 5. The time index t_n is used to label the 1024 single images in the data blocks, the real time can be calculated by $t = t_n \Delta t$ where $\Delta t = 0.488$ ms. The portion of the mixing layer between $0.5 < r/d < 0.8$ is found to be dominated by a 5 and

6 mode structure, *v.* figure 5a. Citriniti and George (1997) have suggested that the structure causing this flow pattern are counter-rotating, streamwise vortex pairs similar to the Bernal-Roshko structures found in the plane mixing layer. Further, a model has been developed describing the dynamics of the large scale structure in the mixing layer which consists of two main events. The first is the passage of the azimuthally coherent ring, or remnant of a ring (*v.* figure 5c) which engulfs fluid from the irrotational outer flow (note that high streamwise-momentum fluid is represented by white in figure 5 while low momentum fluid is black). The second is the advection of fluid by the streamwise vortex pairs which "ride" on the induced velocity field of the rings and are stretched by the high extensional strain region between ring structures (Citriniti and George, 1997). The 6 mode structure of alternating high- and low-momentum fluid in the azimuthal direction (figure 5(a)) is produced by the counter rotating streamwise vortices (streamwise vortex centers are indicated by the plus and minus signs on the figure).

In figure 5(c) the remnants of a circular vortex ring pushes high-momentum fluid through its core. After the ring passes, see figure 5(d), the streamwise vortices are stretched and strengthened by the high extensional strain in the inter-ring regions, see figure 5(e). Finally, the shear layer returns to a state similar to that which started this life-cycle, compare figure 5(a) and figure 5(f), indicating the repetitive nature of the large scale structure life-cycle[1]. The total time from figure 5(a) to figure 5(f) is 9.76 ms which corresponds to a frequency of 100 Hz. This is the same as the peak frequency in the spectrum near the potential core which implies that the POD has indeed extracted the most energetic events in the layer.

4 Conclusions

The Proper Orthogonal Decomposition has been shown to be a useful tool in the analysis of large scale turbulent structure identification. The ability of the POD to extract structure objectively has been shown via visual and numerical means. It is possible to define a life cycle of the large scale structure dynamics in the mixing layer at $x/d = 3$ by using the POD. The first event in the life of the coherent structure is the bursting associated with the passage of an azimuthally coherent structure, or "ring", near the potential core. The induced velocity field between successive rings creates a high strain field that seems to be play an important part in the evolution of counter-rotating streamwise vortex pairs. These concepts are an extension of the model for the shear layer dynamics proposed by Glauser and George (1987).

The higher azimuthal Fourier mode structure, associated with azimuthal modes 3, 5 and 6, are created by streamwise vortex pairs which advect fluid into and out of the potential core of the jet, similar to the action of a side-jet. They are usually found near the outside of the layer ($r/d > 0.5$) and therefore are not convected very

[1]The reader is referred to the movies generated from this data set at the authors (JHC) internet web page accessible via the UB Turbulence Research Laboratory home page at http://www.eng.buffalo.edu/Research/trl/

quickly downstream. They tend to exist for many ring passages. The 0-mode and 4-mode structures appear most often near the potential core and thus are expected to represent the perturbed remnants of the Kelvin-Helmholtz "ring" generated in the shear layer. They are very fast moving and are highly energetic, advecting large amounts of fluid in the short time they are in the layer.

5 Acknowledgments

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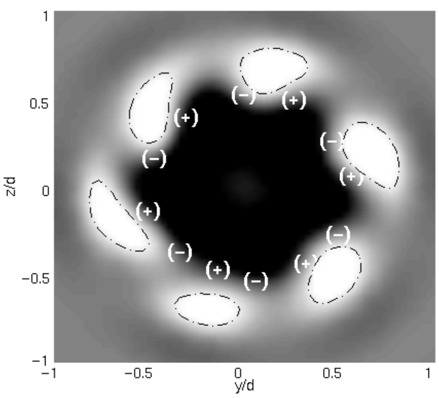
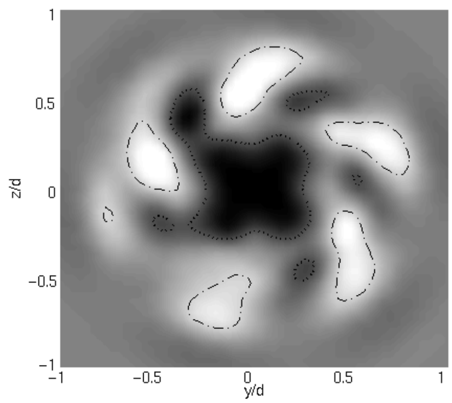
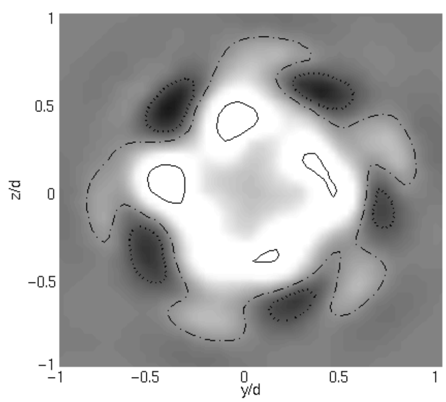


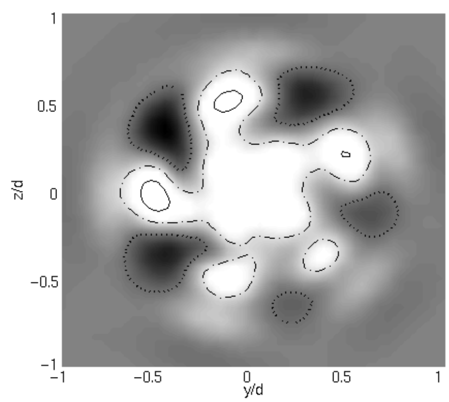
(a) $t_n = 413$



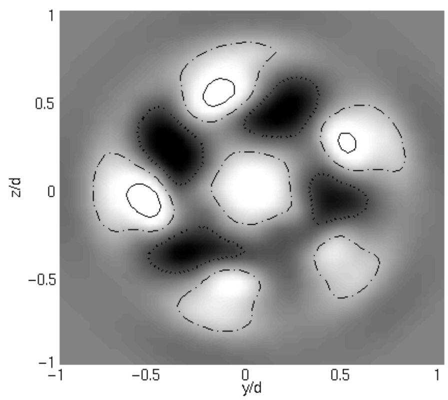
(b) $t_n = 419$



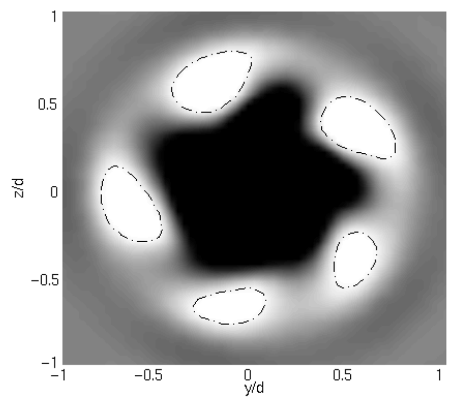
(c) $t_n = 422$



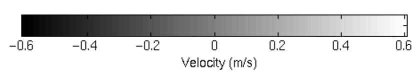
(d) $t_n = 427$



(e) $t_n = 429$



(f) $t_n = 433$



(g)

Figure 5: Two-dimensional projection of the POD reconstructed streamwise velocity field at $x/d = 3$. The temporal index is provided in the individual caption and the color bar denoting velocity above or below the local mean is given in 5(g).