X-WIRE RESPONSE IN TURBULENT FLOWS OF HIGH INTENSITY TURBULENCE AND LOW MEAN VELOCITIES

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Abstract

The angular response of an x-wire is studied at low velocities (0.25 m/s to 1 m/s). It is found that the k-factor in the modified-Cosine Law is strongly velocity dependent. The implications of this on multi-component measurement are explored, and a practical scheme for incorporating it is proposed. At lower velocities a total loss of directional sensitivity is observed which leads to additional errors in x-wire measurements. Expressions are derived for evaluating when cross-flow errors begin to affect x-wires. Also, a "dropout" phenomenon is observed in which certain voltage pairs can not be converted into the velocity components. The implication of this dropout and rectification on the turbulence measurements is also discussed.

Nomemclature

u	instantaneous velocity in vertical direction (m/s)
U	mean velocity in vertical direction (m/s)
v	instantaneous velocity in horizontal direction (m/s)
V	mean velocity in horizontal direction (m/s)
U_{eff}	effective cooling velocity (m/s)
U_o	free stream velocity (m/s)
k	coefficient in eqn. 2
ℓ	wire length (mm)
d	wire diameter (mm)
α	angle between the inclined wire and vertical direction (rad)
ϕ	angle between flow velocity vector and plane normal to wire (rad)

Introduction

It can be shown [1] that for the laminar flow past an infinitely long and uniformally heated cylinder placed obliquely to a uniform undisturbed velocity field, the heat transfer is proportional only to the normal velocity component. This result is known as the Cosine Law since it implies

$$U_{eff} = U_o \cos\phi \tag{1}$$

where ϕ is the angle between the flow velocity vector and the plane normal to the wire as shown in Figure (1).

In most hot wire applications, however, the flow is not laminar and the temperature distribution is not uniform. Nonetheless, the Cosine Law is still the basic relationship on which empirical relations are based. Schubauer and Klebanoff [2] experimentally demonstrated circumstances under which the Cosine Law was valid for finite length wires up to an inclination angle of 70^o. Thus there are situations where the mean velocities are high and the turbulent intensities are small in which Cosine Law may work satisfactorily. There have been, however, few proponents of the Cosine Law for the high intensity turbulent flows since the work of Champagne and Sleicher [3] which showed significant deviation from this simple response. Even so, there are many who have continued to use the empirical relation (1) without being fully aware of its limitaions.

Velocity Dependent Angle Response

Real hot wires depart from Cosine Law because of their finite length and because of end losses to the prongs. The effect of both of these is to make the temperature distribution across the wire non-uniform. Hinze [4] and Webster [5] suggested the following relation

$$U_{eff}/U_o = [\cos^2\phi + k^2\sin^2\phi]^{1/2}, \tag{2}$$

the second term accounting for the cooling caused by the velocity component along the wire. Hinze found the value of k to be between 0.1 to 0.3 whereas Webster found k to be 0.2. Champagne et al. [3] in a systematic study on heat transfer of inclined wires found k to be a function of the wire length to diameter ratio, l/d. They found k= 0.2 for l/d = 250.

Alternatives to equation (2) have been proposed. Fujita and Kavoznay [6] proposed the following expression

$$U_{eff}/U_o = \cos\phi[1+A_1(1-\cos2\phi/\sin\phi)] \quad (3)$$

where A was found to be slightly velocity dependent in the range of 2 m/s to 12m/s.

In addition to its angular sensitivity and the problems it creates at high turbulence intensity, there are additional problems which arise under these conditions. Hot wires respond only to the magnitude of velocity and are insensitive to its direction. As a consequence, the wire is sensitive to components of velocity in the cross-stream direction. The so-called "cross-flow errors". In addition, the wire cannot distinguish when the sign of the velocity vector changes. The phenomenon is known as rectification and can be a major source of error in situations where the flow reversals occur on the wire. Since the advent of the constant temperature anemometer and linearization in the mid 1960's, there has been almost total disregard for the limitations imposed by hot-wire rectification and cross flow errors. An exception to this is the careful study by Tutu and Chevray [7] of the effects of rectification and cross-flow errors on x-wire response.

This paper attempts to summarize our experience at TRL in using x-wires in high intensity turbulent flows and in flows with low mean velocity, specifically, the effect of latter on the velocity dependence of the angular response, the loss of directional sensitivity, and "dropout".

Experimental Results

The tungstun wires used in this study were 2.5 μm in diameter and were gold-plated at the ends to give an effective l/d of 250. These were operated in the constant temperature mode using a DISA 55M10-system. The overheat ratio was 0.4. This low overheat was used to optimize the wire response for its use in an non-isothermal flow and to reduce the velocity at which natural convection effects dominate.

Figure (2) illustrates the angular dependence of a hot-wire at velocities below 1 m/s. These low velocities occur in buoyancy dominated flows such as plumes and at the outer edges of many other free shear flows. It is clear that the angular response is strongly dependent on the velocity U_o. Also note that at the lower velocities and higher inclinations the wire's angular response is significantly reduced. Other investigators have noted this velocity dependence, although the effects were weaker since the velocities were not as low.

Discussion of Results

The phenomenon illustrated in Figure (2) can be attributed to two causes: first, the increased conducton to the end supports as the forced cooling is reduced, and second, the increase in the natural convection losses relative to the forced component with decreasing velocity. The first effect can be attributed to increasing temperature gradient associated with these changes in the wire temperature distribution. The second effect has beeen documented for single wires by Collis and Williams [8] when $Re \lesssim Gr^{1/3}$ (for air). For our wires this would correspond to a velocity of about 4×10^{-2} m/s, or about an order of magnitude below that for which we observe significant effects on the angular response.

An alternative to equation (1) can be taken as

$$U_{eff}/U_o = [\cos^2 \quad + k^2(U_o)\sin^2 \quad]^{1/2} \tag{4}$$

where k is dependent on the total velocity U_o. We were able to express this dependence by the following polynomial

$$k^2(U_o) = B_o + B_1U_o^{1/2} + B_2U_o^{3/2} \tag{5}$$

Due to velocity dependence of k, the solution of (4) requires an iterative procedure. First a guess of U_o is made and the corresponding k is calculated from (5). Then (4) is solved to get a new value of U_o. The procedure is repeated until the two consecutive values differ by a specified tolerance. The simple Cosine Law can be used to make the initial guess of U_o .

Implications for Measurement

Figure (3) shows the velocities which can be resolved by an x-wire. Assuming an ideal x-wire (no prong support interference, symmetric sensitivity), no data can fall outside this cone. These curves were obtained by using the restrictions imposed by equations (2) and (4). For the constant k-factor the measured region is bounded by two straight lines, whereas for our case these lines change with the velocity, thus giving curves. Note the cusp at low velocities which represents a total loss of directional sensitivity near the origin or zero velocity. The dashed lines correspond to the Cosine Law. At high velocities, all three will perform adequately as long as the relative cross-stream velocity is not large.

Cross-flow Errors

The most basic limitation on the hot wire is due to the so-called cross-flow errors which arise from the fact that a vector field (the velocity) is being mapped into a scalar (the cooling velocity) by the wire. It is possible to evaluate when these cross-flow errors become significant by expanding the cooling velocity about the state where the fluctuating velocities are zero. This has been carried out in detail in [4] for a single wire, and will be extended to the x-wire in the following paragraphs. Consider Figure (4) which shows a hot-wire inclined at an angle α to the vertical direction. The effective cooling velocity for this wire is given by

$$u_{eff}^2 = (u \cos\alpha + v \sin\alpha)^2 + k^2(u \sin\alpha - v \cos\alpha)^2 + w^2 \qquad (6)$$

where w is the velocity component normal to the u-v plane, the cross-dlow component. For simplification it will be assumed that α is constant and is equal to 45^o. Therefore,

$$u_{eff}^2 = \frac{1}{2}(u+v)^2 + \frac{k^2}{2}(u-v)^2 + w^2 \qquad (7)$$

Since when using an x-wire the w component is not known, the effective cooling velocity must be assumed to be,

$$u_{eff}^2 = \frac{1}{2}(u_m+v_m)^2 + \frac{k^2}{2}(u_m-v_m)^2 \qquad (8)$$

where the subscript m denotes the velocities measured using a x-wire. Thus the cross-flow errors result due to neglect of w^2 in equation (8).

Tutu and Chevray [7], by assuming a joint-Gaussian pdf for the two velocity components, have calculated the errors in various turbulence moments due to cross-flow and rectification. However, our objective here is to get algebraic expressions which an experimentalist can use to estimate when these errors begin to affect his measurements. To achieve this we equate the actual and assumed forms of the cooling velocity of equations (7) and (8) to obtain

$$(u+v)^2 + k^2(u-v)^2 + 2w^2 = (u_m+v_m)^2 + k^2(u_m-v_m)^2 \qquad (9)$$

Similarly for the second inclined wire ($\alpha = -45^o$) we have

$$(u-v) + k^2(u+v)^2 + 2w^2 = (u_m-v_m)^2 + k^2(u_m+v_m)^2 \tag{10}$$

Subtracting (12) from (11) we have

$$u_m v_m = uv \tag{11}$$

Thus the measured <u>product</u> of u and v is exactly the actual product.

Substituting $u = U + u'$ and $v = V + v'$ into equation (11) and averaging gives

$$\overline{u_m v_m} = \overline{uv} + (UV - U_m V_m) \tag{12}$$

This equation shows that cross-flow errors in the measured shear stress enter only through the measured <u>mean</u> velocities as shown below.

Eliminating V from equation (9) using (12), we can get an expression which is quadratic in u_m^2. The solution of this equation is given by

$$u_m^2 = \frac{1}{2}\left(u^2+v^2+\frac{2w^2}{1+k^2}\right) + \frac{1}{2}\sqrt{\left(u^2+v^2+\frac{2w^2}{1+k^2}\right)-4uv} \tag{13}$$

where the positive root is needed to recover the proper expression for the case when $w^2=0$. To obtain an equation for the instantaneous velocity u_m, equation (13) has to be simplified using several binomial expansions. The final expression for u_m is then decomposed into a mean and a fluctuating part and then time averaged. The algebra, although very lengthy, is straightforward, we here give only the final result:

$$U_m = U\left\{1 + \frac{1}{1+k^2}\left(\frac{\overline{w^2}}{U^2} - \frac{\overline{uw^2}}{U^3} + \frac{\overline{u^2w^2}}{U^4} - \frac{\overline{2w^4}}{U^4}\right)\right\} \tag{14}$$

where $\frac{1}{1+k^2}\frac{\overline{w^2}}{U^2}$ is the leading error term. If similar analysis are carried out for hot-wire placed normal to the flow (Hinze, 1959), one obtains

$$U_m = U\left\{1 + \frac{1}{2}\left(\frac{\overline{w^2}}{U^2} - \frac{\overline{uw^2}}{U^3} + \frac{\overline{u^2w^2}}{U^4} - \frac{1}{4}\frac{\overline{w^2}}{U^4}\right)\right\} \tag{15}$$

It is clear from the above expressions that the cross-flow errors for a x-wire are greater than for a single wire by a factor of $\frac{2}{1+k^2}$. For $\frac{w'}{U}$ = 50%, the mean velocity is overestimated by 12.5% for a normal wire and 20% for a x-wire.

The above procedure can be used to obtain the following expressions for the second moments.

$$\overline{u_m^2} = \overline{u^2}\left\{1 + \frac{1}{1+k^2}\left(\frac{\overline{2uw^2}}{u^2U} - \frac{\overline{2u^2w^2}}{u^2U^2} + \frac{\overline{w^4}}{u^2U^2} - \frac{\overline{w^2}^2}{u^2U^2}\right)\right\} \qquad (16)$$

$$\overline{v_m^2} = \overline{v^2}\left\{1 + \frac{1}{1+k^2}\left[\frac{\overline{vw^2}}{v^2U^2}V - \frac{\overline{uvw^2}}{v^2U^2}V + \frac{1}{1+k^2}\left(\frac{\overline{w^4}-\overline{w^2}^2}{U^2v^2}\right)\right]\right\} \qquad (17)$$

It is clear from these equations that the crossflow errors can significantly affect the measured moments when turbulence intensity is high (>20%). Note that because the higher order terms become increasingly important as the turbulence intensity is increased, these expressions can at best serve as indicators of when crossflow is a problem.

Additional Effects due to Rectification and Dropout

From the preceding it is clear that there are three primary sources of error in a x-wire signal at low velocities, rectification, cross-flow and lack of directional sensitivity at higher inclinations. The problem of rectification is obvious for a single wire in which the flow must reverse its direction for rectification to occur. Tutu and Chevray [8] have pointed out that rectification errors are more subtle and serious for x-wires than for single wires.

The reason for this is illustrated by Figure (5) which shows how a negative u-velocity fluctuation can cause reversal on one wire, even when its magnitude is substantially less than the mean. As a consequence, significant rectification can occur in flows of even modest intensities (~25 %). Since negative fluctuations in the streamwise direction are often highly correlated with fluctuations in the cross-stream direction in shear flows (as in Fig. 5b), rectification can be a real problem, especially since it will not be detected by many data processing schemes in current useage.

Figure (6) shows a joint pdf of the u and v components measured in a plume with 50% local turbulence intensity [10]. The straight lines correspond to the limits of resolution of the cross-wire as shown in Figure (3). Note the squeezing of the contours of constant probability in the region at the lower left where the resolution limits are exceeded.

An additional manifestation of the rectification phenomenon is the occurrence of voltage pairs which could not be resolved into velocity pairs from the angle calibration. In other words, the instantaneous voltage pairs obtained do not lie in the calibrated region of Figure (3) and can not be inverted by equation (4). As a consequence these data must be dropped from the statistics. For such data, the word "dropout" is probably a more accurate description than "rectification". Dropout is usually caused by a high intensity in the u or v component and is especially troublesome when the mean velocity is low. This is because wires are fairly insensitive to direction at low velocities and any small measurement error (electronic noise, prong support interference, velocity component perpendicular to the x-wire plane, wake of one wire or, another, or a velocity or temperature gradient between the wires) can create a large error in the output. The dropout is small at the center of flows such as plumes but has been observed to be as big as 40% at the outer edges.

It is important to note that dropout can not be detected by common analog signal processing schemes (for example summing and differencing circuits). As a consequence, the processed data will be incorrect without the experimenter being aware of the problem. The effects will be most noticeable in the higher moments where the scrambled tails of the distribution are most heavily

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Key Words

Hot-wire anemometer, x-wire, angle calibration, turbulence.

Figure Captions

Figure 1. Geometry for inclined wire showing relation of effective velocity (assumed normal for Cosine Law) to flow velocity.

Figure 2. Angular Response of a Hote Wire.

Figure 3. Velocities which can be resolved by an x-wire.

Figure 4. Sketch showing how flow reversal can occur on an inclined wire at only modest levels of turbulence.

Figure 5. 2-D Probability Contours of u and v velocity components obtained by x-wire at 50% turbulence intensity.

Figure 6. A Hot-Wire inclined at an angle α to the vertical direction.

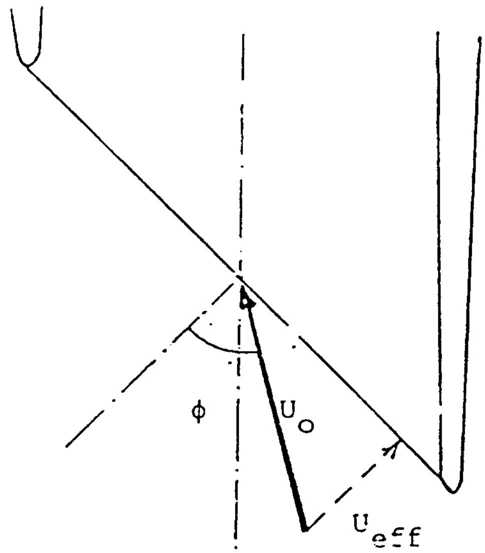
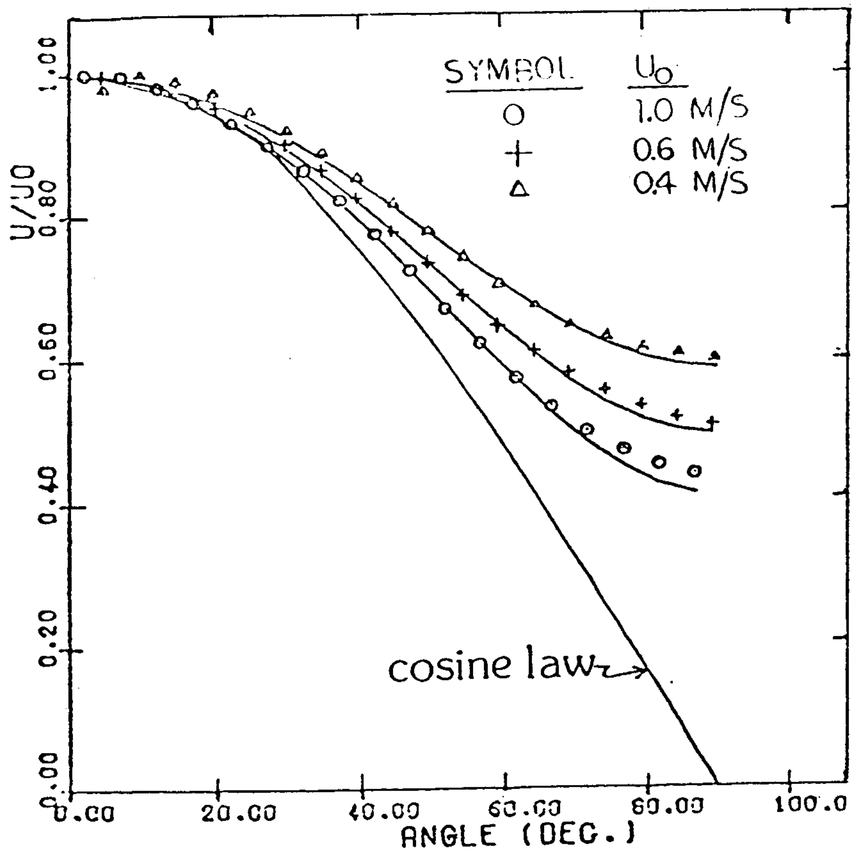
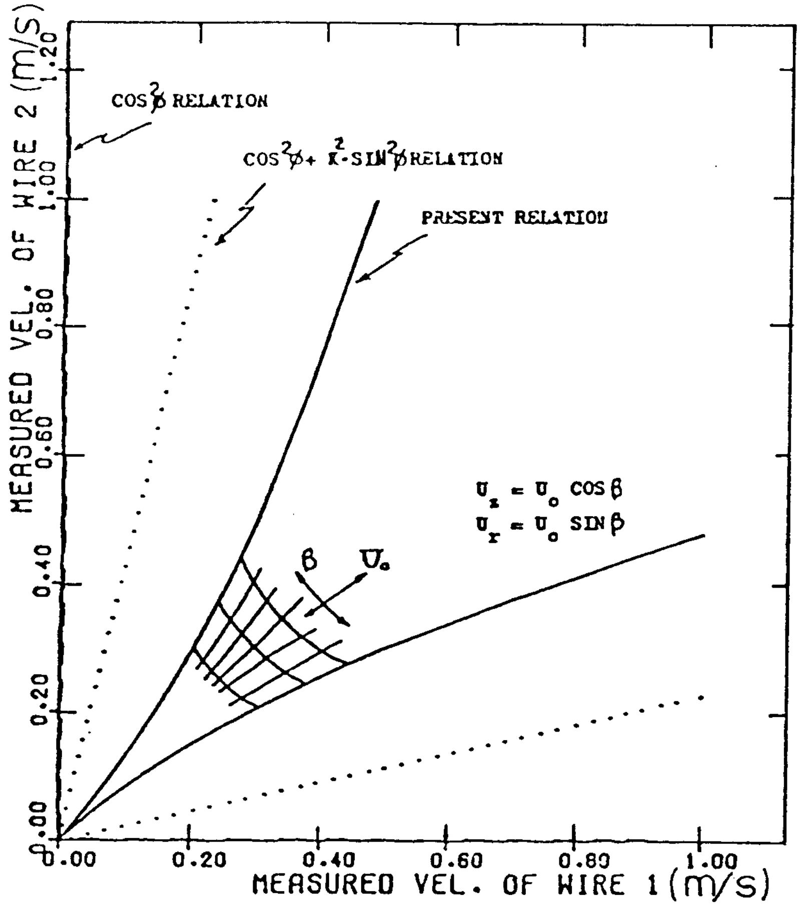


Figure 1.



SYMBOL
Uo
O
1.0 M/S
+
0.6 M/S
Δ
0.4 M/S
U/UO
1.00
0.80
0.60
0.40
0.20
0.00
0.00
20.00
40.00
60.00
80.00
100.0
ANGLE (DEG.)
cosine law

Figure 2.



COS²φ RELATION
COS²φ + K²·SIN²φ RELATION
PRESENT RELATION
$U_z = U_o \cos\beta$
$U_r = U_o \sin\beta$
β
U_o
0.00
0.20
0.40
0.60
0.80
1.00
1.20
MEASURED VEL. OF WIRE 2 (m/s)
MEASURED VEL. OF WIRE 1 (m/s)

Figure 3.

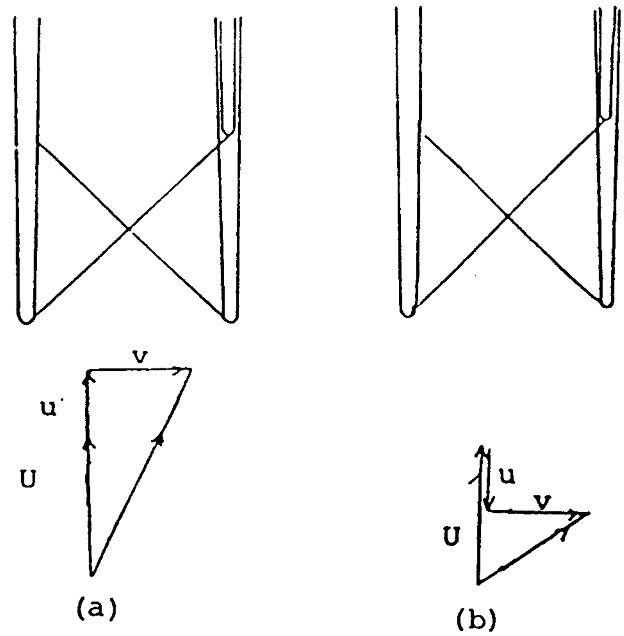


Figure 4.

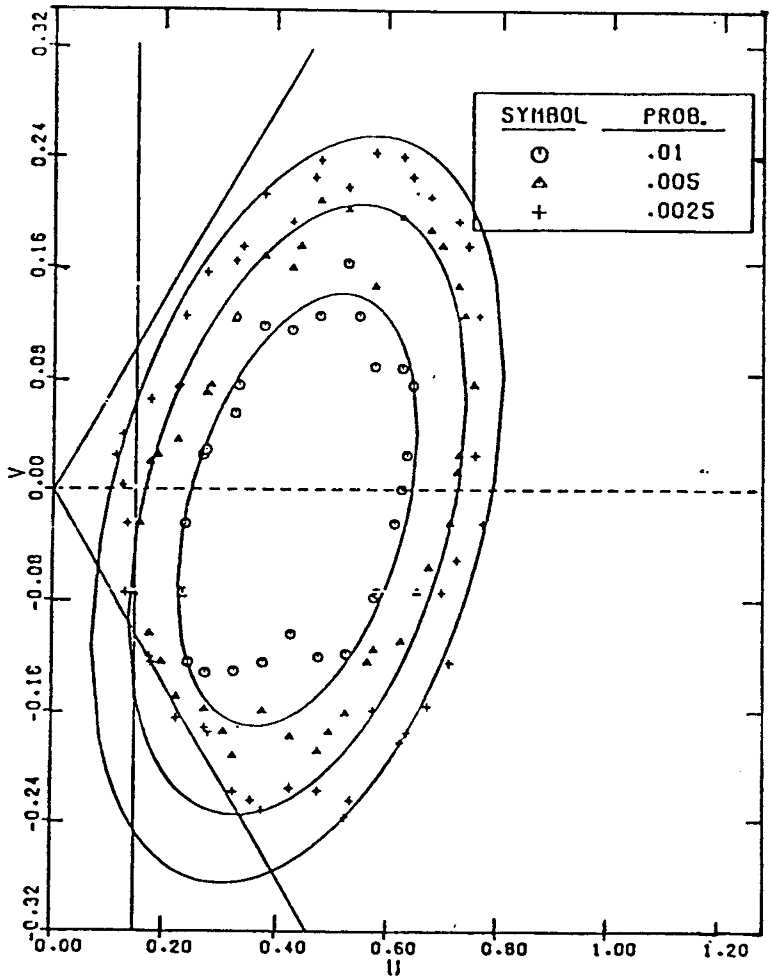

SYMBOL
PROB.
.01
.005
.0025
V
U
0.32
0.24
0.16
0.08
0.00
-0.08
-0.16
-0.24
-0.32
0.00
0.20
0.40
0.60
0.80
1.00
1.20

Figure 5.

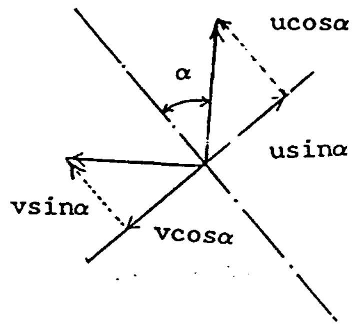


Figure 6.