MEASUREMENTS OF THE TURBULENT ENERGY AND TEMPERATURE BALANCES IN AN AXISYMMETRIC BUOYANT PLUME IN A STABLY STRATIFIED ENVIRONMENT *

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ABSTRACT

Measurements of temperature and velocity were taken in an axisymmetric turbulent hot air plume in a stably stratified environment using hot-wire anemometers. Profiles of the first four moments including Reynolds stress and turbulent heat fluxes are presented. The measurements are shown to satisfy the constraints imposed by the mean momentum and energy equations in differential and integral form. These measurements along with simultaneous measurements of the time derivatives were used to estimate the radial balances of the turbulent kinetic energy and enthalpy. The derivative measurements showed significant deviations from local isotropy, even after corrections for the fluctuating convection velocity effects; these could in part be attributed to the relatively small turbulent Reynolds number and lack of an extensive inertial subrange in the spectra.

NOMENCLATURE

$f(\eta)$ = similarity function for mean velocity

f_1 = centerline value of $f(\eta)$

F = local buoyancy strength

$g(\eta)$ = similarity function for mean temp. diff.

g_1 = centerline value of $g(\eta)$

g_0 = acceleration of gravity

$h(\eta)$ = similarity function for Reynolds stress

q^2 = turbulent kinetic energy

T,t = mean and fluctuating temperature

V,v = mean and fluctuating radial velocity

U,u = mean and fluctuating axial velocity

$w(\eta)$ = similarity function for radial turbulent heat flux

β = coefficient of thermal expansion

ε = dissipation of turbulent kinetic energy

ε_t = dissipation of temperature fluctuation

η = r/z

INTRODUCTION

Turbulent buoyancy induced flows have been measured and studied for a number of years. Some of the early work on plumes was started in 1941 by Schmidt (1). The first detailed study was undertaken in 1952 by Rouse, Yih, and Humphreys (2), and basic texts still use this work as a primary data source for these flows. A comprehensive review was written by Turner (3) in 1973. Since then, additional papers have been published, but there have been few attempts to expand the amount of quantitative information.

George et al. (4) used two-wire probes (one operated in the constant temperature mode and the other operated as a resistance thermometer) to measure mean and RMS fluctuating quantities, velocity-temperature correlations and the joint probability distributions. All data satisfactorily collapsed using the conventional similarity scaling. The shape of the profile of mean temperature displayed good agreement with the results of Rouse et al. but was 20% lower. The mean velocity had a lower centerline value and a wider profile when compared to the data of Rouse et al. The authors also observed, in contrast to the data of Rouse et al., that the mean velocity profile was wider than the temperature profile.+

In an independent effort, Nakagone and Hirata (5) also obtained velocity and temperature data using a two-wire probe. Unfortunately, the mean data is not presented in a manner conducive for quantitative comparison with other experiments. In agreement with the results of George et al., Nakagone and Hirata found the velocity profile to be wider than the temperature profile. Both of the experiments measured RMS temperature and velocity intensities of roughly 37% and 26% respectively

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+Comparison of profiles normalized to centerline values.

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when scaled to centerline mean values.

In 1979, Beuther, Capp, and George (6) presented data obtained from a three-wire probe (two velocity wires and one temperature wire) that were in general agreement with the two previous investigations. However, for the first time, the Reynolds stress and radial heat flux were measured in a buoyancy dominated shear flow. The profiles of $\overline{uv}$ and $\overline{vt}$ agreed well with the radial derivatives of the mean axial velocity and temperature, respectively, lending support to recent attempts at eddy viscosity closure models for such flows.

The complement to this experimental work is one of the main thrusts of turbulence research--to determine a method to close the averaged equations of motion. Using an eddy viscosity model and restricting the turbulent Prandtl number to 1.1 and 2.0, Yih (7) obtained solutions in closed form for the axisymmetric plume. George and Hamilton (8), and Baker, Taulbee and George (9) used numerical techniques in conjunction with an eddy viscosity model to calculate profiles for a wide range of turbulent Prandtl numbers. A turbulent Prandtl number of near unity best predicted the plume profiles. Others have applied two-equation turbulent closure models to buoyant flows (e.g., Tamanini (10), Plumb and Kennedy (11)). Most promising in the long run is the approach of Lumley and his collegues (v. Lumley (12)) who have closed the averaged equations at the fourth order. To complete such closure models, we must rely heavily upon experimental data for the third and fourth order turbulence moments.

EXPERIMENT

The plume was generated by electrically heating air to 300°C and discharging it through a 6.38 cm vertical nozzle into a 6.5 m plume facility which was designed to both protect the plume from cross drafts and minimize interference with the entrainment field. The exit velocity was 0.55 m/s, which resulted in a densimetric Froude number of unity at the plume source. A rapid transition to a fully developed plume was observed. The stratification of the facility was achieved by recirculating the exhausted air outside the enclosure. The experiment was controlled by an on-line minicomputer system which controlled the probe traversing and data acquisition systems, and processed the measurements. The hot-wire probe consisted of two 2.5 μm diameter velocity sensors in an x-array, and a 1 μm temperature sensor. All were spaced within a 1.25 mm radius sphere in order to resolve to within the Kolmogorov microscale. (The centerline value of the Kolmogorov microscale ranges from 0.5mm near the bottom most measurement location to 1.3mm at the top. To resolve the smallest scales, the wires need to be placed within a distance of π times the Kolmogorov microscale of each other. This criterion is met at all but the bottom measurement locations.) The three wires were sampled directly and used to compute the instantaneous Nusselt number for each velocity wire. The Re-Nu calibration relationship (a polynomial) was then "undone" by the computer to obtain the actual velocities, thus eliminating the temperature dependence of the velocity sensors.

EXPERIMENTAL ERRORS

One of the primary sources of error in the experiment is in the velocity resolution at the outer, high intensity regions of the flow. Since the turbulence intensity can exceed 60% in these regions of the plume, large azimuthal components of velocity and even flow reversal are a near certainty. The x-wire itself can only resolve velocities with a solid angle of 90 degrees with respect to the probe axis and the calibration functions used to relate the output voltage of the anemometers to a real value of velocity are accurate within a solid angle of 50 to 70 degrees. Any velocity vector outside this region will be mapped into an unknown location in the measured velocity field or into a nonexistant (imaginary) region of the calibrated velocity field. Generally, these points must be ignored but are not unimportant and account for a significant portion of the velocity statistics (as many as 50% of the points at the outer edge of the plume can fall into this category). Thus, the measurements past r/z = .15 need to be verified by other means before confidence can be placed in them. The problem is analyzed in more detail in ref. 14.

GOVERNING EQUATIONS

The equations of mean motion in cylindrical coordinates for a statistically stationary free convection turbulent flow in a uniform environment can be reduced to similarity equations for which the following solutions apply (see Rouse, et al.(2)):

$$U = f(\eta)\, F^{1/3}\, z^{-1/3} \qquad 1$$

$$V = k(\eta)\, F^{1/3}\, z^{-1/3} \qquad 2$$

$$g_o \beta \Delta T = g(\eta)\, F^{2/3}\, z^{-5/3} \qquad 3$$

$$\overline{uv} = h(\eta)\, F^{2/3}\, z^{-2/3} \qquad 4$$

$$g_o \beta \overline{vt} = w(\eta)\, F\, z^{-2} \qquad 5$$

$$\text{where } F \equiv 2\pi \int_o^\infty g_o \beta (U\Delta T + \overline{ut}) r dr$$

George and Beuther (13) have recently showed that for a stably stratified environment with a power law dependence of the ambient temperature gradient, these similarity relations can again be used if F is now the local buoyancy strength. In our case T_o has a z^{-3} power law dependence and the buoyancy strength can be shown to exhibit a $z^{-1/2}$ dependence (ref. 13). Therefore, measured profiles of all turbulence moments will be presented as appropriately normalized similarity forms.

EXPERIMENTAL RESULTS AND DISCUSSION

In the past, most investigators of turbulent plumes used a Gaussian profile to fit the mean temperature and velocity data. However, there is no physical argument to justify this profile. A more appropriate choice is to use as empirical fits the forms obtained by Yih (7) with an eddy viscosity solution for turbulent Prandtl number equal to 1.1. These forms are not expected to fit as well at the outer region of the plume due to the use of a constant eddy viscosity across the flow. However, in the central core region, they should work well. These expressions are

$$f(\eta) = \frac{f_1}{(1 + A\eta^2)^2} \qquad 6$$

and

$$g(\eta) + \frac{g_1}{(1 + A\eta^2)^3} \qquad 7$$

where in our case f_1, g_1, and A are determined from the data by using a method of least squares.

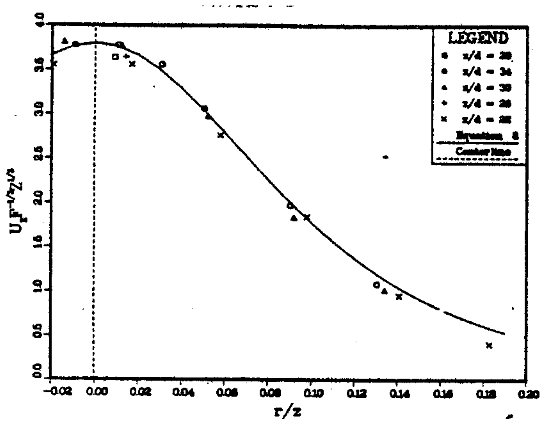

Fig. 1 Mean Velocity

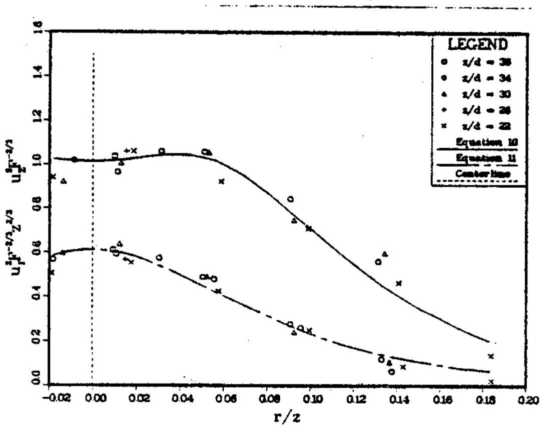

Fig. 2 Mean Square Velocity Fluctuations

For a uniform environment, for which these forms were derived, the constant A is the same for all profiles. However, for a stably stratified environment there is a definite trend which narrows the width of the velocity profile. Figures 1 and 2 present the mean profiles of velocity and temperature for a moderately stratified plume in similarity variables. The data was taken at heights of 22-38 diameters above the plume source. The values of f_1, g_1, and the plume width are summarized in Table I, along with previous data from more uniform environments (Beuther (14)).

The velocity profile is much narrower than that measured previously under less stratified conditions (A=46 vs. 33, Beuther, Capp, and George (6)), and peaks at a higher level (f_1=3.8 vs. 3.6). The temperature profile peaks higher (g_1 = 10.4 vs. 9.5) but retains approximately the same shape. The value of 33 for the width of the temperature profile from the second set of data was chosen as a compromise to agree with the width of the velocity profile. If these two coefficients had been chosen separately, they would have been closer to 30 and 35 for the temperature and velocity, respectively. This fits a trend of a narrowing velocity profile with increased stratification. In fact, for the present set of data, the velocity and temperature profiles have nearly the same shape.

TABLE I

Buoyancy Strength	f_1	A	g_1	A
Uniform (ref. 4)	3.4	28	9.1	28
Not Measured (ref. 6)	3.6	33	9.5	33
$F = 0.011\ z^{-1/2}\ (m^4/s^3)$	3.8	46	10.4	31

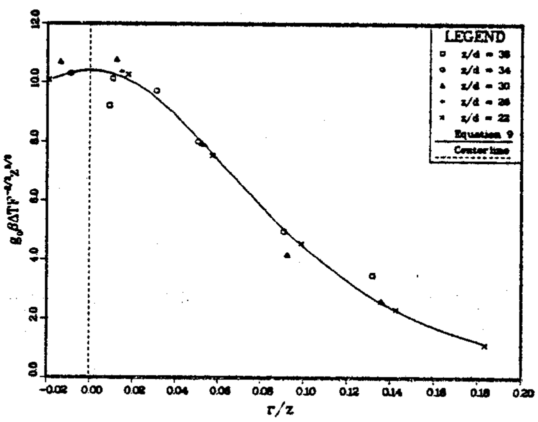

Fig. 3 Mean Temperature Difference

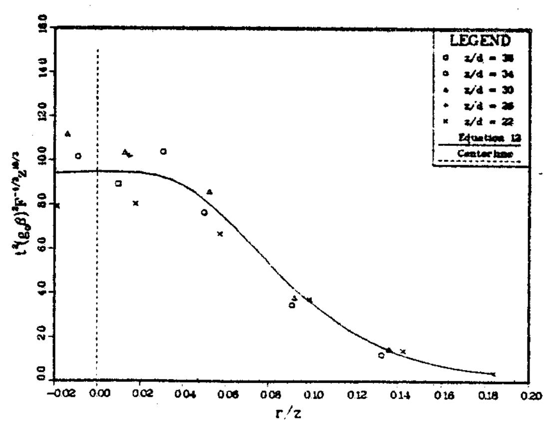

Fig. 4 Mean Square Temperature Fluctuations

Figures 3 and 4 show the velocity and temperature fluctuation data (again normalized to similarity variables). In agreement with references 4, 5, and 6, the centerline vertical velocity fluctuations are 26% of the mean centerline value and showed little change under differing amounts of stratification. The radial velocity fluctuations also remained the same as before at 16% of the mean centerline vertical velocity. The temperature fluctuations were only 30% of the centerline temperature difference, a value considerably lower than that of George et al. and Nakagome and Hirata who obtained values of 38% and 35% respectively. The earlier work in this investigation obtained a value of 30-33% for the teperature fluctuation intensity. (This last result is not found in reference of Beuther, Capp, and George, which presents temperature fluctuations that are in error and too high.) The off-axis peak in the u^2 profile was not observed by other investigators, but this discrepancy is perhaps due to their single wire probes, which mix the u and v fluctuations together. The peak was noted in all profiles measured as part of this investigation and is also predicted by the computational model of Tamanini (10).

The Reynolds stress, $\overline{uv}$, is presented in Figure 5. The shape of this profile follows very closely the derivative of the mean velocity profile in the core region of the plume, explaining why an eddy viscosity model seems to work so well in this flow (v. Baker et al. (9)). The uv correlation has a maximum value of 0.5 near r/z = 0.1.

The vertical and radial turbulent heat flux are presented in Figure 6. The vertical component has a slight off-axis peak at r/z = 0.05 (also not observed by other investigations) and remains relatively constant out to r/z = 0.1. Because the mean vertical heat flux (UΔT) drops off much faster, the turbulent vertical heat flux can be quite significant in the outer regions of the plume. George, et al. estimated that the overall contribution of ut can be as high as 15%, although the data presented here indicates a contribution closer to 10%. This lower value is also associated with a lower correlation coefficient of .59 - .60 vs. .67 for George et al. No explanation for this difference is known. Nakagome and Hirata mesured a very low value of 0.45 for the correlation. This unusually low value could be due to a velocity contamination of the temperature sensor, caused by too high of a current through the wire. The velocity dependence of the temperature wire used in this investigation was practically unmeasurable due to the low current through the wire (150 μa).

Shown with the radial turbulent heat flux, $\overline{vt}$, is a curve proportioned to the radial gradient of the mean temperature.

TABLE II

FITTED CURVES FOR 1ST, 2ND, AND 3RD MOMENTS

U	$3.8/(1+46\eta^2)^2$	8
$g_o\beta\Delta T$	$10.4/(1+31\eta^2)^3$	9
$\overline{u^2}$	$(1+230\eta^2)/(1+46\eta^2)^4$	10
$\overline{v^2}(=\overline{w^2})$	$0.61/(1+59\eta^2)^2$	11
$(g_o\beta)^2\overline{t^2}$	$(0.95+2550\eta^2)/(1+45\eta^2)^6$	12
$\overline{uv}$	$7.45\eta/(1+46\eta^2)^3$	13
$g_o\beta\overline{ut}$	$(1.77+650\eta^2)/(1+46\eta^2)^6$	14
$g_o\beta\overline{vt}$	$17.25\eta/(1+46\eta^2)^3$	15
u^3/u'^3	$0.23+13.5\eta^2+2575\eta^4-35000\eta^6$	16
v^3/v'^3	$3.8\eta+182000\eta^7$	17
t^3/t'^3	$0.58+49.8\eta^2$	18
$ut^2/u't'^2$	$0.3+35\eta^2-5000\eta^6$	19
$vt^2/v't'^2$	$60\eta^2-200\eta^3$	20
$uv^2/u'v'^2$	$-0.5+65\eta^2-7000\eta^6$	21
$u^2v/v'u'^2$	$(-3.1\eta+1540\eta^3)$ EXP (-15η)	22

The analytical expressions for the various moments are given in Table II. By using the fitted curves in Figures 1 through 6, it is possible to plot all the significant terms of the mean momentum and temperature balance equations. This was done by Beuther (14) and the results will be summarized here. Since the plume is not in a uniform

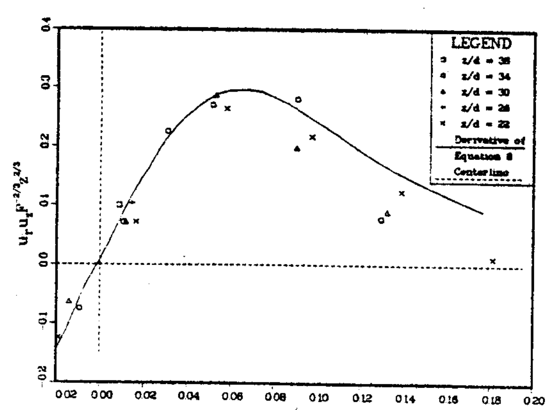

Fig. 5 Reynolds Stress

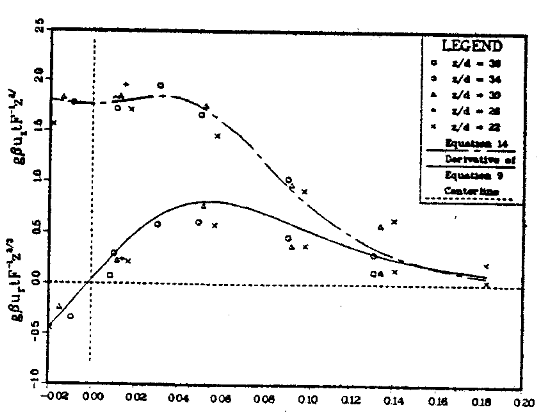

Fig. 6 Turbulent Heat Flux

environment, the calculation of the z derivatives involves the z-dependence of the buoyancy flux. Unfortunately, all the terms do not balance perfectly, especially in the temperature balance. This is partly because of the choice of curve fitting, since although the deviation is small enough that the fitted curves agree well with the data, when combined to form the balances, these slight errors are more noticeable, especially in the outer regions. The ambient temperature gradient term of the temperature balance is an example of this problem. The term d/dz(To) is due entirely to the stratification of the ambient air. In a uniform environment it is absent. Since no attempt was made to choose profiles which minimize this error, the 5-10% error that is typical of these balances is not deemed extremely significant. The measurements do satisfy the equations to within experimental error.

Table II also contains fitted polynomial curves for 7 third moment correlation coefficients. These 7 moments, along with the remaining fourth moments are plotted in Figures 7 through 9. All these collapsed quite well at various heights.

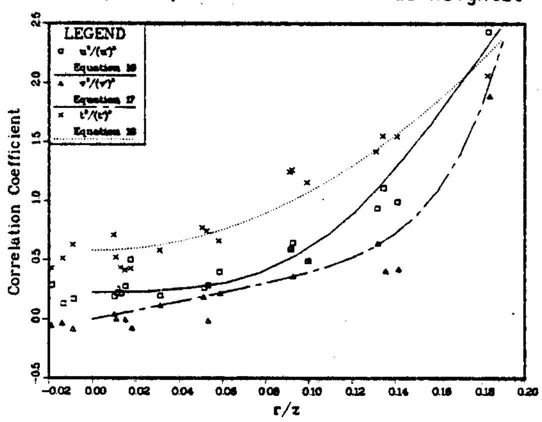

Fig. 7 Third Moment Autocorrelaion Functions

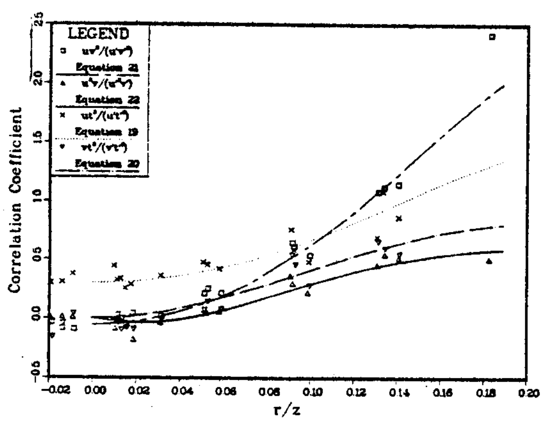

Fig. 8 Third Moment Cross-correlaion Functions

The fourth moments (Figure 9) follow the Gaussian shape quite well in the interior of the plume, but deviate substantially at large values of r/z. For the correlation coefficient of u^4 and v^4, a Gaussian behavior would have a constant value of three. For the u^2v^2 coefficient, the value depends upon the shape of the Reynolds stress, but should be unity at the centerline. The data agree well with the Gaussian assumption near the centerline.

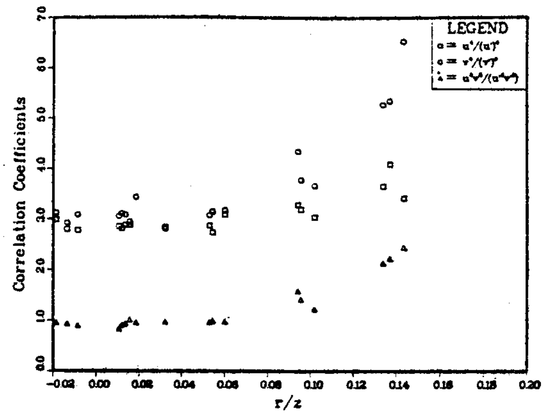

Fig. 9 Fourth Moment Correlation Functions

As was done for the balance of the mean quantities, the terms of the kinetic energy balance and temperature fluctuation balance can be obtained from the equations in Table II. The rate of change of q^2 is due pressure gradient work, transport by turbulent velocity fluctuations, deformation work, work due to buoyancy, and viscous dissipation.

The viscous dissipation term is the rate at which viscous stresses perform deformation work, and is always an energy drain. This term is also the most difficult to measure due to the number of components of spatial gradients. It is customary to assume isotropic relations between these components and thus measure only one or two terms, as shown in equation 23.

$$\varepsilon = 15\nu\overline{\left(\frac{\partial u_1}{\partial z}\right)^2} \qquad 23$$

However, for the range of Reynolds numbers (ul/ = 1000) in this investigation, it has been shown by Beuther (14) that the turbulence in this flow does not obey these isotropic relations. Since all nine components were not measured, it is not possible to directly compute the value of the dissipation. It can also be shown (from the measured ratio of $(du/dz)^2$ to $(dv/dz)^2$ in ref. (14)) that the dissipation as calculated from equation (23) will overestimate the actual value.

The equation for the temperature fluctuations can be described in a similar manner. The rate of change of $t^2/2$ is due to transport by turbulent velocity fluctuations, gradient production, and molecular dissipation. Again, as in the kinetic energy balance, the molecular diffusion term is usually computed with the isotropic relations using the measured value of $(dt/dz)^2$:

$$\varepsilon_t = 3\gamma\ \overline{\left(\frac{\partial t}{\partial z}\right)^2}. \qquad 24$$

Since only one component of the temperature dissipation was measured, no estimate of the degree of anisotropy can be ascertained. However, unlike the kinetic energy balance, all other terms in the temperature fluctuation balance have been measured. Thus, the error in this balance gives an indication of the anisotropy. By this method the temperature derivatives show less anisotropy than the energy dissipation, but the isotropic relation still overestimates the dissipation term (ref. (14)).

Because the actual values of dissipation cannot be measured, the energy balances are computed using the measured data to calculate the convection, turbulent diffusion, and production terms. The remainder of the balance will be defined to be the dissipation. Although the kinetic energy balance also has an unknown pressure transport term included in this remainder, it is believed to be small compared to the dissipation term over much of the flow. However, this hypothesis cannot be substantiated. It was originally hoped that the directly measureable terms (all but pressure) could be measured accurately enough to estimate the magnitude of the pressure transport from the energy balance, but this was impossible due to the low Reynolds number of the flow and the anisotropy at small scales.

The kinetic energy and temperature fluctuation balances can also be computed from the results of Table II. In both balances, the turbulent diffusion terms integrate to zero as they should, and at the outer region of the plume, production is balanced by dissipation and the turbulent diffusion by mean convection. However, this predicted dissipation of turbulent kinetic energy is much less than that obtained from equation 23. The pressure transport terms are not believed to be significant enough to account for much of the discrepancy in the kinetic energy dissipation. The differences in the measured and calculated temperature derivatives are less severe than that of the velocity derivatives. It is disturbing that these measured and calculated values differ so greatly, especially at the centerline (ref. 14 shows measured values more than a factor of two too large). At larger values of r/z it is expected that the higher moments are in serious error due to the drop-out, but their accuracy at the centerline should be to within 10%. Thus, the only explanation for the large values of dissipation is the anisotropy of the flow.

SUMMARY

Vertical and radial velocity components and the temperature were measured in a turbulent axisymmetric hot air plume in a stably stratified environment. Profiles of uv and vt agreed well with the radial derivatives of the mean axial velocity and temperature, respectively. Measurements of higher order moments were combined to form the balances of the turbulent kinetic energy and temperature fluctuations. The measurements lend support to earlier attempts (refs. (7,8,9,10)) to model such flows.

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REFERENCES

1. Schmidt, W., "Turbulent Propagation of a Stream of Air" ZAMM, V. 21, 6.

2. Rouse, H., C.S. Yih and H.W. Humphreys, "Gravitational Convection from a Boundary Source", Tellus, V. 4, 201-210, 1952.

3. Turner, J.S., Buoyancy Effects in Fluids, Cambridge University Press, Cambridge, England, 1973.

4. George, W.K., Jr., R.L. Alpert and F. Tamanini, "Turbulence Measurements in an Axisymmetric Buoyant Plume", Int. J. Heat Mass Trans., V. 20, 1145-1154, 1977.

5. Nakagome H. and M. Hirata, "The Structure of Turbulent Diffusion in an Axi-symmetrical Thermal Plume", Heat Transfer and Turbulent Buoyant Convection, (Spalding and Afgan, eds.) McGraw-Hill, Ny., 367-372, 1977.

6. Beuther, P. D., Capp, S. P., Goerge, W. K., Jr., "Momentum, and Temperature Balance Meeasurements in an Axisymmetric Turbulent Plume", ASME Publication 79-HT-42, 1979.

7. Yih, C. S., "Turbulent Buoyant Plumes", The Physics of Fluids, V. 20, 8, 1977.

8. Hamilton, C. F. and W. K. George, Jr., "Eddy Viscosity Calculations of Turbulent Buoyant Plumes", Bull. Am. Phys. Soc., Series II, 21, 10, 1225, 1976.

9. Baker, C. B., D. B. Taulbee, and W. K. George, Jr., "Eddy Viscosity Calculations of Turbulent Buoyant Plumes", presentation at ASME/AIChE National Heat Trans. Conf., San Diego, CA., August 1979.

10. Tamanini, F., "The Effect of Buoyancy on the Turbulence Structure of Vertical Round Jets", ASME J. of Heat Trans., V. 100, 4, 1979.

11. Plumb, O. A., and L. A. Kennedy, "Application of a K-E Turbulence Model to Natural Convection from a Vertical Isothermal Surface", J. of Heat Trans., Trans. ASME, Series C., V. 99, pp 79-85, 1977.

12. Lumley, J. L. Presentation - von Karman Institute 1975.

13. George, W. K., Jr. and Beuther, P. D., "A Similarity Analysis of Turbulent Buoyant Plumes In a Stratified Environment," to be presented at the 9th U.S. National Congress of Applied Mechanics, June 1982.

14. Beuther, P. D., "Experimental Investigation of the Axisymmetric Turbulent Buoyant Plume", Ph.D. Dissertation, SUNY/Buffalo, Buffalo, NY., 1980. (Also, report TRL-108, Turbulent Research Lab, Faculty of Engr. and Applied Sciences, SUNY/Buffalo.)